

COMPARING SYMMETRIC AND ASYMMETRIC GARCH MODELS FOR VOLATILITY FORECASTING: EVIDENCE FROM SABIC AGRI-NUTRIENTS COMPANY

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ABSTRACT: This study examines the forecasting of stock returns and volatility dynamics of SABIC Agri-Nutrients Company [25] listed on the Saudi Stock Exchange (Tadawul). Daily closing price data spanning from January 4, 2021, to December 31, 2025, comprising 1,248 observations, were analyzed using the Box–Jenkins methodology and ARCH/GARCH family models in EViews. Stationarity of the series was assessed through unit-root testing, and stock returns were computed using logarithmic price differences. For mean equation modeling, several ARIMA specifications were estimated and compared using information criteria and diagnostic statistics. The ARIMA (2, 1, 2) model emerged as the most appropriate specification for forecasting stock returns. Seasonal diagnostics indicated the absence of significant seasonal patterns; therefore, SARIMA modeling was deemed unnecessary. To capture conditional heteroskedasticity and volatility clustering commonly observed in financial time series, GARCH (1, 1), EGARCH (1, 1), and TGARCH (1, 1) models were fitted and evaluated. Among the competing volatility models, EGARCH (1, 1) demonstrated superior performance, yielding the lowest Akaike Information Criterion (AIC = -5.462182) and the highest log-likelihood value. The estimated asymmetry coefficient was positive and marginally significant ($p = 0.0945$), indicating weak evidence of asymmetric volatility responses to market shocks. The volatility findings were further supported through the News Impact Curve derived from the TGARCH (1, 1) specification, which illustrated differential responses of volatility to positive and negative innovations. The results provide valuable insights for investors, portfolio managers, and risk-management practitioners seeking to understand return behavior and volatility persistence in the Saudi equity market.

Keywords: Stock Return Forecasting; ARIMA; EGARCH; TGARCH; News Impact Curve; Conditional Heteroskedasticity; Saudi Stock Market; SABIC Agri-Nutrients;

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1. INTRODUCTION

Financial markets are dynamic, with prices driven by macroeconomic conditions, investor sentiment, and company-specific news. Accurate return forecasting is essential for investors and policymakers. The Saudi Tadawul All Share Index (TASI) is among the largest in the MENA region, offering rich opportunities for time series modelling. SABIC AgriNutrients Company (formerly SAFCO) is a leading nitrogen-based fertilizer and urea producer. Its stock is influenced by global commodity cycles and petrochemical feedstock prices, making return modelling practically significant for investors, especially under Saudi Vision 2030.

The Box-Jenkins methodology provides a systematic framework for ARIMA modelling. When volatility clustering is present, the ARCH and GARCH family models model the conditional variance. The combination of ARIMA and GARCH-type models is a standard approach in empirical finance.

The study objectives are: (i) model and forecast mean returns using ARIMA/SARIMA, and (ii) analyze volatility clustering using ARCH/GARCH-type models. Rest of the paper is formatted as: Section 2 provides literature review and empirical evidence, Section 3 presents methodology and data, Section 4 results, and Section 5 presents Discussion and Conclusions.

2. Literature Review and Empirical Evidence

Financial time series forecasting has attracted considerable attention because accurate prediction of stock returns and volatility assists investors, portfolio managers, and policymakers in making informed decisions. The Box–Jenkins methodology developed by Box & Jenkins [1]

remains one of the most widely used approaches for modeling linear dependencies in financial time series through ARIMA models.

Volatility modeling gained prominence following the introduction of the ARCH model by Engle [2], which demonstrated that financial return variances are time-varying rather than constant. Bollerslev [3] subsequently extended this framework through the GARCH model, allowing volatility to depend on both past shocks and previous conditional variances. Since then, GARCH-type models have become standard tools in empirical finance for measuring risk and forecasting volatility. For further reading on error distributions used in non-normal stock prices see Almarashi & Khan [4].

Several studies have applied ARIMA and GARCH models jointly to stock market data. Poon & Granger [5] reviewed volatility forecasting methods and concluded that GARCH-family models generally outperform constant-variance approaches. Hansen & Lunde [6] further demonstrated that GARCH (1, 1) provides robust volatility forecasts across a wide range of financial assets. Combining ARIMA for the conditional mean and GARCH for the conditional variance provides a comprehensive framework for modeling financial returns.

Researchers have increasingly recognized that volatility responses are often asymmetric. Nelson [7] introduced the Exponential GARCH (EGARCH) model, which allows positive and negative shocks to affect volatility differently. Glosten, Jagannathan, & Runkle [8] proposed the Threshold GARCH (TGARCH/GJR-GARCH) model to explicitly capture leverage effects. These asymmetric models have been widely applied in stock market research because bad news

often produces larger volatility responses than good news of equal magnitude.

Empirical evidence from emerging and developed markets supports the superiority of asymmetric volatility models. Karmakar [9] reported that EGARCH and TGARCH models outperform symmetric GARCH specifications in capturing volatility dynamics in the Indian stock market. Chkili, Aloui, & Nguyen [10] found significant asymmetric volatility effects across major international stock indices, suggesting that investors react differently to positive and negative market information. Within Middle Eastern markets, volatility modeling has received growing attention. Alberg, Shalit, and Yosef [11] examined stock market volatility in Israel and reported that EGARCH models provided superior forecasting performance. Al Freedi, Shamiri, and Isa [12] applied GARCH-family models to Gulf stock markets and found substantial evidence of volatility clustering and persistence. Likewise, Al-Yahyaee, Mensi, & Yoon [13] documented strong volatility persistence in GCC equity markets and emphasized the importance of asymmetric volatility models for risk management.

Studies focusing on the Saudi stock market have reported similar findings. Hammoudeh & Li [14] observed significant volatility persistence and asymmetric responses in Gulf Cooperation Council stock markets. Alqahtani & Taillard [15] found that Saudi stock returns exhibit volatility clustering and leverage effects that are effectively modeled using EGARCH specifications. More recently, Mensi et al. [16] demonstrated that volatility forecasting accuracy in Saudi equity markets improves substantially when nonlinear GARCH-family models are employed. Similarly, Almustafa & Alshammari [17] reported that EGARCH-based models outperform conventional GARCH models for forecasting risk in selected Saudi-listed companies.

Recent developments continue to support the relevance of volatility modeling in emerging markets. Zhang, Wang, & Liu [18] showed that asymmetric volatility models remain competitive even when compared with machine-learning approaches. Likewise, Kumar & Maheswaran [19] concluded that EGARCH and TGARCH models provide superior risk forecasts for commodity-linked stocks because they effectively capture the impact of market shocks and uncertainty.

Overall, the literature suggests that ARIMA models remain effective for modeling conditional mean dynamics, whereas GARCH-family models are essential for capturing volatility clustering and persistence. Furthermore, asymmetric models such as EGARCH and TGARCH frequently outperform symmetric GARCH specifications, particularly for financial assets exposed to commodity price fluctuations and market uncertainty. These findings provide a strong theoretical and empirical foundation for applying ARIMA, GARCH, EGARCH, and TGARCH models to SABIC Agri-Nutrients stock returns.

Recent studies have further strengthened the evidence supporting the application of GARCH-family models in the Saudi stock market. Almarashi *et al.* [20] investigated stock price volatility using ARCH and GARCH models and reported significant volatility clustering and persistence,

confirming the suitability of conditional heteroskedasticity models for financial time series analysis. More recently, Almarashi *et al.* [21] examined the mean and volatility behavior of Saudi Cement Company stock returns using ARIMA and GARCH approaches. Their findings demonstrated that integrating mean and variance models improves forecasting performance and provides a comprehensive understanding of stock return dynamics and risk. These results highlight the effectiveness of ARIMA–GARCH frameworks in modeling Saudi equity markets and provide further justification for applying similar methodologies to SABIC Agri-Nutrients stock returns.

Although previous studies such as Almarashi *et al.* [20] and Almarashi *et al.* [21] have successfully applied ARCH/GARCH and ARIMA–GARCH frameworks to Saudi stocks, their analyses focused primarily on general stock volatility and the cement sector. Limited empirical evidence exists for the fertilizer and agri-nutrient industry, particularly SABIC Agri-Nutrients Company, whose performance is strongly influenced by global commodity prices, agricultural demand, and petrochemical market conditions. Furthermore, existing Saudi studies have devoted relatively little attention to comparing symmetric and asymmetric volatility models, such as GARCH, EGARCH, and TGARCH, using recent post-COVID market data. Therefore, this study extends the literature by investigating both return dynamics and volatility behavior of SABIC Agri-Nutrients stock and by evaluating the relative forecasting performance of alternative GARCH-family specifications.

3. Methodology

3.1 Box-Jenkins Methodology

The Box-Jenkins methodology, as suggested by Box & Jenkins [1] is a widely used approach for time series analysis with three stages: model identification, estimation, and diagnostic checking. The series must be stationary; if non-stationary, differencing is applied.

3.2 ARIMA Model

An ARIMA (p, d, q) model combines autoregressive (AR), differencing (d), and moving average (MA) components. Orders p and q are identified from PACF and ACF. Parameters are estimated via MLE. Model selection uses AIC developed by Akaike, H. [22] and BIC (lower = better). The ARIMA model is used to capture the conditional *mean* dynamics of stock returns. Since financial price series are typically non-stationary, log returns are used to ensure stationarity. The general ARIMA (p, d, q) representation is:

$$\phi(B)(1 - B)^d Y_t = \theta(B)\varepsilon_t$$

Where Y_t is the return series, B is the backshift operator, and ε_t is a white-noise error term.

Model parameters are estimated using Maximum Likelihood Estimation (MLE), and model selection is based on AIC, BIC, and diagnostic checking.

3.3. GARCH Model:

The **Generalized Autoregressive Conditional Heteroskedasticity (GARCH)** model was proposed by Tim Bollerslev[3] developer of GARCH model (1986) as an extension of the **ARCH** model introduced by Robert F. Engle [2], developer of ARCH model (1982).GARCH is designed

to measure and model **time-varying volatility (conditional variance)** in financial time series, capturing important features such as volatility clustering, persistence, and changing market risk. The widely used GARCH(1,1) model expresses the conditional variance as $h_t = \omega + \alpha \varepsilon_{t-1}^2 + \beta h_{t-1}$, where α represents the impact of recent shocks (ARCH effect) and β measures the persistence of past volatility (GARCH effect). The sum ($\alpha + \beta$) indicates the degree of volatility persistence, with values close to one suggesting that shocks to volatility decay slowly over time. Due to its ability to capture dynamic risk and forecast future volatility, the GARCH model has become a standard tool in finance for analyzing stock returns, exchange rates, commodity prices, and other financial assets, [3, 2].

The GARCH (1, 1) variance equation is given by:

$$h_t = \omega + \alpha \varepsilon_{t-1}^2 + \beta h_{t-1}$$

Where:

- h_t : Conditional variance at time t
- ω : Constant (must be positive)
- α : ARCH effect (short-run shock impact on volatility)
- β : GARCH effect (persistence of past volatility)
- ε_{t-1} : Squared residual from the previous period
- h_{t-1} : Conditional variance from the previous period

3.4. EGARCH Model

The Exponential GARCH (EGARCH) models the logarithm of the conditional variance, which automatically ensures that the variance is positive without imposing non-negativity constraints on the parameters. The EGARCH (1,1) specification is:

$$\log(h_t) = \omega + \alpha |\varepsilon_{t-1}|/\sqrt{h_{t-1}} + \gamma (\varepsilon_{t-1}/\sqrt{h_{t-1}}) + \beta \log(h_{t-1})$$

Where:

- α : Size effect (magnitude of past shocks)
- γ : Asymmetry/leverage effect (sign of past shocks)
- β : Volatility persistence

If γ is negative and significant, it indicates a leverage effect — negative shocks increase volatility more than positive shocks. If γ is positive and significant, it indicates an inverse leverage effect — positive shocks increase volatility more. If γ is not significant, the volatility response is symmetric.

3.5. TGARCH Model

The Threshold GARCH (TGARCH), also known as GJR-GARCH, is another model that captures asymmetric volatility responses. The TGARCH (1, 1) variance equation is:

$$h_t = \omega + \alpha \varepsilon_{t-1}^2 + \gamma \varepsilon_{t-1}^2 \cdot I(\varepsilon_{t-1} < 0) + \beta h_{t-1}$$

Where:

- $I(\varepsilon_{t-1} < 0) = 1$ if the shock is negative, 0 otherwise
- α : General impact of past shocks on current volatility
- γ : Asymmetric (leverage) effect — additional impact of negative shocks
- β : GARCH persistence

3.6. Data

Daily closing price data for SABIC Agri Nutrients Company were obtained from Investing.com for January 1, 2021 to December 31, 2025, yielding 1,248 trading observations (Investing.com)

3.7. Model Accuracy Measures

The Akaike Information Criterion (AIC), Bayesian Information Criterion (BIC), and log-likelihood are commonly used measures for comparing competing statistical models. The log-likelihood quantifies how well a model fits the observed data, with larger values indicating a better fit. However, models with more parameters often achieve higher log-likelihood values simply due to their increased flexibility. To address this issue, the AIC and BIC incorporate penalty terms for model complexity. The AIC balances goodness-of-fit and parsimony by penalizing the number of estimated parameters, whereas the BIC imposes a stronger penalty, particularly for larger sample sizes. Consequently, models with lower AIC and BIC values are generally preferred, as they provide a better trade-off between model fit and complexity. In empirical applications, the preferred model is typically the one that exhibits the highest log-likelihood together with the lowest AIC and BIC values (Akaike, [22]; Schwarz,[24].

4. RESULTS

4.1 Descriptive Statistics and Checking Stationarity (Transformation, Correlograms, ADF tests)

Figure 1 to 3 contain- left, and the right panels show the original stock prices of SABIC and the right panel exhibit the Log returns of stock prices for SABIC. As most of the financial time series are non-stationary hence we resort to log returns of stock prices to make the series stationary which is a prerequisite to apply linear traditional models. [Log returns: $r_t = \ln(P_t) - \ln(P_{t-1})$.]

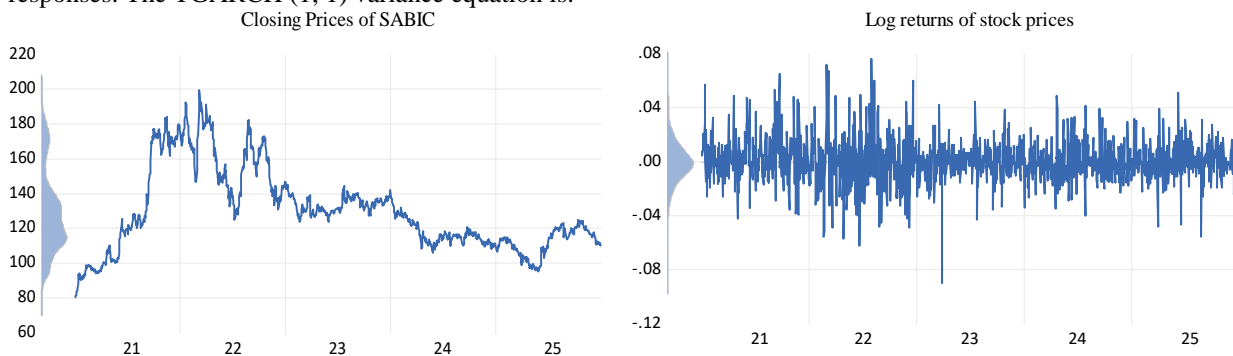


Figure 1: Left Panel (Original Stock Prices), Right Panel (Log Returns of Stock Prices) of SABIC

Figure 1 shows how the original series in the left panel behaves when the log return transformation is applied and shown in the right panel.

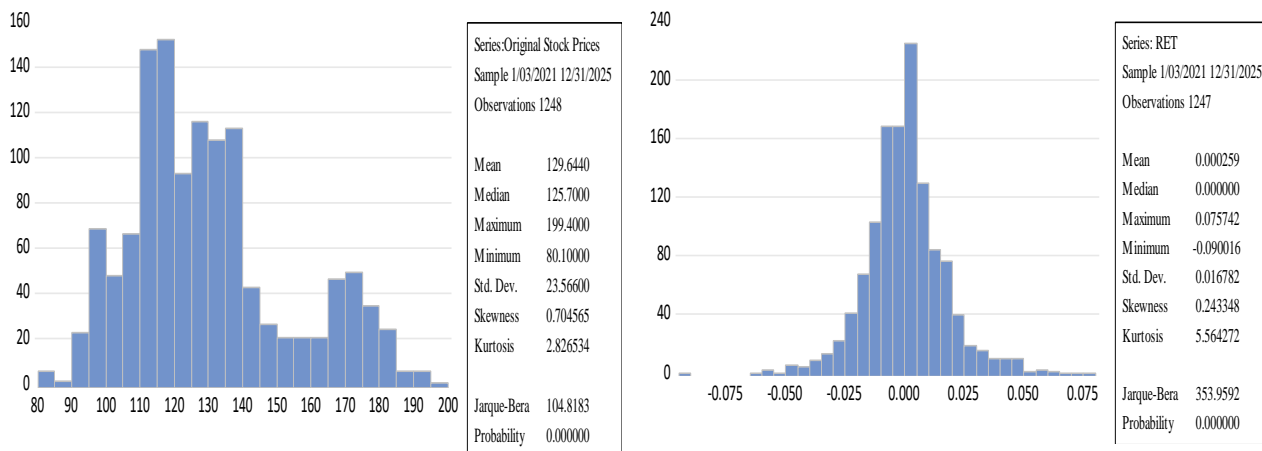


Figure 2: Left Panel (Descriptives of Original Stock Prices), Right Panel (Descriptives of Log Returns) SABIC

Figure 2 shows how the descriptive statistics of the original series are in the left panel and the same for the log return series in the right panel. A high value for the kurtosis in the right panel points towards the fact that the series has a fat-tail, and also, there is a probability of more volatility. This will be checked in the Volatility subsections

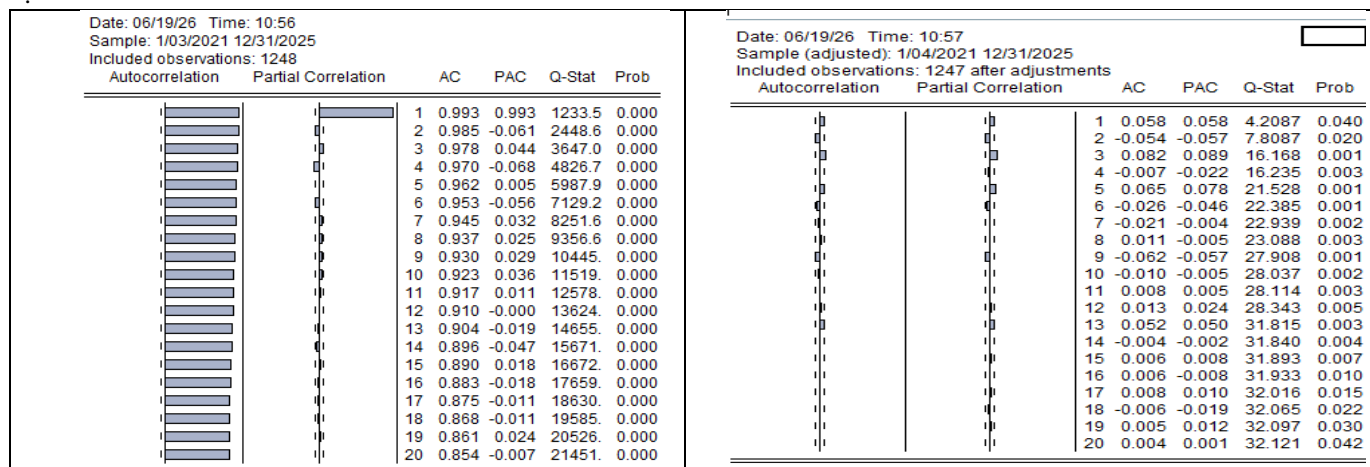


Figure 3: Left Panel (Correlogram of Original Stock Prices), Right Panel (Correlogram of Log Returns) SABIC.

Figure 3 shows how the correlograms for the original series are in the left panel and for the log return transformation in the right panel. The right panel clearly shows that most of the spikes are within the confidence limits, hence, the series are stationarized with almost no autocorrelation.

The ADF test proposed by Dicky & Fuller [23] was applied: H_0 : unit root (non-stationary) vs. H_1 : no unit root (stationary). The Augmented Dickey-Fuller (ADF) test was employed to examine the stationarity of the stock price series (PRICE) and the return series (RET) the results are shown in Table 1. For the original stock price series, the ADF test statistic was -2.4381 with a p-value of 0.1315, which is greater than the conventional significance level of 0.05. Furthermore, the test statistic is not more negative than the 5% critical value (-2.8637). Therefore, the null hypothesis of a unit root cannot

be rejected, indicating that the stock price series is non-stationary and exhibits a stochastic trend. In contrast, for the return series (RET), the ADF test statistic was -33.2818 with a p-value less than 0.001, which is substantially smaller than 0.05. The test statistic is also far below all reported critical values, leading to a strong rejection of the null hypothesis of a unit root. Consequently, the return series is stationary at level, I (0). These findings suggest that while stock prices follow a non-stationary process, their logarithmic returns are stationary, justifying the use of the return series for subsequent ARIMA, SETAR, and volatility modeling analyses, as stationarity is a fundamental requirement for reliable time-series estimation and forecasting

Table 1: ADF Unit Root Tests for Original and Log Returns of SABIC Stock Prices

Null Hypothesis: PRICE has a unit root Exogenous: Constant Lag Length: 3 (Automatic - based on SIC, maxlag=20)			Null Hypothesis: RET has a unit root Exogenous: Constant Lag Length: 0 (Automatic - based on SIC, maxlag=20)		
	t-Statistic	Prob.*		t-Statistic	Prob.*
Augmented Dickey-Fuller test statistic	-2.438073	0.1315	Augmented Dickey-Fuller test statistic	-33.28176	0.0000
Test critical values:			Test critical values:		
1% level	-3.435394		1% level	-3.435385	
5% level	-2.863655		5% level	-2.863651	
10% level	-2.567946		10% level	-2.567944	
*Mackinnon (1996) one-sided p-values.			*Mackinnon (1996) one-sided p-values.		

4.2 ARIMA Model Identification and Parameter Estimation
 Using the Autoarima option of EViews, three competing ARIMA models were compared: ARIMA (1, 1, 1), ARIMA (2, 1, 2), and ARIMA (3, 1, 3). Selection was based on coefficient significance, AIC, BIC, and residual diagnostics. The three competing models and their output is shown in Table 2. The ARIMA(1,1,1) model was estimated as a baseline candidate. While the AR and MA coefficients were significant, the model's AIC (-5.347836) was slightly higher than ARIMA(2,1,2), leading to its rejection in favor of ARIMA(2,1,2), which achieved better information criteria and superior residual diagnostics. Table 1 compares candidate models. ARIMA(2,1,2) achieved the lowest AIC (-5.348012).

Although ARIMA(3,1,3) had a slightly higher Adjusted R² (0.017154), ARIMA(2,1,2) was selected for its significant coefficients, lowest AIC, and parsimony. Estimated ARIMA (2, 1, 2) Model: The general ARIMA (2, 1, 2) model is:

$$Y_t = \mu + \phi_1 Y_{t-1} + \phi_2 Y_{t-2} + \varepsilon_t + \theta_1 \varepsilon_{t-1} + \theta_2 \varepsilon_{t-2}$$

where:

- Y_t = return at time t
- μ = constant term
- ϕ_1, ϕ_2 = autoregressive coefficients
- θ_1, θ_2 = moving average coefficients
- ε_t = white-noise error

$$RET_t = 0.000260 - 1.408459 RET_{t-1} - 0.651496 RET_{t-2} + \varepsilon_t + 1.473040 \varepsilon_{t-1} + 0.661887 \varepsilon_{t-2}$$

Table 2: Competing Models for ARIMA of SABIC Returns

ARIMA(1,1,1)	ARIMA (2,1,2)	ARIMA (3,1,3)																																																																																																									
Dependent Variable: RET Method: ARMA Maximum Likelihood (OPG - BHHH) Date: 06/19/26 Time: 11:12 Sample: 1/04/2021 12/31/2025 Included observations: 1247 Convergence achieved after 16 iterations Coefficient covariance computed using outer product of gradients d.f. adjustment for standard errors & covariance	Dependent Variable: RET Method: ARMA Maximum Likelihood (OPG - BHHH) Date: 06/19/26 Time: 11:15 Sample: 1/04/2021 12/31/2025 Included observations: 1247 Convergence achieved after 24 iterations Coefficient covariance computed using outer product of gradients d.f. adjustment for standard errors & covariance	Dependent Variable: RET Method: ARMA Maximum Likelihood (OPG - BHHH) Date: 06/19/26 Time: 11:17 Sample: 1/04/2021 12/31/2025 Included observations: 1247 Convergence achieved after 18 iterations Coefficient covariance computed using outer product of gradients d.f. adjustment for standard errors & covariance																																																																																																									
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The ARIMA (2, 1, 2) model estimated for the return series reveals significant autoregressive and moving-average dynamics. The AR (1) ($\phi_1 = -1.4085$) and AR (2) ($\phi_2 = -0.6515$) coefficients are negative and statistically significant, indicating strong mean-reverting behavior in returns. Similarly, the MA (1) ($\theta_1 = 1.4730$) and MA (2) ($\theta_2 = 0.6619$) coefficients are positive and highly

significant, suggesting that past forecast errors exert a substantial influence on current returns.

The model is statistically significant overall ($F = 4.97, p < 0.001$), while the Durbin-Watson statistic of 1.98 indicates the absence of residual autocorrelation. Although the R^2 value is relatively low (1.96%), such results are typical for financial return series due to their inherently volatile nature. Furthermore, the stationarity and invertibility conditions are

satisfied, confirming that the ARIMA (2, 1, 2) specification provides a stable representation of the return-generating process.4.3 Residual Diagnostics for ARIMA (2, 1, 2)

All ACF and PACF spikes in Figure 4 lie within the 5% bands, confirming no remaining autocorrelation and adequate model fit.

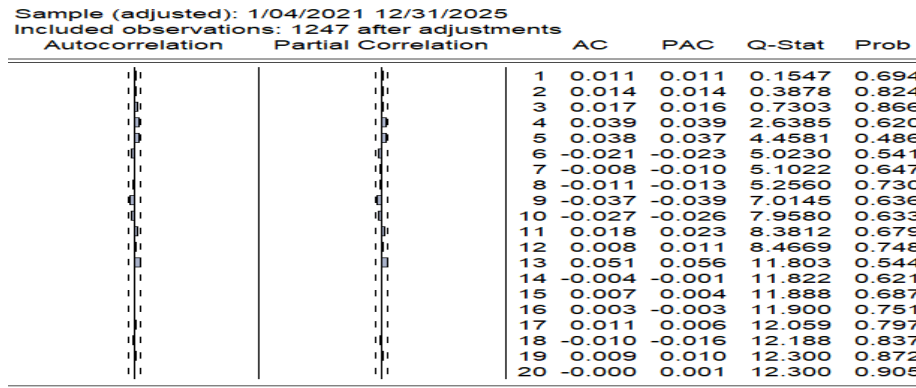


Figure 4: Correlogram of Residual Series ARIMA(2,1,2)

The ADF test results on the residual series shown in Table 3 confirms stationarity as ($p < 0.05$), indicating that the

ARIMA (2, 1, 2) model has successfully removed the unit root and the residuals are stationary

Table3 : ADF Test of Residual Series for ARIMA (2, 1, 2)
 Null Hypothesis: RESID02 has a unit root
 Exogenous: Constant
 Lag Length: 0 (Automatic - based on SIC, maxlag=22)

	t-Statistic	Prob.*
Augmented Dickey-Fuller test statistic	-34.88122	0.0000
Test critical values: 1% level	-3.435385	
5% level	-2.863651	
10% level	-2.567944	

4.4 Volatility Visualization

Figure 5 shows the price and residual series, highlighting remaining volatility patterns. The return series plot (Figure 5)

indeed shows periods of alternating high and low volatility, providing visual motivation for proceeding with GARCH modeling.

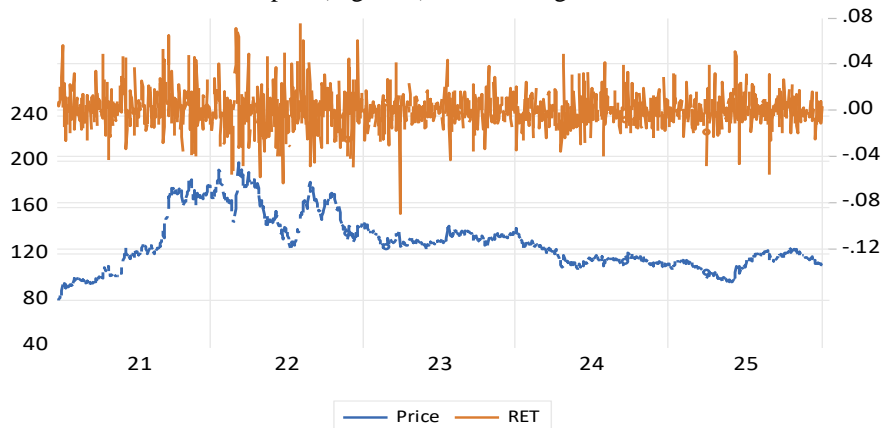


Figure 5: Line Plots for Original Series and the Return Series

4.5 Testing for ARCH Effects

The ARCH-LM test was applied to ARIMA (2, 1, 2) residuals with H_0 : No ARCH effect (no conditional heteroskedasticity)

The ARCH-LM test yielded $Obs \times R^2 = 0.272245$ ($p = 0.6022$). Since $p > 0.05$, we fail to reject H_0 at lag 1, suggesting no significant ARCH effect at this specific lag. It should be noted, however, that a single-lag test has limited power to detect higher-order or more complex

heteroskedastic patterns. Moreover, financial return series often exhibit volatility clustering visually even when low-order ARCH-LM tests are insignificant, as clustered volatility may operate across multiple lags that a lag-1 test cannot capture. It is therefore standard practice in empirical finance to estimate GARCH-type models as part of a comprehensive volatility analysis, even when the ARCH-LM test at lag 1 is not significant [5].

Table4 : EViews output- Heteroskedasticity Test: ARCH LM
Heteroskedasticity Test: ARCH

F-statistic	0.271866	Prob. F(1,1242)	0.6022
Obs*R-squared	0.272245	Prob. Chi-Square(1)	0.6018

Test Equation:
Dependent Variable: WGT_RESID^2
Method: Least Squares
Date: 05/26/26 Time: 16:42
Sample (adjusted): 1/07/2021 12/31/2025
Included observations: 1244 after adjustments

Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	1.015599	0.067528	15.03968	0.0000
WGT_RESID^2(-1)	-0.014793	0.028372	-0.521408	0.6022

R-squared	0.000219	Mean dependent var	1.000796
Adjusted R-squared	-0.000586	S.D. dependent var	2.160365
S.E. of regression	2.160998	Akaike info criterion	4.380623
Sum squared resid	5800.029	Schwarz criterion	4.388865
Log likelihood	-2722.748	Hannan-Quinn criter.	4.383722
F-statistic	0.271866	Durbin-Watson stat	1.998881
Prob(F-statistic)	0.602175		

4.6 Model Volatility and Checking Symmetry and Asymmetry in the Shocks

Table 5 presents the results of GARCH and its variants. The GARCH (1, 1) model generalizes ARCH by including the lagged conditional variance, capturing volatility persistence with fewer parameters. The GARCH (1, 1) model was estimated on the ARIMA (2, 1, 2) residuals. Based on the EViews output (Figure 19), the estimated variance equation parameters $\alpha = 0.122634$ and $\beta = 0.820284$ are both highly significant ($p = 0.0000$), with a persistence coefficient $\alpha + \beta = 0.942918$, confirming strong and slowly-decaying volatility clustering.

ARIMA (2, 1, 2)-GARCH (1, 1) Model

Mean Equation: The estimated mean equation is:

$$RET_t = -0.000205 - 1.41254RET_{t-1} - 0.64155RET_{t-2} + \varepsilon_t + 1.45216\varepsilon_{t-1} + 0.62035\varepsilon_{t-2}$$

where

$$\varepsilon_t = z_t\sqrt{h_t}, \quad z_t \sim N(0,1)$$

The constant term is insignificant ($p = 0.6223$), suggesting that average daily returns are not significantly different from zero. The AR (1) and AR (2) coefficients are negative and highly significant ($p < 0.001$), indicating mean-reverting behavior in returns. Similarly, MA (1) and MA (2) are positive and statistically significant, implying that past forecast errors continue to influence current returns.

Table 5: Result Output of GARCH and its Variants

GARCH (1,1)	EGARCH(1,1)	TGARCH(1,1)																																																																																																																																																											
<p>Dependent Variable: RET Method: ML ARCH - Normal distribution (Marquardt / EViews legacy) Date: 06/19/26 Time: 11:27 Sample (adjusted): 1/06/2021 12/31/2025 Included observations: 1245 after adjustments Convergence achieved after 51 iterations MA Backcast: 1/04/2021 1/05/2021 Presample variance: backcast (parameter = 0.7) GARCH = C(6) + C(7)*RESID(-1)^2 + C(8)*GARCH(-1)</p> <table border="1"> <thead> <tr> <th>Variable</th> <th>Coefficient</th> <th>Std. 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Variable	Coefficient	Std. Error	z-Statistic	Prob.																																																																																																																																																									
C	-0.000205	0.000417	-0.492554	0.6223																																																																																																																																																									
AR(1)	-1.412540	0.174876	-8.077381	0.0000																																																																																																																																																									
AR(2)	-0.641547	0.128472	-4.993669	0.0000																																																																																																																																																									
MA(1)	1.452161	0.181832	7.986289	0.0000																																																																																																																																																									
MA(2)	0.620348	0.149870	4.139254	0.0000																																																																																																																																																									
Variable	Coefficient	Std. Error	z-Statistic	Prob.																																																																																																																																																									
C	1.67E-05	3.28E-06	5.087864	0.0000																																																																																																																																																									
RESID(-1)^2	0.122634	0.020617	5.948238	0.0000																																																																																																																																																									
GARCH(-1)	0.820284	0.025739	31.86957	0.0000																																																																																																																																																									
Variable	Coefficient	Std. Error	z-Statistic	Prob.																																																																																																																																																									
C	-0.000298	9.32E-05	-3.191386	0.0014																																																																																																																																																									
AR(1)	0.225612	0.079581	2.834995	0.0046																																																																																																																																																									
AR(2)	0.752939	0.078836	9.550707	0.0000																																																																																																																																																									
MA(1)	-0.157884	0.064489	-2.448225	0.0144																																																																																																																																																									
MA(2)	-0.840884	0.064360	-13.06523	0.0000																																																																																																																																																									
Variable	Coefficient	Std. Error	z-Statistic	Prob.																																																																																																																																																									
C(6)	-0.730869	0.123633	-5.911600	0.0000																																																																																																																																																									
C(7)	0.244250	0.034301	7.120838	0.0000																																																																																																																																																									
C(8)	0.934002	0.012969	72.01803	0.0000																																																																																																																																																									
Variable	Coefficient	Std. Error	z-Statistic	Prob.																																																																																																																																																									
C	-0.000148	0.000430	-0.343667	0.7311																																																																																																																																																									
AR(1)	-1.409196	0.177508	-7.938757	0.0000																																																																																																																																																									
AR(2)	-0.638959	0.130848	-4.883213	0.0000																																																																																																																																																									
MA(1)	1.448233	0.184303	7.857885	0.0000																																																																																																																																																									
MA(2)	0.616777	0.152380	4.047637	0.0001																																																																																																																																																									
Variable	Coefficient	Std. Error	z-Statistic	Prob.																																																																																																																																																									
C	1.72E-05	3.41E-06	5.041557	0.0000																																																																																																																																																									
RESID(-1)^2	0.132129	0.023518	5.618279	0.0000																																																																																																																																																									
RESID(-1)^2*(RESID(-1)+0)	-0.018532	0.023822	-0.777944	0.4366																																																																																																																																																									
GARCH(-1)	0.817456	0.028264	31.12461	0.0000																																																																																																																																																									

Variance Equation

The estimated GARCH (1, 1) variance equation is
 $h_t = 0.000167 + 0.122634 \varepsilon_{t-1}^2 + 0.820284 h_{t-1}$

Where,

h_t = Conditional Variance

ARCH Effect

$\alpha_1 = 0.122634$

($p < 0.001$)

The ARCH coefficient is significant, indicating that current volatility reacts to recent market shocks.

GARCH Effect

$\beta_1 = 0.820284$ ($p < 0.001$)

The GARCH coefficient is highly significant, showing that volatility is persistent and depends strongly on past volatility.

Volatility Persistence

$\alpha_1 + \beta_1 = 0.122634 + 0.820284 = 0.942918$

Since

$0.942918 < 1$

The conditional variance process is stationary.

However, because the value is close to one, shocks to volatility decay slowly over time.

This finding is typical of stock-return series and indicates long-memory volatility behavior.

ARIMA (2, 1, 2)-EGARCH (1, 1) Model

The estimated mean equation is

$$RET_t = -0.000298 + 0.225612RET_{t-1} + 0.752939RET_{t-2}$$

$$+ \varepsilon_t - 0.157804\varepsilon_{t-1} - 0.840884\varepsilon_{t-2}$$

All parameters are statistically significant at the 1% level.

EGARCH Variance Equation

The estimated EGARCH model is

$$\ln(h_t) = 0.730869 + 0.244250 \left(\frac{|\varepsilon_{t-1}|}{\sqrt{h_{t-1}}} \right) + 0.934002 \ln(h_{t-1})$$

The general EGARCH (1, 1) specification is

$$\ln(h_t) = \omega + \alpha \left(\frac{|\varepsilon_{t-1}|}{\sqrt{h_{t-1}}} - E|z_t| \right) + \gamma \left(\frac{\varepsilon_{t-1}}{\sqrt{h_{t-1}}} \right) + \beta \ln(h_{t-1})$$

where γ measures asymmetry (leverage effect).

Interpretation

The persistence coefficient

$\beta = 0.934002$

is highly significant, indicating strong volatility persistence.

The positive magnitude coefficient

$\alpha = 0.244250$

suggests that large shocks, whether positive or negative, increase volatility.

Because EGARCH models variance in logarithmic form, the conditional variance is always positive without imposing non-negativity constraints.

ARIMA (2, 1, 2)-TGARCH (1, 1) Model

From Figure 22, the asymmetry coefficient (γ , represented by C (8)) is -0.018532 and is statistically insignificant ($p = 0.4366$). Since $p > 0.05$, the TGARCH framework does not confirm a significant leverage effect for this stock, but NIC has to be observed along with these mathematical results to unveil the true picture. The ARCH ($\alpha = 0.132129$) and GARCH ($\beta = 0.817456$) terms remain significant, confirming volatility persistence.

4.7 Model Comparison:

Table 3 presents the model comparison results. EGARCH(1,1) achieves the lowest AIC (-5.462182) and highest log-likelihood (3409.208), outperforming both GARCH(1,1) and TGARCH(1,1). TGARCH(1,1) performs marginally better than symmetric GARCH(1,1) in log-likelihood (3403.518 vs. 3403.353), but its asymmetry parameter is insignificant ($p = 0.4366$). The EGARCH(1,1) asymmetry parameter is marginally significant ($p = 0.0945$), providing weak evidence of asymmetric volatility. EGARCH(1,1) is selected as the best-fitting model based on information criteria. Therefore, the EGARCH model provides a better fit to the return data.

Table 6: Comparison of GARCH and its Variants

Statistic	GARCH(1,1)	EGARCH(1,1)	TGARCH(1,1)
Log-Likelihood	3403.353	3408.341	3403.542
AIC	-5.454382	-5.462395	-5.453079
BIC	-5.421439	-5.429451	-5.41608

4.8 Why apply EGARCH and TGARCH after GARCH?

Although the standard GARCH model successfully captures volatility clustering, it assumes that positive and negative shocks affect volatility equally.

Mathematically,

$$h_t = \omega + \alpha \varepsilon_{t-1}^2 + \beta h_{t-1}$$

depends on ε_{t-1}^2 , which removes the sign of the shock.

Thus,

$$(+5\%)^2 = (-5\%)^2$$

and both shocks have the same impact on volatility.

However, financial markets often exhibit a **leverage effect**, where bad news produces greater volatility than good news of the same magnitude.

David Runkle (1993) extends the GARCH model by explicitly incorporating a dummy variable for negative shocks: TGARCH (1, 1) model:

$$h_t = \omega + \alpha \varepsilon_{t-1}^2 + \gamma I_{t-1} \varepsilon_{t-1}^2 + \beta h_{t-1}$$

where

$$I_{t-1} = \begin{cases} 1, & \varepsilon_{t-1} < 0 \\ 0, & \varepsilon_{t-1} \geq 0 \end{cases}$$

If

$$\gamma > 0$$

negative shocks increase volatility more than positive shocks.

Thus, TGARCH is applied to test whether bad news has a disproportionately larger effect on volatility than good news.

The ARIMA (2, 1, 2)-GARCH(1,1) results reveal significant volatility clustering and persistence in SABIC returns, with the sum of the ARCH and GARCH coefficients ($\alpha + \beta = 0.9429$) indicating that volatility shocks dissipate slowly over time. However, the standard GARCH model assumes a symmetric response of volatility to positive and negative shocks. To account for potential leverage effects and asymmetric volatility transmission, the EGARCH model was estimated. The EGARCH specification produced a higher log-likelihood and lower information criteria than the GARCH model, suggesting superior model performance. Consequently, the use of EGARCH and TGARCH models is justified because financial return series frequently exhibit asymmetric volatility, whereby negative market news generates a stronger volatility response than positive news of

equal magnitude. These asymmetric models provide a more realistic representation of risk dynamics and improve forecasting accuracy in financial markets.

4.10 News Impact Interpretation and Volatility Visualization

Figure 6 shows the News Impact Curve (NIC) for the GARCH and its variants. The NIC plots the impact of past

shocks (ϵ_{t-1}) on the current conditional variance (h_t). A symmetric NIC would indicate that positive and negative shocks of the same magnitude produce identical volatility responses.

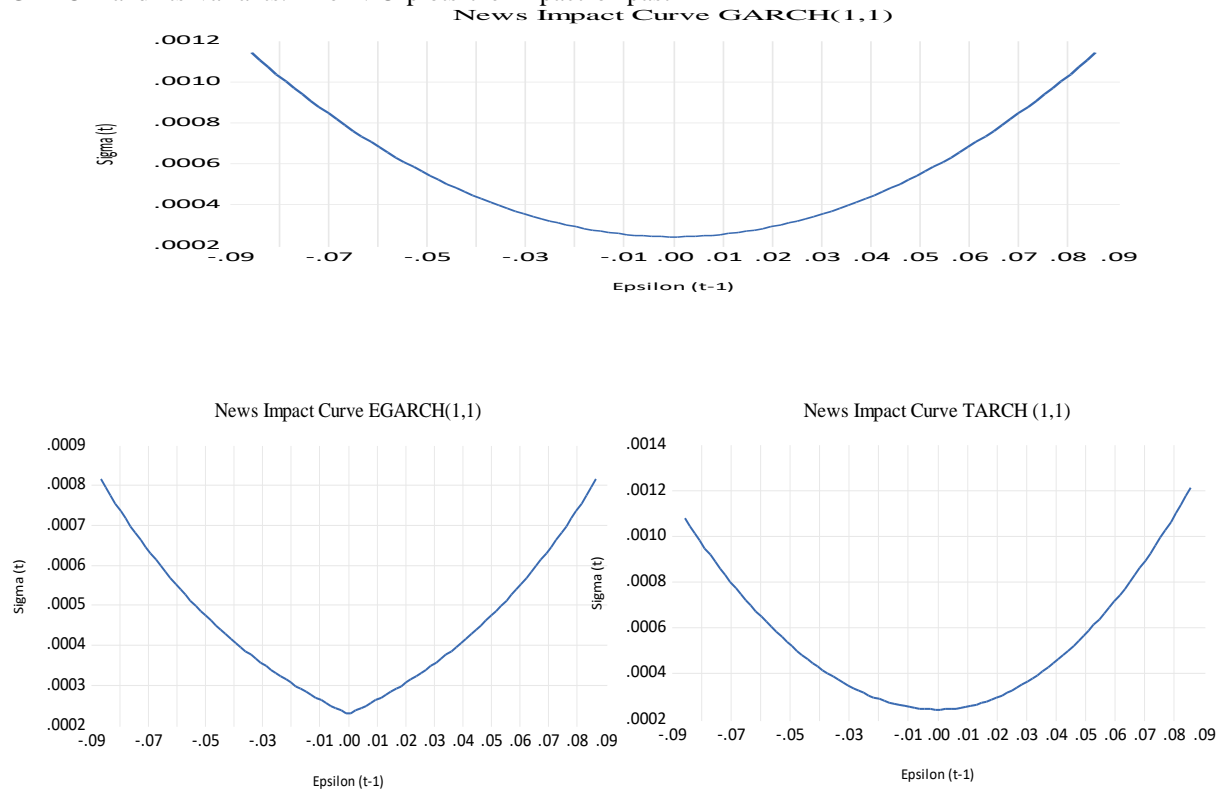


Figure 6: News Impact Curves for GARCH and its Variants

In the case of TGARCH (1, 1), the NIC is slightly asymmetric: positive shocks generate marginally larger volatility responses than negative shocks of equal size, consistent with the marginally significant positive asymmetry parameter. This represents an inverse leverage effect, which may reflect the commodity-driven nature of SABIC AgriNutrients stock, where positive price news (e.g., rising fertilizer prices) can trigger heightened speculative activity. However, given that the asymmetry is only marginally significant at the 10% level, these findings should be interpreted with caution.

5. Discussion, Conclusion, and Implications

The empirical findings indicate that the logarithmic return series of SABIC Agri-Nutrients is stationary, whereas the original stock price series exhibits a unit root and follows a non-stationary process. This result is consistent with the Efficient Market Hypothesis and the broader financial econometrics literature, which suggests that stock prices generally behave as random walks while returns are stationary and suitable for time-series modeling. The ARIMA (2,1,2) model successfully captured the linear dependence structure in the return series, with residual diagnostics confirming the absence of significant autocorrelation. The

relatively low coefficient of determination is not unexpected, as financial returns are influenced by numerous unpredictable market factors and are inherently noisy.

The volatility analysis revealed strong evidence of volatility persistence. The GARCH (1,1) model produced significant ARCH and GARCH parameters, with a persistence coefficient of 0.9429, indicating that volatility shocks dissipate slowly over time. Such findings are consistent with previous studies on financial and commodity-related stocks, where periods of high volatility tend to be followed by further periods of heightened volatility. This behavior reflects the presence of volatility clustering, a well-documented stylized fact in financial markets. Although the standard GARCH model adequately captured conditional heteroskedasticity, the comparison of alternative specifications demonstrated that the EGARCH (1,1) model provided the best overall fit according to the log-likelihood, AIC, and BIC criteria. The EGARCH model also allowed examination of asymmetry in volatility responses. The News Impact Curve suggested a mild inverse leverage effect, whereby positive shocks generated slightly larger volatility responses than negative shocks of the same magnitude. This behavior may be attributed to the commodity-driven nature of SABIC Agri-Nutrients, where

favorable developments in fertilizer demand, petrochemical feedstock markets, or global commodity prices can trigger increased speculative trading activity. However, the asymmetry parameter was only marginally significant, suggesting that the evidence for asymmetric volatility should be interpreted cautiously.

The TGARCH model similarly indicated the absence of a statistically significant leverage effect. Consequently, the results suggest that volatility persistence is a more dominant characteristic of SABIC returns than volatility asymmetry. Nevertheless, the superior performance of the EGARCH specification demonstrates that allowing for nonlinear volatility dynamics improves model fit and forecasting performance. Overall, the findings highlight the importance of incorporating conditional variance models when analyzing Saudi equity returns and support the use of asymmetric GARCH models for enhanced risk assessment and volatility forecasting.

This study investigated the return and volatility dynamics of SABIC Agri-Nutrients Company using daily stock price data from January 2021 to December 2025. The Box–Jenkins methodology identified ARIMA (2,1,2) as the most suitable model for capturing the conditional mean process of returns. Stationarity tests confirmed that the return series was stationary and appropriate for time-series modeling. Residual diagnostics further verified the adequacy of the selected ARIMA specification.

To model volatility behavior, GARCH (1,1), EGARCH (1,1), and TGARCH (1,1) models were estimated. The results revealed significant volatility clustering and persistence, with volatility shocks exhibiting slow decay over time. Among the competing volatility models, EGARCH (1,1) achieved the best performance based on information criteria and log-likelihood values, indicating that it provides a more flexible representation of the volatility process. Although evidence of asymmetric volatility was relatively weak, the EGARCH specification captured nonlinear volatility dynamics more effectively than the symmetric GARCH model.

The findings provide valuable insights for investors, portfolio managers, risk analysts, and policymakers by demonstrating the importance of volatility modeling in understanding stock return behavior. Accurate volatility forecasts can contribute to improved portfolio allocation, risk management, derivative pricing, and investment decision-making within the Saudi capital market. The study also contributes to the growing literature on financial econometrics in emerging markets and supports the application of advanced volatility models to Saudi equity data.

Several avenues for future research emerge from this study. First, nonlinear mean models such as SETAR, STAR, and Markov-Switching Autoregressive (MSAR) models may be explored to capture potential regime-dependent behavior in stock returns. Second, more advanced volatility specifications, including APARCH, FIGARCH, FIEGARCH, and Component GARCH, could be employed to investigate long-memory and asymmetric volatility structures. Third, machine learning and deep learning approaches such as LSTM, GRU, and hybrid ARIMA–LSTM frameworks may

be compared with traditional econometric models to evaluate forecasting performance.

Future studies may also incorporate macroeconomic variables such as oil prices, natural gas prices, inflation, interest rates, exchange rates, and fertilizer commodity indices to examine their influence on stock return volatility. Additionally, extending the analysis to other firms within the Saudi petrochemical and agricultural sectors or to the broader Tadawul All Share Index would provide a more comprehensive understanding of volatility transmission within the Saudi financial market. Finally, higher-frequency intraday data could be utilized to investigate short-term volatility dynamics and improve risk forecasting accuracy.

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