# STATISTICAL PROCESS CONTROL OF DEFECT PROPORTIONS IN AN ENERGY-DRINK BOTTLING PROCESS: A PRELIMINARY VARIABLE-SIZE P-CHART AND ATTRIBUTE CONFORMANCE STUDY

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***ABSTRACT****—This preliminary case study evaluated defect proportions in an energy-drink bottling process using a variable-subgroup-size p-chart and an attribute conformance assessment. Secondary quality records covered 13 consecutive production subgroups comprising 4,203,816 bottles and 336 defective units. The pooled defect proportion was 0.00007993, equivalent to 0.00799% or 79.93 defective units per million, with an exact 95% binomial confidence interval of 71.61-88.95 per million. Variable p-chart limits were calculated for each subgroup because production volumes differed. All subgroup proportions remained within their corresponding three-sigma limits, and the Pearson chi-square dispersion statistic (6.09, df = 12, p = 0.911; dispersion ratio = 0.508) showed no evidence of extra-binomial variation in the observed period. The pooled defect proportion used only 7.27% of the company-provided maximum threshold of 0.11%, giving a 13.76-fold numerical margin. These findings indicate preliminary statistical stability and conformance to the internal defect threshold. However, the short monitoring window, aggregated secondary data, and absence of continuous critical-to-quality measurements limit generalization. Conventional Cp and Cpk indices were therefore not computed because only binary defective/non-defective records were available. Continued Phase II monitoring, verified measurement systems, defect-type stratification, and collection of fill-volume and closure-torque data are recommended.*

**Keywords**— Attribute data, Beverage bottling, Defect proportion, P-chart, Process conformance, Statistical process control

## INTRODUCTION

High-volume beverage bottling requires consistent control of filling, capping, sealing, labeling, and packaging operations because small defect proportions can translate into substantial scrap, rework, customer complaints, and regulatory risk. Statistical Process Control (SPC) supports prevention-oriented quality management by distinguishing routine process variation from assignable variation and by providing timely evidence for corrective action. A systematic review of food-industry applications identified reduced process variation and improved regulatory conformance as major benefits of SPC, while limited statistical knowledge and resistance to change were persistent barriers [1].

Sustainable SPC deployment depends on more than chart construction. Food-sector roadmaps emphasize management support, measurement-system adequacy, employee involvement, organizational readiness, and a structured implementation cycle [2, 3]. Evidence from Science International also indicates that manufacturing firms commonly adopt detection-oriented methods such as SPC and acceptance sampling, whereas preventive methods and capability studies are less consistently used [4]. More broadly, quality assessment can reduce waste and rework and support operational sustainability when it is integrated into routine decision-making [5].

For binary outcomes classified as defective or non-defective, the p-chart is a standard Shewhart chart [6]. When subgroup sizes differ, each subgroup requires its own upper and lower control limits. This is especially important in high-quality processes because low nonconforming proportions can challenge the usual binomial-to-normal approximation and can produce misleading limits when sample sizes are inadequate [7], [8]. In the present data, subgroup sizes are large enough that the expected number of defects under the pooled rate exceeds five in every subgroup, supporting a preliminary normal-approximation p-chart; nevertheless, exact or improved limits remain appropriate sensitivity options for future monitoring.

Bottling studies have also shown the value of process sensors, process control, and multivariate monitoring for maintaining fill and production consistency [9-11]. These approaches complement attribute charts because continuous critical-to-quality variables, such as fill volume and closure torque, allow direct assessment of process centering and spread. Capability indices such as Cp and Cpk are designed for such continuous measurements and should not be inferred from aggregated binary defect counts [12].

The original company records provided only production volume and total defective units for 13 consecutive subgroups. Accordingly, the present study was redesigned to align the statistical method with the available data. It aimed to (1) estimate the pooled defect proportion and its uncertainty, (2) assess short-term statistical stability using a variable-size p-chart, (3) compare observed performance with the company-provided allowable defect threshold, and (4) identify limitations and data requirements for a stronger continuing quality-improvement program. Because the chart limits were estimated from the same short dataset, the analysis is explicitly treated as a preliminary Phase I assessment rather than definitive long-term validation.

## MATERIAL AND METHODS

### Research design and data source

A quantitative retrospective case-study design was used. The unit of analysis was the production subgroup or inspection period. Thirteen consecutive subgroups were selected from the bottling company's quality records. For each subgroup i, the available variables were the number inspected ($n_i$) and the number classified as defective ($d_i$). The source records did not contain bottle-level identifiers, defect-type counts, machine settings, operator or shift identifiers, fill-volume measurements, closure-torque measurements, or measurement-system studies. The company name was withheld to protect operational confidentiality.

The company specified a maximum allowable defect proportion of 0.0011 (0.11%, or 1,100 defective units per million). This value was used as an internal conformance benchmark. Its technical, contractual, or regulatory basis was not independently verified; therefore, it was not treated as a universal industry specification.

### Defect proportion and variable-size p-chart

The subgroup defect proportion and pooled center line were calculated as follows:

$$p_i = d_i / n_i$$

$$p\text{-bar} = \text{sum}(d_i) / \text{sum}(n_i)$$

Because subgroup sizes varied, the three-sigma limits were calculated separately for each subgroup:

$$UCL_i = p\text{-bar} + 3\ \text{sqrt}[p\text{-bar}(1 - p\text{-bar}) / n_i]$$

$$LCL_i = \max\{0, p\text{-bar} - 3\ \text{sqrt}[p\text{-bar}(1 - p\text{-bar}) / n_i]\}$$

A subgroup was flagged when its observed proportion fell outside the corresponding control limits. The sequence was also inspected for simple non-random patterns, including sustained runs or monotonic trends. The analysis followed Shewhart principles [6] and considered the low-defect context discussed in improved attribute-chart research [7], [8].

### Uncertainty, dispersion, and conformance assessment

An exact two-sided 95% binomial confidence interval was computed for the pooled defect proportion. To check whether subgroup-to-subgroup variability was greater than expected under a common binomial rate, the Pearson statistic was calculated using the pooled p estimate, with k - 1 degrees of freedom. The dispersion ratio was Pearson chi-square/(k - 1); a value above one would suggest extra-binomial variation, whereas a value below one indicates no overdispersion in the analyzed period. This diagnostic was interpreted cautiously because only 13 subgroups were available.

Attribute conformance was summarized by comparing the pooled defect proportion with the company threshold. Threshold utilization was defined as p-bar/p_limit, and the numerical performance margin as p_limit/p-bar. These ratios are descriptive and are not substitutes for Cp, Cpk, or other variable-data capability indices. The latter require a measured continuous characteristic, verified specification limits, a suitable distributional model, and a statistically stable process [12].

## RESULTS

### Overall defect performance

The 13 subgroups contained 4,203,816 inspected bottles and 336 defective bottles. The pooled defect proportion was 0.00007993, equivalent to 0.00799% or 79.93 defective units per million. The exact 95% confidence interval was 71.61-88.95 per million. The upper confidence limit remained far below the company threshold of 1100 per million.

**Table 1:Summary of overall defect performance and conformance**

| Indicator | Result |
|---|---|
| Subgroups | 13 |
| Bottles inspected | 4,203,816 |
| Defective bottles | 336 |
| Pooled defect proportion | 0.00007993 |
| Defective units per million | 79.93 |
| Exact 95% CI (per million) | 71.61-88.95 |
| Company threshold | 0.11% (1100/million) |
| Threshold utilization | 7.27% |
| Numerical margin | 13.76 times below threshold |

### P-chart stability

All 13 subgroup proportions were within their subgroup-specific control limits. The highest observed proportion occurred in subgroup 8 (98.80 per million), followed by subgroup 13 (95.54 per million); neither point approached its respective upper control limit sufficiently to constitute an out-of-control signal. The lower control limit was positive for 12 subgroups and was truncated to zero only for subgroup 7, which had the smallest production size. This corrects the earlier all-zero lower-limit presentation and is a direct consequence of using variable subgroup sizes.

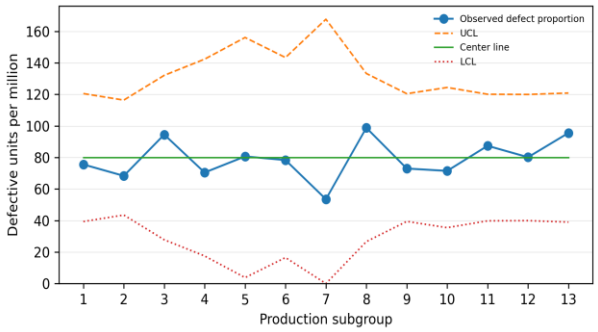


***Figure 1.*** **Variable-subgroup-size p-chart for the 13 production periods. Limits are expressed as defective units per million**.

The Pearson chi-square dispersion statistic was 6.09 with 12 degrees of freedom (p = 0.911); the dispersion ratio was 0.508. Thus, the data showed no evidence of extra-binomial variation over the observed period. No sustained run or monotonic trend was apparent, although the small number of subgroups limits the power of pattern rules. The chart should therefore be interpreted as preliminary evidence of short-term stability, not proof that the process will remain stable under future operating conditions.

**Table 2.: Production subgroups and recalculated variable-size p-chart limits**

| No. | n | d | p (ppm) | LCL-UCL (ppm) |
|---|---|---|---|---|
| 1 | 436,800 | 33 | 75.55 | 39.35-120.51 |
| 2 | 541,632 | 37 | 68.31 | 43.49-116.37 |
| 3 | 264,576 | 25 | 94.49 | 27.79-132.07 |
| 4 | 184,560 | 13 | 70.44 | 17.50-142.36 |
| 5 | 124,008 | 10 | 80.64 | 3.77-156.09 |
| 6 | 178,728 | 14 | 78.33 | 16.49-143.37 |
| 7 | 93,456 | 5 | 53.50 | 0.00-167.66 |
| 8 | 253,032 | 25 | 98.80 | 26.61-133.24 |
| 9 | 438,552 | 32 | 72.97 | 39.43-120.43 |
| 10 | 363,816 | 26 | 71.46 | 35.46-124.39 |
| 11 | 446,280 | 39 | 87.39 | 39.78-120.07 |
| 12 | 449,232 | 36 | 80.14 | 39.91-119.94 |
| 13 | 429,144 | 41 | 95.54 | 38.99-120.87 |

### Attribute conformance

The pooled defect proportion represented 7.27% of the company-provided maximum. Equivalently, the threshold was 13.76 times the observed pooled rate. Every subgroup also remained below the 1,100-per-million threshold. On the available attribute data, the process therefore conformed to the internal defect-rate requirement during the study period. This conclusion is intentionally stated as attribute

conformance rather than process capability because no continuous critical-to-quality variable was measured.

## DISCUSSION

The results show that a variable-size p-chart can convert aggregated production records into a clear statistical monitoring baseline. The pooled rate of approximately 80 defective units per million was low relative to the internal maximum, and the confidence interval indicates that sampling uncertainty does not materially alter that comparison. The p-chart further suggests that the observed fluctuations were compatible with common-cause variation during the 13 periods. These findings are consistent with the broader food-industry literature describing SPC as a means of reducing variation and moving quality assurance from end-product sorting toward process-based prevention [1, 2].

The corrected lower control limits are methodologically important. A p-chart with changing n does not generally have one fixed upper limit and one fixed lower limit. Larger subgroups produce narrower limits, while smaller subgroups produce wider limits. Setting every lower limit to zero would understate the chart's sensitivity to unusually low defect proportions and would obscure the effect of subgroup size. At the same time, low defect proportions warrant caution because normal-approximation charts may perform poorly when expected counts are very small [7, 8]. In this dataset, the expected number of defects under the pooled rate ranged from 7.47 to 43.29, supporting the preliminary approximation. Future automated monitoring may nonetheless use exact binomial or improved p-chart limits as a robustness check.

The absence of an out-of-control point does not identify why defects occurred, nor does it establish that all process stages were optimal. Aggregated defect counts cannot distinguish underfilling, leakage, cap misalignment, sealing failure, or packaging damage. They also cannot test whether machines, shifts, operators, suppliers, or environmental conditions influence the result. Bottling-process research illustrates the value of direct sensor measurements and process control [9], while soft-drink and beverage studies show how multivariate monitoring can capture quality information throughout production [10, 11]. A stronger improvement program should therefore connect the attribute chart with defect-type Pareto analysis, maintenance and shift records, and continuous measures of fill volume and cap torque.

\It is also necessary to separate statistical control from specification conformance. Statistical control concerns the predictability of variation; conformance concerns whether performance satisfies a stated requirement. A process can be stable but economically or technically unacceptable, or capable relative to a specification but unstable over time. The present data support preliminary stability and threshold conformance, but they do not support Cp or Cpk. Those indices require continuous measurements and meaningful upper and lower specification limits [12]. Removing unsupported Cp/Cpk statements strengthens the alignment among the research question, data type, formulas, and conclusions.

### Practical implications

For routine use, the company should preserve the variable subgroup size in the chart calculation rather than applying a single limit to all periods. Each data point should be accompanied by production date, shift, line, machine, product variant, defect category, and relevant maintenance events. Signals should trigger a documented reaction plan: verify data integrity, contain potentially affected products, inspect the process, identify the assignable cause, implement corrective action, and confirm recovery. Management support and measurement-system readiness are essential for sustaining this cycle [2, 3].

Continuous critical-to-quality data should be added after gauge repeatability and reproducibility or another appropriate measurement-system assessment. Fill volume, cap application torque, seam or seal integrity, and line speed can then be monitored using suitable variable or multivariate control charts. Cp and Cpk may be estimated only after the process is stable and the measurement and distribution assumptions are justified. Such staged development moves the quality system from simple defect counting toward preventive control, which is the direction encouraged by SQC research in manufacturing [4].

### Limitations

Five limitations constrain interpretation. First, only 13 consecutive subgroups were available, which is a short baseline for Phase I limit estimation and pattern detection. Second, the analysis used aggregated secondary records; bottle-level observations and the accuracy of defect classification were not independently audited. Third, no defect-type, machine, shift, operator, supplier, maintenance, or environmental variables were available for root-cause analysis. Fourth, no continuous measurements were provided, preventing valid Cp and Cpk estimation. Fifth, the company threshold was accepted as an internal benchmark without independent verification of its regulatory or contractual origin. These limitations mean that the findings describe the analyzed period and should not be generalized automatically to other lines, products, or time periods.

## CONCLUSION

The revised analysis found 336 defective bottles among 4,203,816 inspected units, producing a pooled defect proportion of 0.00799% (79.93 per million). All 13 observations were within correctly calculated variable-size p-chart limits, and the dispersion diagnostic showed no evidence of extra-binomial variation. The exact confidence interval was also far below the company-provided 0.11% threshold. The energy-drink bottling process therefore showed preliminary short-term statistical stability and attribute conformance during the observed periods.

The evidence does not justify conventional process-capability indices because the records were binary and aggregated. The most defensible conclusion is that the observed defect proportion conformed to the internal threshold, not that a continuous process characteristic achieved a particular Cp or Cpk. Continued Phase II p-chart monitoring, exact-limit sensitivity checks, defect-type stratification, a formal reaction plan, measurement-system verification, and collection of continuous fill-volume and closure-torque data are recommended. These steps will provide a stronger basis for root-cause analysis, capability assessment, and sustained quality improvement.

### Conflict of interest

The author declares no conflict of interest.

**Data availability**

The analyzed data were derived from confidential company production records. The aggregated subgroup values used in the analysis are reported in Table 2.

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