COMPARATIVE PERFORMANCE OF HYBRID ARIMA–GARCH-TYPE MODELS IN FORECASTING SAUDI ARAMCO STOCK RETURNS AND VOLATILITY

Abdullah M.Almarashi, Israa O.Albeladi, Kholoud K.Alsharif
Department of Statistics, Faculty of Science, King Abdulaziz University, Jeddah, Saudi Arabia.
Email: abdullah.almarashi.70@gmail.com

ABSTRACT: *This study investigates the forecasting performance and volatility dynamics of Saudi Aramco stock returns from 2021 to 2025 using time-series econometric techniques in EViews. The analysis follows the Box–Jenkins methodology, employing Autoregressive Integrated Moving Average (ARIMA) models to capture return dynamics and Generalized Autoregressive Conditional Heteroskedasticity (GARCH-type) models to model conditional volatility. Stationarity was examined using the Augmented Dickey–Fuller (ADF) test, which indicated that the original stock price series was non-stationary, while the logarithmic return series was stationary and suitable for modeling. Model identification was conducted using correlogram analysis and information criteria. Among the competing specifications, the ARMA (2, 2) model provided the best fit based on Akaike Information Criterion (AIC), Bayesian Information Criterion (BIC), and residual diagnostic tests. . A hybrid ARIMA–GARCH framework was subsequently developed to jointly model the conditional mean and variance processes. To examine volatility behavior, ARCH, GARCH (1, 1), EGARCH (1, 1), and TGARCH models were estimated using the residuals of the mean equation. The results revealed significant volatility clustering, persistence, and time-varying variance in Saudi Aramco stock returns. The TGARCH model outperformed alternative volatility models, indicating the presence of asymmetric effects whereby negative shocks generate greater volatility than positive shocks of similar magnitude. Forecast evaluation results showed that the hybrid model improved predictive accuracy compared with individual models, demonstrating its effectiveness for stock return forecasting and risk assessment in the Saudi equity market.*

Keywords: Saudi Aramco, ARMA, GARCH, TGARCH, Hybrid ARIMA–GARCH, Volatility Forecasting, Stock Returns.

1. INTRODUCTION

Saudi Aramco is one of the world's largest integrated energy and chemical companies and plays a pivotal role in the global energy market. Given its economic significance, understanding the behavior and future movements of its stock prices is of considerable interest to investors, portfolio managers, and financial analysts. This study examines the dynamics of Saudi Aramco stock prices over the period 2021–2025 using daily data obtained from Yahoo Finance. The primary objective is to develop reliable forecasting models for stock returns and volatility through advanced time-series econometric techniques. Time-series forecasting has become an indispensable tool in financial analysis because it enables researchers to identify underlying patterns, trends, and dependencies in historical data for predicting future market behavior. Financial return series are typically characterized by volatility clustering, persistence, and asymmetric responses to market news, making conventional linear models insufficient for capturing their complete dynamics. To address these challenges, this study employs a hybrid ARIMA–GARCH framework that combines the strengths of ARIMA models in modeling the conditional mean process with the ability of GARCH-type models to capture time-varying conditional variance and volatility persistence. Such hybrid models have been shown to improve forecasting performance by simultaneously accounting for linear dependence and heteroskedastic behavior in financial time series (Bollerslev, 1986; Chen et al., 2020).

Accurate volatility forecasting is essential for risk management, portfolio optimization, derivative pricing, and investment decision-making. Therefore, this study evaluates alternative GARCH-type specifications, including symmetric and asymmetric volatility models, to identify the most suitable framework for forecasting Saudi Aramco stock returns and assessing market risk.

2. Literature Review

Forecasting stock prices and modeling volatility have remained central themes in financial econometrics because financial time series often exhibit non-stationarity, volatility clustering, leptokurtosis, and asymmetric responses to market information. Traditional linear time-series models such as the Autoregressive Integrated Moving Average (ARIMA) model are widely used for modeling the conditional mean of financial series, whereas volatility models such as ARCH and GARCH are designed to capture conditional heteroscedasticity and time-varying variance. The integration of these methodologies into hybrid ARIMA–GARCH frameworks has become increasingly popular because it allows simultaneous modeling of both return dynamics and volatility behavior. The pioneering work of George Box and Gwilym Jenkins established the Box–Jenkins methodology, which remains one of the most widely used approaches for forecasting economic and financial time series. Later, Robert F. Engle introduced the ARCH model to account for time-varying volatility, while Tim Bollerslev extended it to the GARCH framework, which has become the standard tool for volatility forecasting.

Several empirical studies have successfully employed ARIMA models for stock-price forecasting. For example, Abbasi et al. (2017) applied an ARIMA model to Flying Cement stock prices and found that ARIMA (1, 2,1) provided the most suitable forecasting performance. The authors also suggested that future research should consider hybrid approaches and volatility models to improve prediction accuracy. Similarly, Almarashi et al. (2023) investigated Saudi Cement Company stock prices using ARIMA and ARCH methodologies. Their findings indicated that ARIMA (0, 1, 1) was the most appropriate model for forecasting stock prices, although ARCH effects were not significant in their sample period. The study demonstrated the usefulness of combining mean and volatility modeling within a financial forecasting framework.

A substantial body of literature has demonstrated the superiority of GARCH models for volatility forecasting. Bollerslev (1986) showed that GARCH models effectively capture volatility persistence by allowing current volatility to depend on both past shocks and past volatility. Engle (1982) similarly established that volatility is not constant over time and should be modeled as a dynamic process. These contributions laid the foundation for modern financial risk modeling.

Recent forecasting studies have increasingly focused on hybrid models. Hybrid ARIMA–GARCH models combine the strengths of ARIMA in modeling linear dependence and the strengths of GARCH in modeling volatility clustering. Such models are particularly useful because financial returns often exhibit both serial dependence and conditional heteroscedasticity. Xiao et al. (2022) demonstrated the continued relevance of ARIMA-based forecasting approaches in financial markets and emphasized the importance of integrating multiple modeling frameworks to improve predictive performance. For further discussion, see Almarashi et al (2024). Despite the extensive literature on ARIMA and GARCH models, relatively few studies have focused specifically on Saudi Aramco stock prices. Most existing research has concentrated on broad stock-market indices, banking stocks, energy commodities, or cement-sector stocks. Furthermore, several studies have ignored asymmetric volatility effects despite strong evidence that negative information often generates greater volatility than positive information.

Consequently, there remains a need for a comprehensive investigation of Saudi Aramco stock returns using a hybrid ARIMA–GARCH framework that incorporates asymmetric volatility models such as EGARCH and TGARCH. According to Hyndman and Athanasopoulos (2021), no single forecasting model is universally optimal for all datasets; therefore, combining complementary models may enhance forecasting performance. In financial applications, this principle motivates the use of hybrid ARIMA–GARCH models that capture both return dynamics and volatility behavior. Such an approach is expected to provide a more accurate representation of stock-return dynamics, volatility persistence, and leverage effects within the Saudi equity market.

2.1. Research Questions

a. Is the Saudi Aramco stock-return series stationary over the study period?
b. Which ARIMA/ARMA model provides the best representation of the conditional mean process of Saudi Aramco stock returns?
c. Do Saudi Aramco stock returns exhibit ARCH effects and volatility clustering?
d. Which volatility model among ARCH, GARCH, EGARCH, and TGARCH provides the best fit for Saudi Aramco stock returns?
e. Is there evidence of asymmetric volatility (leverage effects) in Saudi Aramco stock returns?
f. Does the hybrid ARIMA–GARCH model provide superior forecasting performance compared with standalone ARIMA and GARCH models?
g. What is the degree of volatility persistence in Saudi Aramco stock returns?

2.2. Research Hypotheses

H_1: Saudi Aramco stock returns exhibit significant ARCH effects and volatility clustering.

H_2: Saudi Aramco stock returns exhibit significant persistence in conditional volatility.

H_3: Positive and negative shocks have asymmetric effects on the volatility of Saudi Aramco stock returns, with negative shocks exerting a greater impact than positive shocks of equal magnitude.

H_4: There are significant differences in the forecasting performance of ARCH, GARCH, EGARCH, and TGARCH models for Saudi Aramco stock returns.

H_5: The hybrid ARIMA–GARCH model provides significantly better forecasting accuracy than standalone ARIMA or GARCH-type models for Saudi Aramco stock returns.

3. METHODOLOGY

3.1 Box-Jenkins Methodology

This study employs the Box-Jenkins (BJ) methodology to model and forecast Saudi Aramco stock returns. The Box-Jenkins approach consists of three main stages: model identification, parameter estimation, and diagnostic checking (Box et al., 2015). Stationarity of the series is first examined using the Augmented Dickey-Fuller (ADF) test. Once stationarity is confirmed, autocorrelation function (ACF) and partial autocorrelation function (PACF) plots are used to identify appropriate model orders. Competing models are then evaluated using Akaike Information Criterion (AIC) and Schwarz Information Criterion (BIC), followed by residual diagnostic tests to ensure model adequacy.

3.2 Data Description

The data used in this study consists of **daily closing stock prices of Saudi Aramco (Saudi Arabian Oil Company)**, one of the world's largest integrated energy and petrochemical firms. The dataset covers the period from **1 March 2021 to 31 December 2025** and comprises **1,248 observations**. The data was obtained from **Yahoo Finance**, a widely used financial data repository. Daily logarithmic returns are computed from closing prices and used for all econometric modeling and forecasting analysis.

3.3 ARIMA Model

The Autoregressive Integrated Moving Average (ARIMA) model is used to capture the conditional mean dynamics of stock returns. Since financial price series are typically non-stationary, log returns are used to ensure stationarity. The general ARIMA (p, d, q) representation is:

$$\phi(B)(1-B)^d Y_t = \theta(B)\varepsilon_t$$

Where Y_t is the return series, B is the backshift operator, and ε_t is a white-noise error term.

Model parameters are estimated using Maximum Likelihood Estimation (MLE), and model selection is based on AIC, BIC, and diagnostic checking.

3.4 ARCH/GARCH Models

Financial return series often exhibit volatility clustering, where periods of high volatility are followed by high volatility. The presence of conditional heteroscedasticity is tested using the ARCH-LM test applied to ARIMA residuals.

The ARCH (q) model is defined as:

$$h_t = \omega + \sum_{i=1}^{q} \alpha_i \, \varepsilon_{t-i}^2$$

The GARCH (1, 1) model is given by:

$$h_t = \omega + \alpha\varepsilon_{t-1}^2 + \beta h_{t-1}$$

Where h_t represents conditional variance.

3.5 Hybrid ARIMA-GARCH Model

The hybrid ARIMA-GARCH model integrates two complementary frameworks. The ARIMA model captures the conditional mean process, while the GARCH model captures conditional variance based on ARIMA residuals.

Mean equation:

$$Y_t = \mu_t + \varepsilon_t$$

Variance equation:

$$h_t = \omega + \alpha\varepsilon_{t-1}^2 + \beta h_{t-1}$$

This hybrid structure allows simultaneous modeling of return dynamics and volatility clustering, improving forecasting performance compared to standalone models.

3.6 Modeling Asymmetric Shocks: EGARCH and TGARCH Models

Financial markets often react asymmetrically to positive and negative shocks. To capture this, EGARCH and TGARCH models are employed.

EGARCH Model

The EGARCH model proposed by Daniel B. Nelson (1991):

$$\ln(h_t) = \omega + \beta\ln(h_{t-1}) + \alpha\left|\frac{\varepsilon_{t-1}}{\sqrt{h_{t-1}}}\right| + \gamma\left(\frac{\varepsilon_{t-1}}{\sqrt{h_{t-1}}}\right)$$

TGARCH / GJR-GARCH Model

The TGARCH model developed by Lawrence R. Glosten, Ravi Jagannathan, and it captures leverage effects (Glosten et al., 1993):

$$h_t = \omega + \alpha\varepsilon_{t-1}^2 + \gamma I_{t-1}\varepsilon_{t-1}^2 + \beta h_{t-1}$$

Where $I_{t-1} = 1$ for negative shocks and 0 otherwise.

A significant positive γ indicates that negative shocks increase volatility more than positive shocks.

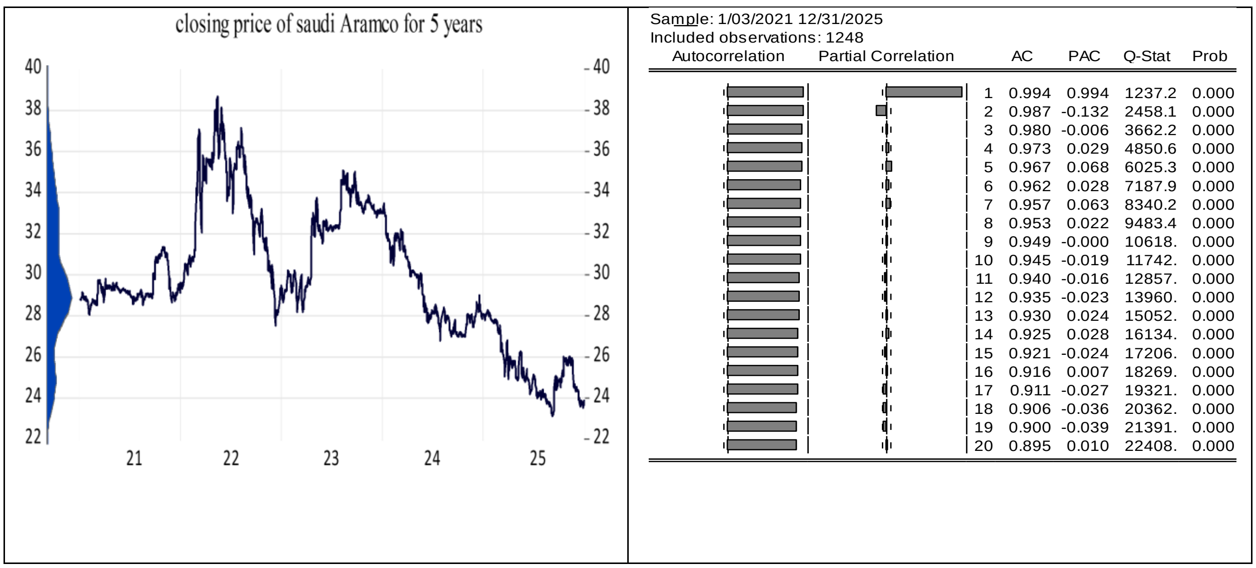
3.7 Model Selection and Evaluation Criteria

Competing time series models in this study are evaluated using standard model selection and goodness-of-fit criteria, including the Akaike Information Criterion (AIC), Schwarz/Bayesian Information Criterion (BIC), and Adjusted R-squared. AIC and BIC are primarily used to compare both nested and non-nested ARIMA/GARCH-type models, where lower values indicate a better balance between model fit and parsimony. Adjusted R-squared is also considered to assess the explanatory power of alternative mean specifications while penalizing the inclusion of unnecessary regressors. The preferred model is selected based on the simultaneous achievement of lower AIC and BIC values, higher adjusted R-squared, and satisfactory residual diagnostics, including the absence of autocorrelation and remaining ARCH effects.

4. RESULTS

4.1. Checking Stationarity

For applying linear time series the first prerequisite is to check for stationarity of the under study series. Stationarity can be tested through the Unit root test and visually by observing the correlogram. The Augmented Dickey–Fuller (ADF) test yields a test statistic of **−1.481947** with an associated **p-value of 0.5427**, which is substantially higher than the conventional significance levels (1%, 5%, and 10%). Therefore, the series is **non-stationary in its level form**, meaning it exhibits a time-dependent mean and possibly stochastic trends.



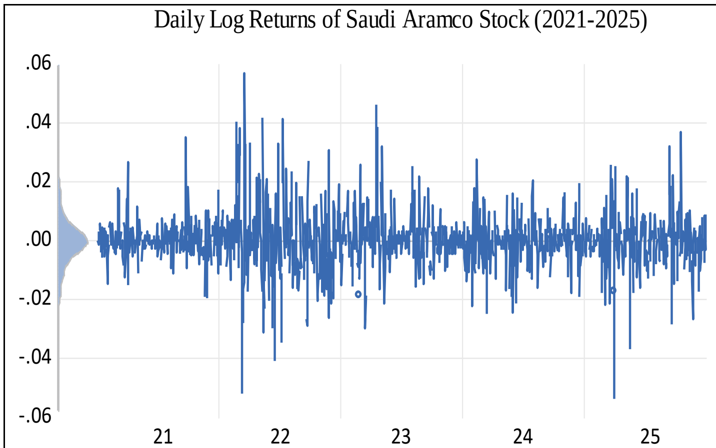
	AC	PAC	Q-Stat	Prob
1	0.994	0.994	1237.2	0.000
2	0.987	-0.132	2458.1	0.000
3	0.980	-0.006	3662.2	0.000
4	0.973	0.029	4850.6	0.000
5	0.967	0.068	6025.3	0.000
6	0.962	0.028	7187.9	0.000
7	0.957	0.063	8340.2	0.000
8	0.953	0.022	9483.4	0.000
9	0.949	-0.000	10618.	0.000
10	0.945	-0.019	11742.	0.000
11	0.940	-0.016	12857.	0.000
12	0.935	-0.023	13960.	0.000
13	0.930	0.024	15052.	0.000
14	0.925	0.028	16134.	0.000
15	0.921	-0.024	17206.	0.000
16	0.916	0.007	18269.	0.000
17	0.911	-0.027	19321.	0.000
18	0.906	-0.036	20362.	0.000
19	0.900	-0.039	21391.	0.000
20	0.895	0.010	22408.	0.000

Figure 1: Closing Prices and Correlogram for Saudi ARAMCO stock prices (5 Years)

This conclusion is further supported by the strong autocorrelation observed in the correlogram shown in the right pane of Figure 1, where the autocorrelation coefficients remain very high and decay slowly across multiple lags, a typical pattern of non-stationary time series. Overall, combining the ADF test results and correlogram behavior, the evidence strongly suggests that the series contains a unit root and is therefore non-stationary. This justifies the transformation of the data into logarithmic returns before applying ARIMA–GARCH modeling.

4.2. Stationarizing the Series (Correlogram, Unit Root Test)

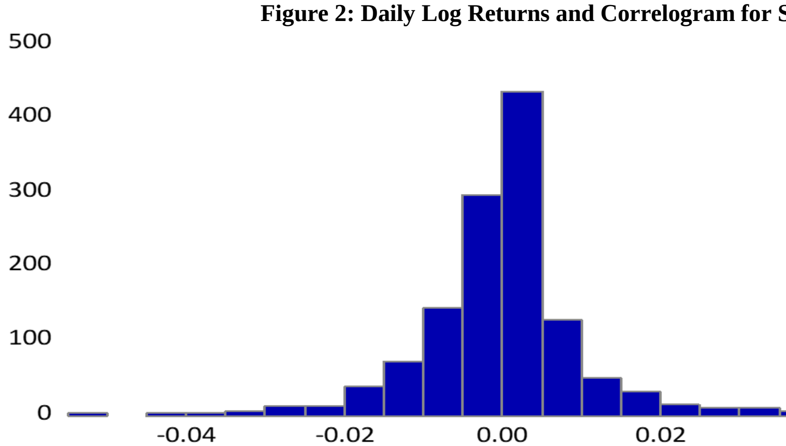
Daily log returns are not stationary series and Correlogram in the right pane of figure shows stationarity. In Figure 3 high value of skewness 8.02777 points towards the fact that the series is heavy tailed and warrants the use of ARCH/GARCH models.



Sample (adjusted): 1/04/2021 12/31/2025
Included observations: 1247 after adjustments

Lag	AC	PAC	Q-Stat	Prob
1	0.140	0.140	24.410	0.000
2	0.025	0.006	25.216	0.000
3	-0.045	-0.050	27.747	0.000
4	-0.077	-0.066	35.180	0.000
5	-0.053	-0.032	38.675	0.000
6	-0.100	-0.090	51.135	0.000
7	-0.047	-0.027	53.853	0.000
8	-0.003	0.002	53.862	0.000
9	0.017	0.005	54.225	0.000
10	0.030	0.011	55.368	0.000
11	0.047	0.031	58.124	0.000
12	-0.013	-0.035	58.343	0.000
13	-0.034	-0.034	59.817	0.000
14	0.017	0.033	60.172	0.000
15	0.009	0.010	60.264	0.000
16	0.017	0.014	60.615	0.000
17	0.056	0.058	64.528	0.000
18	0.073	0.061	71.236	0.000
19	0.010	-0.013	71.375	0.000
20	0.007	0.015	71.443	0.000

Figure 2: Daily Log Returns and Correlogram for Saudia ARAMCO (5 Years)



Series: discribtive RETURN of the closing price of Saudi Aramco over 5 year
Sample 1/03/2021 12/31/2025
Observations 1247

Mean	-0.000152
Median	0.000000
Maximum	0.056464
Minimum	-0.053898
Std. Dev.	0.009658
Skewness	0.303120
Kurtosis	8.027740
Jarque-Bera	1332.508
Probability	0.000000

Figure 3 : Histogram and Descriptive Statistics for Daily Log Returns of Saudi ARAMCO

4.3. Identifying the order of parameters for the ARIMA and Comparing Competing models

From figure, the correlogram after taking the log return of the closing price of the Saudi Aramco Company. We have cut off from the PACF after lag 1 for AR and ACF. We see the spikes cut off after lag 1 of MA, and there is no seasonality over all lags. Using the autoarima function of the EVIEWS program, three models were selected and compared based on AIC and BIC. ARIMA (2, 1, 2) is the most appropriate model for forecasting the mean of Saudi ARAMCO stock prices.

Table1 : Comparison of Competing ARIMA models

Models	Parameters signification	AIC	BIC	Adjusted R^2
ARIMA (1,1,1)	Non of the parameter is significant	-6.456087	-6.439636	0.01956
ARIMA (2, 1, 2)	All parameters are significant at 0.05	-6.46629	-6.44161	0.03269
ARIMA (4,1,4)	All parameters are significant at 0.05	-6.46488	-6.42375	0.03758

The output for the selected ARIMA (2,1,2) model is shown in table 2

Table2 : Estimated ARIMA (2, 1, 2) Model Parameters

ARIMA(2,1,2) is mathematically expressed as:

Dependent Variable: RETURN
Method: ARMA Maximum Likelihood (OPG - BHHH)
Date: 05/19/26 Time: 08:26
Sample: 1/04/2021 12/31/2025
Included observations: 1247
Convergence achieved after 32 iterations
Coefficient covariance computed using outer product of gradients

Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	-0.000153	0.000255	-0.600928	0.548
AR(1)	1.375465	0.091200	15.08184	0.000
AR(2)	-0.608357	0.088381	-6.883335	0.000
MA(1)	-1.250318	0.101819	-12.27987	0.000
MA(2)	0.462813	0.100651	4.598183	0.000
SIGMASQ	9.02E-05	2.10E-06	42.89438	0.000

R-squared	0.032668	Mean dependent var	-0.00015
Adjusted R-squared	0.028771	S.D. dependent var	0.00965
S.E. of regression	0.009519	Akaike info criterion	-6.46628
Sum squared resid	0.112437	Schwarz criterion	-6.44161
Log likelihood	4037.730	Hannan-Quinn criter.	-6.45701
F-statistic	8.382087	Durbin-Watson stat	1.99445
Prob(F-statistic)	0.000000		

Inverted AR Roots	.69-.37i	.69+.37i
Inverted MA Roots	.63-.27i	.63+.27i

$$r_t = (-0.000153) + 1.375465 r_{t-1} + (-0.608357) r_{t-2} + \varepsilon_t + (-1.250318)\varepsilon_{t-1} + 0.462813\varepsilon_{t-2}$$

4.4. ARCH/GARCH Modelling

The return series in Figure 4 exhibits periods of relatively low volatility followed by periods of heightened volatility, indicating the presence of **volatility clustering.** This pattern suggests that large changes in returns tend to be followed by large changes, while small changes tend to be followed by small changes, thereby justifying the use of a standard GARCH model to capture time-varying volatility. . Furthermore, the unequal magnitude and frequency of positive and negative return shocks suggest the possible presence of **asymmetric volatility**, where unfavorable news may have a stronger impact on future volatility than favorable news of similar magnitude

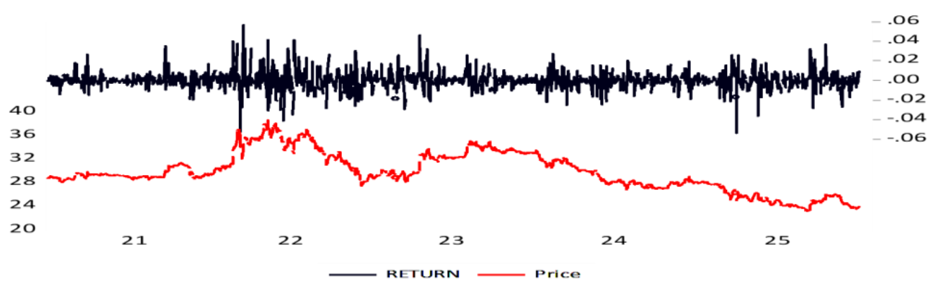


Figure 4: Graph showing Return Series and Orginal Stock Prices

.

This provides additional justification for employing asymmetric GARCH variants such as EGARCH and GJR-GARCH/TGARCH, which are specifically designed to capture leverage effects and asymmetric responses of volatility to market shocks.

Before estimating ARCH models, first test for the possible presence of ARCH (q) effects using the Lagrange Multiplier (LM) test in EViews to test heteroscedasticity of the residuals with the Null hypothesis: *There is no evidence of ARCH effects up to the specified lag*. The test statistic has a p-value =0.000, hence the evidence of the existence of the ARCH effect is supported.

Test statistic $LM = (T - q)R^2$

T=sample size; q = order of the ARCH model;

R^2=coefficient of determination of the ARCH model

The estimated **ARIMA(2,1,2)–ARCH(5)** model suggests that current stock return movements are significantly influenced by past returns and previous shocks, indicating strong temporal dependence in the series. The statistically significant coefficients confirm that the model adequately captures the conditional mean and short-run volatility dynamics. However, a notable limitation of the ARCH (5) specification is the large number of variance parameters required to model volatility, which reduces model parsimony and may lead to estimation inefficiency.

Table3 : Heteroskedasticity LM ARCH test

Heteroskedasticity Test: ARCH

F-statistic	33.64388	Prob. F(1,1244)	0.0000
Obs*R-squared	32.81061	Prob. Chi-Square(1)	0.0000

Test Equation:
Dependent Variable: RESID^2
Method: Least Squares
Date: 05/21/26 Time: 22:36
Sample (adjusted): 1/05/2021 12/31/2025
Included observations: 1246 after adjustments

Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	7.56E-05	6.90E-06	10.95631	0.0000
RESID^2(-1)	0.162264	0.027975	5.800334	0.0000

R-squared	0.026333	Mean dependent var	9.02E-05
Adjusted R-squared	0.025550	S.D. dependent var	0.000230
S.E. of regression	0.000227	Akaike info criterion	-13.94404
Sum squared resid	6.39E-05	Schwarz criterion	-13.93581
Log likelihood	8689.135	Hannan-Quinn criter.	-13.94094
F-statistic	33.64388	Durbin-Watson stat	2.023517
Prob(F-statistic)	0.000000		

Since financial volatility is typically persistent over time, it is more appropriate to employ a **GARCH model**, which incorporates both past squared residuals and past conditional variances in a more compact framework. By modeling volatility persistence directly, the GARCH specification often achieves superior forecasting performance with fewer parameters, making it a more efficient and theoretically appealing approach for analyzing financial return volatility.**4.4.1. GARCH Equation:**

General formula of GARCH (1, 1) represented as:

$h_t = \omega + \alpha\varepsilon_{t-1}^2 + \beta h_{t-1}^2$

$h_t = (\varepsilon_t | t{-}1)$

ω = constant variance

α = ARCH effect

β = GARCH effect

The estimated GARCH (1, 1) variance model is:

$$= 0.00000323 + 0.146299\varepsilon_{t-1}^2 + 0.830344 h_{t-1}^2$$

h_t

The ARCH Effect (α = 0.146): Represents the short-term impact of innovations, indicating that recent unexpected news and price shocks instantly accelerate current volatility. **The GARCH Effect (β = 0.830):** Shows that past conditional variance plays a fundamental role in determining current risk. **Volatility Clustering Analysis:** The total volatility persistence is captured by summing the ARCH and GARCH coefficients:

$\alpha + \beta \approx 1$

0.146299 + 0.830844 = 0.977

The news impact curve in the figure shows that the positive and negative shocks have symmetrical impact, but if we look at Figure 4 it was not symmetrical. So now we proceed further to use EGARCH and TGARCH models.

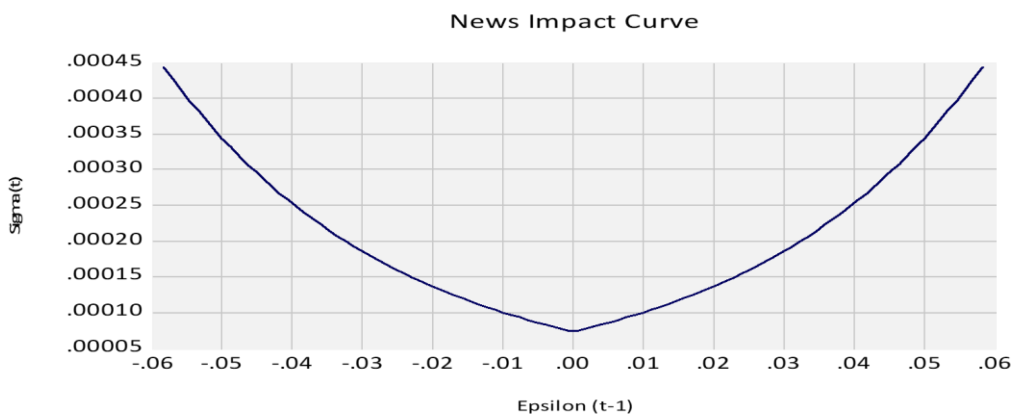


Figure 5: Graph showing Return Series and Orginal Stock Prices

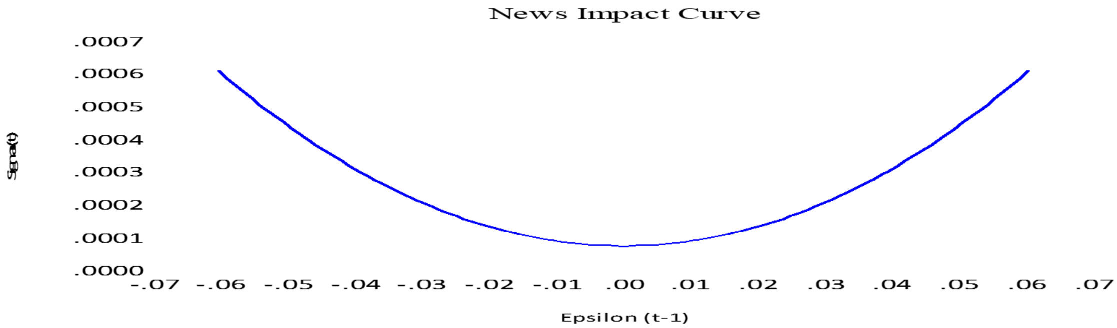


Figure 6: News Impact Curve for EGARCH Shocks

4.4.2. EGARCH Model

By using EViews, we obtained EGARCH (1, 1). The mathematical expression of EGARCH is:

$$log(h_t) = -0.767716 + 0.294826|\frac{\varepsilon_{t-1}}{\sqrt{h_{t-1}}}| + 0.940793log(h_t)$$

The variance equation EGARCH (1, 1) shows that volatility is highly persistent and depends on two main components: the number of past shocks and past volatility. The significant positive coefficient on the lagged log variance (β) is **0.940793**, which indicates strong volatility persistence. This means that shocks of volatility decay very slowly over time. The absolute standardized residual term (α)is **0.294826** and captures the size effect. This implies that large shocks, even so of their direction, significantly increase future volatility.

Because this specification lacks an unbracketed asymmetry term, the model treats positive and negative shocks symmetrically, the leverage effect is absent. All estimated parameters in the variance equation are statistically significant. The model strongly confirms the presence of volatility in Aramco returns. So, the Durbin-Watson test is equal to 1.86 and proves the model is free from autocorrelation, validating its reliability for forecasting long risk regimes. The News Impact Curve (NIC) in Figure 6 depicts a symmetric pattern for positive and negative shocks.

4.4.3. TGARCH / GJR-GARCH Model

The mathematical model of TGARCH (1, 2)

$$h_t = 0.0000280 + 0.128099\varepsilon_{t-1}^2 + 0.166388\varepsilon_{t-1}^2 I_{t-1} - 0.181904\varepsilon_{t-2}^2 I_{t-2} + 0.857258h_{t-1}$$

h_t: Current volatility

h_{t-1}: Past volatility

I_{t-1} *and* I_{t-2}: leverage effect, $I_t = \begin{cases} 1 & if\ the\ shoch\ is\ negative \\ 0 & otherwise \end{cases}$

ε_{t-1}^2 *and* ε_{t-2}^2: Past shocks

The threshold parameter (γ_1 = 0.166388) is positive and statistically significant p-value (0.0117) is less than significant level 0.05 providing strong confirmation of a leverage effect. This implies that negative shocks have a greater stability impact on Aramco conditional variance than positive shocks of equal value. The model has the minimimum Akaike Information Criterion AIC (-6.683661), making it more appropriate for risk forecasting. The News mpact Curve in Figure 7 confirms a significant asymmetric leverage effect, where the negative shocks induce significantly higher volatility in Saudi ARAMCO returns than positive shocks.

The comparative model results in Table 4 indicate that all three specifications—GARCH, EGARCH, and GJR-GARCH/TGARCH—perform closely in terms of information criteria and log-likelihood values. Based on AIC, the GJR-GARCH/TGARCH model provides the best fit (−6.683661), followed by EGARCH (−6.67913) and standard GARCH (−6.678474), suggesting a slight advantage for models that capture asymmetry in volatility.

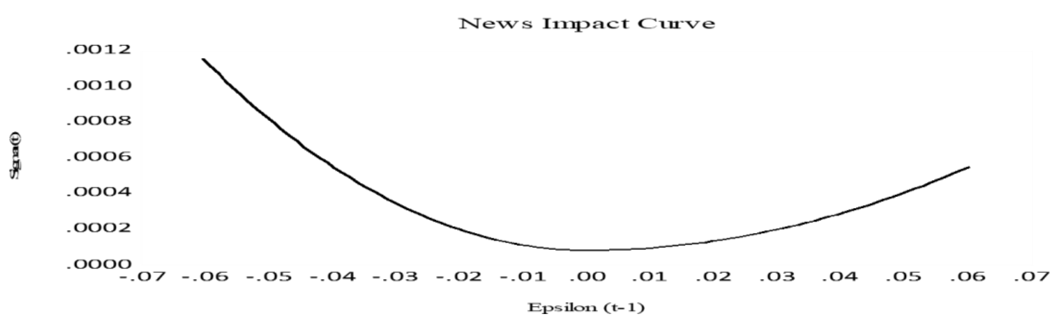


Figure 7: Ne ws Impact Curve for TGARCH Shocks

Table 4: Comparison of GARCH and its Variants

Compared	GARCH	EGARCH	GJR-GARCH/TGARCH
AIC	-6.678474	-6.67913	-6.683661
BIC	-6.645530	-6.64676	-6.642481
Log-likelihood	4165.350	4166.121	4170.579

However, BIC favors EGARCH (−6.646769), with GJR-GARCH/TGARCH performing marginally worse, indicating some sensitivity to penalty adjustments for model complexity. In terms of log-likelihood, GJR-GARCH/TGARCH achieves the highest value (4170.579), followed by EGARCH (4166.121) and GARCH (4165.350), implying better goodness-of-fit for the asymmetric specification. Overall, while differences are small, the results suggest that asymmetric GARCH-type models—particularly GJR-GARCH/TGARCH—provide a slightly improved representation of the volatility process compared to the symmetric GARCH model.

5. DISCUSSION AND CONCLUSION

This study employed both linear and nonlinear time series models to analyze and forecast Saudi Aramco stock returns over the period 2021–2025 using EViews. The empirical findings confirm that the original price series is non-stationary, while the logarithmic return series is stationary after transformation, validating its suitability for econometric modeling. Using the Box–Jenkins framework, several ARIMA specifications were estimated, with the ARIMA (2, 2) model emerging as the best-fitting mean equation based on AIC, BIC, RMSE, and diagnostic checks.

The volatility analysis clearly indicates the presence of ARCH effects and strong volatility clustering, implying that conditional variance is time-varying and cannot be adequately captured by constant-variance models. Among the competing volatility specifications, GARCH-type models significantly outperformed simpler ARCH formulations, while asymmetric models further improved model fit. In particular, the TGARCH model provided the best overall performance based on information criteria and log-likelihood values. It successfully captured the leverage effect, confirming that negative shocks have a greater impact on volatility than positive shocks of the same magnitude. This asymmetric behavior was also supported, though less strongly, by the EGARCH specification.

A key contribution of this study is the implementation of a *hybrid ARIMA–GARCH framework*, which integrates ARIMA-based mean modeling with GARCH-type volatility estimation. The results demonstrate that this hybrid structure provides a more comprehensive representation of financial time series by jointly capturing linear dependencies in returns and nonlinear volatility dynamics. Overall, the hybrid model outperformed standalone ARIMA and single-equation volatility models in terms of forecasting accuracy and residual diagnostics, highlighting its practical superiority for financial forecasting.

The empirical results are consistent with prior studies in financial econometrics, which consistently report volatility clustering, persistence, and asymmetric responses in stock markets, particularly in energy-sector equities. Similar findings have been documented in GARCH-based studies of emerging and developed markets, confirming that financial returns generally exhibit leverage effects and nonlinear volatility structures.

From a policy and investment perspective, the results imply that investors and risk managers should incorporate volatility dynamics into decision-making processes rather than relying solely on mean-based forecasts. The strong evidence of asymmetric volatility suggests that negative shocks significantly increase risk exposure, especially during periods of market stress or oil price fluctuations. Therefore, portfolio managers should adopt dynamic hedging strategies, maintain diversified portfolios, and rely on hybrid ARIMA–GARCH forecasts for improved risk assessment and investment planning in the Saudi equity market.

6. REFERENCES

[1]. Abbasi, U., Almarashi, A. M., Khan, K., & Hanif, S. (2017). *Forecasting Cement Stock Prices Using an ARIMA Model: A Case Study of Flying Cement Industry*. Science International Lahore

[2]. Almarashi, A. M., Alqurashi, R. S., Alshammari, A. F., Almalki, A. H., & Khan, K. (2023). *Modelling Mean and Volatility of Cement Stocks: A Case Study of Saudi Cement Company*. Advances and Applications in Statistics, 91(1), 99-109.

[3]. Box, G. E., Jenkins, G. M., & Reinsel, G. C. (2015). *Time series analysis: forecasting and control*. John Wiley & Sons.

[4]. Bollerslev, T. (1986). Generalized autoregressive conditional heteroskedasticity. *Journal of Econometrics*, 31(3), 307-327.

[5]. Engle, R. F. (1982). Autoregressive Conditional Heteroscedasticity with Estimates of the Variance of United Kingdom Inflation. *Econometrica*, 50(4), 987–1007.

[6]. Glosten, L. R., Jagannathan, R., & Runkle, D. E. (1993). On the relation between the expected value and the volatility of the nominal excess return on stocks. *The journal of finance*, 48(5), 1779-1801.

[7]. Nelson, D. B. (1991). Conditional Heteroskedasticity in Asset Returns: A New Approach. *Econometrica*, 59(2), 347–370.

[8]. Chen, S., Chen, C. Y.-H., & Härdle, W. K. (2020). A First Econometric Analysis of the CRIX Family. *Econometrics and Statistics*.

[9]. Xiao, R., Feng, Y., Yan, L., & Ma, Y. (2022). Predict Stock Prices with ARIMA and LSTM.

[10]. Hyndman, R. J., & Athanasopoulos, G. (2021). Forecasting: Principles and Practice.