NONLINEAR AND ASYMMETRIC VOLATILITY IN SAUDI STOCK RETURNS: EVIDENCE FROM THE SETAR MODEL WITH ARIMA BENCHMARKING

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ABSTRACT: *This study investigates the volatility dynamics and asymmetric behavior of stock returns for Alrajhi Bank, one of the largest financial institutions in Saudi Arabia. Daily closing price data covering the period from January 3, 2021, to December 31, 2025, were obtained from Investing.com and transformed into logarithmic returns. The Box-Jenkins methodology was employed to model the conditional mean process using Autoregressive Integrated Moving Average (ARIMA) models, while volatility behavior was analyzed using ARCH, GARCH, and Exponential GARCH (EGARCH) models. In addition, a Self-Exciting Threshold Autoregressive (SETAR) model was estimated to examine potential nonlinear dynamics in stock returns. The empirical results indicate that ARIMA (1, 0, 1) provides the most suitable specification for modeling the mean process. ARCH-LM testing confirms the presence of heteroskedasticity and volatility clustering in the return series. Among the competing volatility models, EGARCH (1, 1) demonstrates superior performance based on information criteria and goodness-of-fit measures. The significant negative asymmetry coefficient confirms the existence of the leverage effect, indicating that negative shocks have a stronger impact on future volatility than positive shocks of equal magnitude. The findings highlight the importance of accounting for asymmetric volatility when modeling stock returns in the Saudi stock market and provide useful implications for investors, portfolio managers, and risk analysts.*

Keywords: Volatility Clustering, ARIMA, GARCH, EGARCH, SETAR, Leverage Effect.

`1. INTRODUCTION

Financial markets are characterized by uncertainty, dynamic price movements, and changing volatility patterns. Understanding stock return behavior is essential for investors, portfolio managers, and policymakers because volatility directly affects investment decisions and risk management strategies. Financial return series often exhibit stylized features such as volatility clustering, leptokurtosis, and asymmetric responses to market information. The Saudi stock market has experienced substantial growth during the past decade and represents one of the largest financial markets in the Middle East. Among the listed firms, *Alrajhi Bank* occupies a prominent position due to its large market capitalization, high liquidity, and active trading volume. Consequently, its stock returns provide an appropriate framework for examining volatility dynamics in the Saudi financial market.

Traditional linear time series models, such as ARIMA, are commonly used to model the conditional mean of financial series. However, these models are unable to capture time-varying volatility. To address this limitation, Engle (1982) introduced the ARCH model, which was later extended by Bollerslev (1986) through the GARCH model. Furthermore, asymmetric volatility models such as EGARCH allow positive and negative shocks to have different impacts on future volatility.

The primary objective of this study is to model stock return volatility for Al Rajhi Bank and investigate the presence of asymmetric volatility effects using ARIMA and GARCH-type models. In addition, a SETAR model is employed to explore possible nonlinear dynamics in stock returns.

2. Literature Review

Modeling and forecasting financial market volatility has received considerable attention in empirical finance because volatility is closely associated with risk management, portfolio allocation, option pricing, and financial decision-making. Financial return series often exhibit stylized characteristics such as volatility clustering, leptokurtosis, persistence, and asymmetric responses to market information (Cont, 2001). The development of volatility modeling began with the Autoregressive Conditional Heteroskedasticity (ARCH) model proposed by Engle (1982), which allows the conditional variance of a time series to depend on past squared error terms. Although ARCH models successfully capture volatility clustering, they often require a large number of lagged terms to adequately describe volatility dynamics. To overcome this limitation, Bollerslev (1986) introduced the Generalized ARCH (GARCH) model, which incorporates both past shocks and past conditional variances, providing a more parsimonious representation of volatility persistence. Despite their usefulness, standard GARCH models assume that positive and negative shocks have identical effects on future volatility. However, empirical evidence suggests that negative news often produces a stronger impact on volatility than positive news of equal magnitude. This phenomenon is commonly known as the leverage effect (Black, 1976). To accommodate this asymmetry, Nelson (1991) developed the Exponential GARCH (EGARCH) model, which allows positive and negative innovations to influence volatility differently while ensuring a positive conditional variance without imposing non-negativity constraints on model parameters.

Several studies have demonstrated the effectiveness of ARIMA-based models in forecasting stock market prices and returns. Abbasi et al. (2017) examined the forecasting performance of ARIMA models using stock price data from Flying Cement and reported that the ARIMA (1, 2, 1) specification produced the most accurate forecasts among the competing models. The study highlighted the capability of ARIMA models to capture linear temporal dependencies in stock prices while recommending the incorporation of volatility and hybrid forecasting techniques to enhance predictive performance. In a related study, Almarashi et al. (2023) analyzed the stock prices of Saudi Cement Company using a combination of ARIMA and ARCH frameworks. Their results identified ARIMA(0,1,1) as the most suitable model for modeling the conditional mean process. Although the ARCH component was found to be statistically insignificant, the study emphasized the importance of jointly examining mean and variance dynamics when forecasting financial time series. Collectively, these studies support the applicability of ARIMA-based approaches in stock market forecasting while underscoring the potential benefits of integrating volatility models to account for the heteroscedastic nature of financial data.

Numerous studies have demonstrated that stock returns exhibit significant volatility clustering, where periods of high volatility tend to be followed by high volatility and periods of low volatility tend to be followed by low volatility (Mandelbrot, 1963; Bollerslev, 1986). Such behavior violates the assumption of constant variance and necessitates the use of conditional heteroskedasticity models. Several empirical studies have confirmed the superiority of asymmetric GARCH models in financial applications. Glosten et al. (1993) introduced the Threshold GARCH (TGARCH) model and demonstrated that bad news tends to generate larger volatility responses than good news. Similarly, Engle and Ng (1993) proposed diagnostic procedures for identifying asymmetries in volatility processes and found significant leverage effects in equity markets.

In addition to volatility clustering and asymmetry, financial markets may also exhibit nonlinear dynamics. Linear models such as ARIMA are often insufficient for describing complex market behavior because the underlying data-generating process may change across different market conditions. Tong (1990) introduced Self-Exciting Threshold Autoregressive (SETAR) models, which permit different autoregressive structures depending on whether a threshold variable exceeds a specified value. SETAR models have been widely used to capture regime-switching behavior and nonlinear adjustment mechanisms in financial and economic time series. Within the Saudi stock market context, several studies have reported strong volatility persistence and asymmetric volatility behavior. Almarashi (2023) investigated Saudi stock market returns and found that EGARCH models outperformed symmetric GARCH specifications in capturing leverage effects and volatility persistence. Similar findings have been reported for emerging financial markets where investor sentiment and market uncertainty contribute to asymmetric volatility responses.

The Self-Exciting Threshold Autoregressive (SETAR) model, introduced by Howell Tong, was developed to capture nonlinear and regime-dependent behavior in time series data that cannot be adequately modeled using linear autoregressive approaches (Tong, 1983). Subsequently, Ruey S. Tsay proposed practical procedures for testing and estimating threshold autoregressive models, facilitating their application in economic and financial research (Tsay, 1989). Empirical studies have shown that SETAR models effectively capture asymmetric market dynamics and regime-switching behavior, often providing improved forecasting performance over linear models in financial time series (Hansen & Mikkelsen, 2005; Chen & Lee, 2021).

Although considerable research has focused on volatility modeling in developed markets, relatively few studies have investigated nonlinear dynamics and asymmetric volatility simultaneously within the Saudi banking sector. Therefore, the present study contributes to the literature by combining ARIMA, SETAR, ARCH, GARCH, and EGARCH models to analyze the return dynamics of Al Rajhi Bank stock and evaluate the presence of volatility clustering, persistence, leverage effects, and nonlinear market regimes.

Although extensive research has examined stock market volatility using ARCH and GARCH-type models, several gaps remain in the literature, particularly within the context of the Saudi stock market. First, many previous studies have focused primarily on symmetric volatility models such as ARCH and GARCH, which assume that positive and negative market shocks affect volatility equally. However, empirical evidence suggests that financial markets often exhibit asymmetric volatility behavior, where negative information has a stronger effect on volatility than positive information of similar magnitude. Second, there is a scarcity of studies that simultaneously integrate ARIMA, SETAR, ARCH, GARCH, and EGARCH models within a single analytical framework. Combining these approaches enables the examination of linear dynamics, nonlinear regime-switching behavior, volatility clustering, persistence, and asymmetric volatility effects in a comprehensive manner.

Therefore, the present study addresses these gaps by examining Al Rajhi Bank stock returns using an integrated framework consisting of ARIMA, SETAR, ARCH, GARCH, and EGARCH models. The study contributes to the existing literature by evaluating both linear and nonlinear return dynamics while simultaneously investigating volatility persistence and leverage effects in one of the most important financial institutions in Saudi Arabia.

3. Methodology

3.1 Data

Daily closing prices of Al Rajhi Bank were collected from Investing.com for the period January 3, 2021, to December 31, 2025, yielding 1,248 observations. Logarithmic returns were calculated as:

$$r_t = \ln(P_t) - \ln(P_{t-1})$$

where:

P_t = closing price at time t

r_t = continuously compounded return

3.2 ARIMA Model

The ARIMA model is expressed as:

$$\phi(B)(1-B)^d Y_t = \theta(B)\varepsilon_t$$

Model selection was based on:

a. AIC
b. BIC
c. RMSE
d. Residual diagnostics

3.3 SETAR Model

The SETAR model allows different autoregressive structures across regimes separated by a threshold value:

$$r_t = \begin{cases} \alpha_1 + \beta_1 r_{t-1} + \varepsilon_t, & r_{t-1} < \gamma \\ \alpha_2 + \beta_2 r_{t-1} + \varepsilon_t, & r_{t-1} \geq \gamma \end{cases}$$

where γ represents the threshold value.

3.4 ARCH/GARCH Models

The ARCH (q) model is defined as:

$$\sigma_t^2 = \omega + \sum_{i=1}^{q} \alpha_i \, \varepsilon_{t-i}^2$$

The GARCH (1, 1) model is:

$$h_t = \omega + \alpha\varepsilon_{t-1}^2 + \beta h_{t-1}$$

3.5 EGARCH Model

The EGARCH (1, 1) model is:

$$\log(h_t) = \omega + \alpha\left|\frac{\varepsilon_{t-1}}{\sqrt{h_{t-1}}}\right| + \gamma\left(\frac{\varepsilon_{t-1}}{\sqrt{h_{t-1}}}\right) + \beta\log(h_{t-1})$$

A negative value of γ indicates the presence of the leverage effect.

4. Results

4.1 Descriptive Analysis

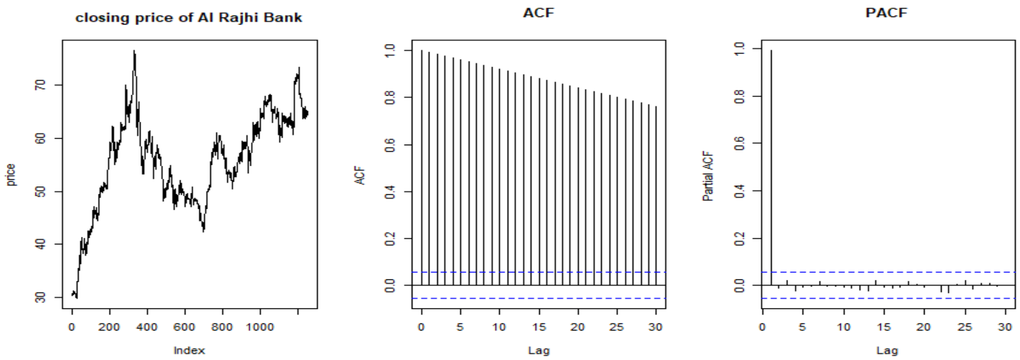

Figure 1: Closing Prices for Alrahi Bank with ACF and PACF

The three panels of Figure 1 exhibit the closing prices of *Al Rajhi Bank*. The first panel on the left hand shows non-stationary data with clear presence of trend and no evidence of seasonality alongside high and low volatility. The stationarity of the series is formally confirmed by the Augmented Dickey-Fuller (ADF) test, where a p-value of 0.0713 indicates a failure to reject the null hypothesis of non-stationarity. Furthermore, the Autocorrelation Function (ACF) decays slowly, which indicates strong autocorrelation and Partial Autocorrelation Function (PACF) cut off after the first lag, which is consistent with the result. First prerequisite to apply linear models is to stationarize the data.

4.2 Stationarity Analysis: The closing prices data was transformed into return series to stationarize the data using the following formula:

$$r_t = \ln(P_t) - \ln(P_{t-1})$$

Figure 2 exhibit the return series alongwith ACF and PACF.

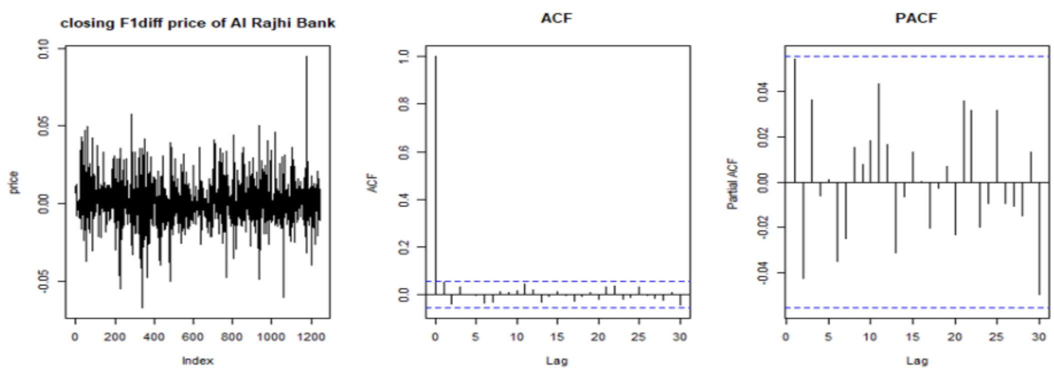

Figure 2: Returns of Alrajhi Bank with ACF and PACF of returns

The left-hand panel of Figure 2 shows that the series is now stationary (mean-reverting). The stationarity of the series is formally confirmed by the Augmented Dickey-Fuller (ADF) test, where a p-value of 0.00 indicates to reject the null hypothesis of non-stationarity. The ACF plot also decays exponentially pointing towards the fact that there exists no autocorrelation and the returns series is not stationary.

4.3 Orders of ARIMA parameters:

From Figure 2 both the Autocorrelation Function (ACF) and Partial Autocorrelation Function (PACF), which are commonly known as correlograms, exhibit cut off, in middle and right-hand panels suggest that ARIMA(0, 0, 0) without drift is appropriate for modelling mean for the current study, as all coefficients fall within the statistical boundaries (± 0.055). However, the Automatic ARIMA procedure in EViews suggests ARIMA (1, 0, 1) as an alternative specification. Table 1 presents the two candidate models together with their information criteria and forecasting accuracy measures. Although both models produces relatively similar forecasting accuracy, but ARIMA (1,0,1) showed slightly lowe AIC and BIC values indicating a better overall fit.

Table 1: Comparing Competing ARIMA Models

Model	AIC	BIC	RMSE
ARIMA (0,0,1)	-5.6416	-5.6334	0.014400
ARIMA (1,0,1)	-5.6425	-5.6302	0.014387

ARIMA (1, 0, 1) was selected as the preferred model.

4.4 Nonlinearity of Returns:

The scatter plot of the variable and its lagged values in Figure 3 exhibits a concentration of observations around a central region, with comparatively greater dispersion as the magnitude of the series increases, which suggests that the relationship may not be strictly linear across the entire range of observations.

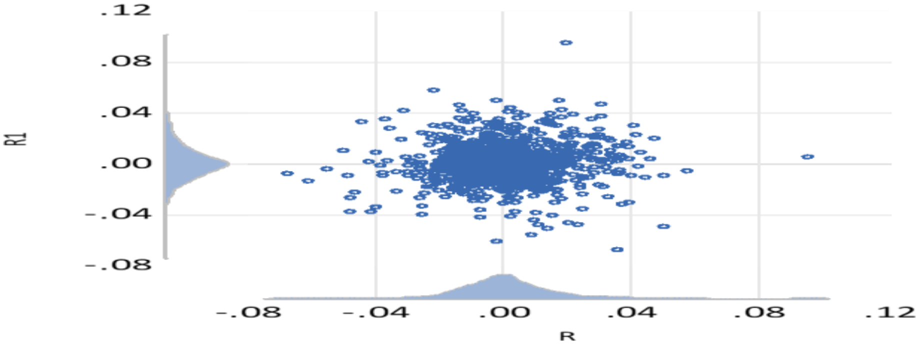

Figure 3: Scatter plot of Returns and Lagged Returns

The Self-Exciting Threshold Autoregressive (SETAR) model is applied to study nonlinearity in returns and the lagged dependent variable is used as the threshold variable. The following equations were obtained using EVIEWS:

$$r_t = \begin{cases} -0.001346 - 0.080786 r_{t-1} + \varepsilon_t, & r_{t-1} < 0.002431498 \\ +0.002847 - 0.023005 r_{t-1} + \varepsilon_t, & r_{t-1} \geq 0.002431498 \end{cases}$$

The results reveal that the threshold separates high and low return market conditions, indicating that stock return dynamics behave differently across market regimes.In addition, using the Bai-Perron sequential threshold testing; the procedure found exactly one statistically significant threshold at (0.0024). Therefore, the model has two regimes; the behavior of the dependent variable differs depending on whether r_{t-1} above or below (0.0024).

4.5 ARCH/GARCH Effect Testing:

The ARCH-LM test was highly significant ($p < 0.001$), confirming volatility clustering and justifying ARCH-family modeling. ARCH model suggested ARCH (3) as the most appropriate model to model volatility but to gain a parsimonious model, the GARCH model was applied to model volatility.

The estimated GARCH (1,1) **variance equation** is given by:

$$h_t = 0.000135 + 0.150000 \varepsilon_{t-1}^2 + 0.599931 h_{t-1}$$

The variance equation indicates that current volatility depends on both previous shocks and past volatility. The positive ARCH and GARCH coefficients confirm the presence of volatility clustering and persistence in Al Rajhi Bank closing price returns.

$\alpha + \beta = 0.150000 + 0.599931 = 0.749931$

Since the sum of the ARCH and GARCH coefficients is less than one but relatively high, volatility shocks tend to persist over time, indicating moderate volatility persistence in the return series.

To check the asymmetric/leverage effect of shocks/innovations, the EGARCH (1, 1) model was applied. The EGARCH (1, 1) estimation reveals a statistically significant asymmetry parameter, γ = -0.061892 with a p-value of 0.0089. The negative sign confirms the presence of the leverage effect, indicating that negative shocks generate higher volatility than positive shocks of the same magnitude. The estimated EGARCH (1,1) **variance equation** is given by:

$$\log(h_t) = -1.792729 + 0.411212 \left| \frac{\varepsilon_{t-1}}{\sqrt{h_{t-1}}} \right| - 0.061892 \left(\frac{\varepsilon_{t-1}}{\sqrt{h_{t-1}}} \right) + 0.826370 \log(h_{t-1})$$

News Impact Curve for the EGARCH (1, 1) model shown in Figure 4 clearly captures the asymmetric/leverage effect of shocks.

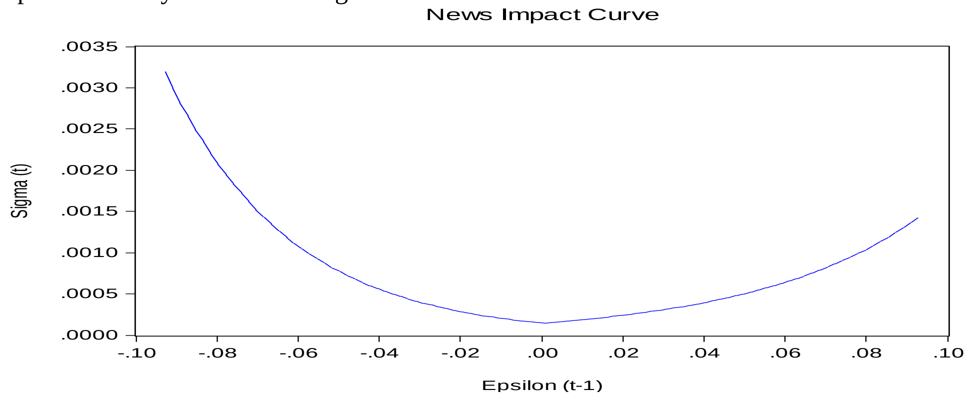

Figure 4: News Impact Curve for EGARCH (1, 1) Model

4.6. Comparison of Volatility Models:

The comparison of ARCH (3), GARCH (1, 1), and EGARCH (1, 1) models in Table 2 indicates a progressive improvement in capturing the volatility dynamics of the stock return series. The ARCH (3) model successfully accounts for short-run volatility clustering by modeling current volatility as a function of past squared shocks; however, its relatively higher AIC and BIC values suggest a less efficient fit. The GARCH (1,1) model improves upon ARCH by incorporating both past shocks and past conditional variance, enabling it to capture volatility persistence more effectively, as reflected in its lower information criteria. Among all models, the EGARCH (1, 1) model provides the best fit, evidenced by the lowest AIC and BIC values. Its logarithmic variance specification and significant negative asymmetry parameter (γ= -0.0619) reveal the presence of a leverage effect, where negative shocks have a stronger impact on future volatility than positive shocks of the same magnitude. Consequently, the EGARCH model is the most appropriate specification for this dataset, as it captures both volatility persistence and asymmetric responses to market information.

Table 2: Comparison of Volatility Models

Model	Mean Equation	Variance Equation	Key Parameters	AIC	BIC	Main Findings
ARCH (3)	ARIMA (1,0,1)	Depends on 3 lagged squared residuals	α_1=0.164, α_2=0.177, α_3=0.171	Higher	Higher	Captures short-run volatility clustering
GARCH (1,1)	ARIMA (1,0,1)	Depends on lagged residuals and lagged variance	α=0.1500, β=0.5999	Lower than ARCH	Lower than ARCH	Captures volatility persistence
EGARCH (1,1)	ARIMA (1,0,1)	Log variance specification	γ= -0.0619, β=0.8264	Lowest	Lowest	Captures the leverage effect and asymmetric volatility

5. Discussion and Conclusion

This study investigated stock return dynamics and volatility behavior for Al Rajhi Bank using ARIMA, SETAR, ARCH, GARCH, and EGARCH models. The findings reveal that stock returns exhibit volatility clustering, nonlinear behavior, and asymmetric volatility effects. ARIMA (1,0,1) was found to be the most suitable model for the conditional mean process, while EGARCH (1,1) provided the best representation of conditional volatility. The significant negative asymmetry parameter confirms the presence of a leverage effect, indicating that adverse market news produces larger increases in volatility than favorable news. These findings contribute to a better understanding of risk dynamics in the Saudi stock market and provide useful information for investors, analysts, and policymakers.

This study contributes to the literature by providing a comprehensive analysis of Saudi banking stock returns using ARIMA, SETAR, ARCH, GARCH, and EGARCH models. Unlike many previous studies that focus solely on volatility modeling, the present research simultaneously examines linear dynamics, nonlinear threshold behavior, and volatility clustering, persistence, and leverage effects. The findings offer practical implications for investors, portfolio managers, and policymakers seeking to understand risk dynamics in the Saudi financial market.

6. Limitations and Future Research

This study focuses on a single financial institution and a specific time period. Future research may extend the analysis to multiple sectors within the Saudi stock market. Alternative asymmetric volatility models such as TGARCH, APARCH, and PGARCH may also be considered. Furthermore, machine learning approaches, including Artificial Neural Networks (ANNs) and Long Short-Term Memory (LSTM) networks, could be explored to improve forecasting performance.

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