A NEW SUBCLASS OF BI-UNIVALENT FUNCTIONS

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ABSTRACT. *A new subclass of bi-univalent analytic functions is introduced and discussed. The second and third coefficient bounds are obtained. Two particular cases are deduced. The Fekete-Szego inequality for the subclass is calculated.*

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1 INTRODUCTION

Let A denote the class of all analytic functions which are defined in the unit disc $U = \{z : |z| < 1\}$ and can be written in the form;

$$f(z) = z + \sum_{n=2}^{\infty} a_n z^n, \ a_n \in \mathbb{C}.$$

Let S be the class of all normalized analytic univalent functions in A, and let P be a class of all functions with positive real part for which $\varphi \in P$ if $\varphi(w) = \dfrac{1+w}{1-w}$, $w(0) = 0$, and $|w(z)| < 1$ which maps the unit disk U onto a region starlike with respect to 1 and is symmetric with respect to the real axis. In this article, S^* and K respectively denote the subcasses of starlike and convex functions in S. An analytic function f is subordinate to an analytic function g, written $f \prec g$, if there is an analytic function w with $|w(z)| \le |z|$ such that $f = (g(w))$. If g is univalent, then $f \prec g$ if and only if $f(0) = g(0)$ and

$f(U) \subseteq g(U)$. A function $f \in S_s^*(\alpha)$ is strongly starlike of order $\alpha (0 < \alpha \le 1)$ if $\left| arg\left(\dfrac{zf'(z)}{f(z)}\right) \right| < \dfrac{\alpha\pi}{2}$, for $z \in U$. Alternatively, $f \in S_s^*(\alpha)$ if $\dfrac{zf'(z)}{f(z)} \prec \left(\dfrac{1+z}{1-z}\right)^\alpha$. A subclass $S_s(\varphi)$ of starlike functions with respect to symmetric points satisfies the condition $\dfrac{zf'(z)}{f(z)-f(-z)} \prec \varphi(z)$, for all $z \in U$. And a subclass $K_s(\varphi)$ of S is a convex function with respect to symmetric points satisfies the condition $\dfrac{(zf'(z))'}{(f(z)-f(-z))'} \prec \varphi(z)$, for all $z \in U$.

The Koebe one-quarter theorem [2] ensures that the image of U under every univalent function $f \in A$ contains a disk of radius $1/4$. Thus every univalent function f has an inverse f^{-1} satisfying

$$f^{-1}(f(z)) = z, (z \in U) \quad and$$

$$f^{-1}(f(w)) = w, (|w| < r_0(f), r_0(f) \ge 1/4),$$

where (1.1)

$$f^{-1}(w) = w - a_2 w^2 + (2a_2^2 - a_3)w^3 - \cdots,$$

A function $f \in A$ is said to be bi-univalent in U if both f and f^{-1} are univalent in U. Let Σ denote the class of bi-univalent functions defined in the unit disk U. In 1967, Lewin[1] introduced the class Σ of bi-univalent functions and he proved the second coefficient for a function f in (1.1). Several authors have studied many valuable and interesting results of bi-univalent functions such as Brannan and Taha[7], Ma-Minda[16], Srivastava et al.[4, 5, 19], Frasin and Aouf [3] and others, even with various generalizations as appeared in many literatures and articles[10-18].

Lemma 1.1 *[8] If $p \in P$ such that $Re(p(z) > 0$ and*

$$p(z) = 1 + \sum_{i=2}^{\infty} p_i z^i, \textit{ then } |p_i| \le 2.$$

In this paper, the followings mappings K_0 and K_1 are defined by

$$K_0(f(z)) = \frac{f(z) - f(-z)}{2} = z + \sum_{k=1}^{\infty} a_{2k+1} z^{2k+1},$$

$$K_1(f(z)) = \frac{f'(z) - f'(-z)}{2} = 2\sum_{k=1}^{\infty} k a_{2k} z^{2k-1},$$

And

$$D_\lambda f(z) = f'(z) + \lambda z f''(z)$$

$$= 1 + \sum_{k=1}^{\infty} (k+1)(1+k\lambda) a_{k+1} z^k.$$

Lemma1.2[16].*If $f(z) = 1 + c_1 z + c_2 z^2 + \cdots$ is an analytic function with positive real part in U, then*

$$|c_3 - \nu c_2^2| \le \begin{cases} -4\nu + 2 & if \quad \nu \le 0 \\ 2 & if \quad 0 \le \nu \le 1 \\ 4\nu - 2 & if \quad 1 \le \nu \end{cases}$$

When $\nu > 1$ or $\nu < 0$, the equality holds if and only if $p_1(z) = \frac{1+z}{1-z}$ or one of its rotations. If $0 < \nu < 1$, then the equality holds if and only if $p_1(z) = \frac{1+z^2}{1-z^2}$. If $\nu = 0$, the equality holds if and only if $p_1(z) = (\frac{1+\gamma}{2})\frac{1+z}{1-z} + (\frac{1-\gamma}{2})\frac{1-z}{1+z}$, for $(0 \le \gamma \le 1)$ or one of its rotations. If $\nu = 1$, the equality holds when the reciprocal of one of the functions such that the equality holds in the case of $\nu = 0$. Also the above upper bound is sharp, and it can be improved as follows when $0 < \nu < 1$:

$$|c_3 - \nu c_2^2| + \nu |c_1| \le 2 \quad (0 < \nu \le \frac{1}{2})$$

And $|c_3 - \nu c_2^2| + (1-\nu)|c_1| \le 2, \quad (\frac{1}{2} < \nu < 1)$

within the above operators, we introduce a new generlization of some classes of strongly starlike and strongly convex funtions.

Definition 1.3 . *A function* $f \in A$ *is in the class* $SC(\lambda,\varphi)$ *for* $0 \le \lambda \le 1$ *if*

$$\frac{z(D_\lambda f(z))}{(1-\lambda)K_0(f(z)) + \lambda z K_1(f(z))} \prec \varphi(z).$$

The class $SC(\lambda,\varphi)$ is a generalization of various subclasses of strongly starlike and convex functions with respect to symmetric points, it is easy to note that if $\lambda = 0$ then $SC((0,\varphi)) \equiv S_S(\varphi)$ due to [9] Also, if $\lambda = 1$ then $SC(1,\varphi(z)) \equiv K_S(\varphi)$ due to [9]. Our object of this paper is introducing a new subclass of a function f in the class Σ and finding estimates on the coefficients $|a_2|$ and $|a_3|$ for them.

2 The main results.

Consider the analytic function with positive real part $\varphi \in P$ is given by

$$\varphi(w) = 1 + \varphi_1 z + \varphi_2 z^2 + \dots$$

And the p, q, u *and* v are defined by

$$p(z) = \left(\frac{1+u(z)}{1-u(z)}\right) = 1 + p_1 z + p_2 z^2 + \dots$$

and $q(z) = \left(\frac{1+v(z)}{1-v(z)}\right) = 1 + q_1 z + q_2 z^2 + \dots$

if and only if

$$u(z) = \left(\frac{p(z)-1}{p(z)+1}\right) = \frac{1}{2}\left[p_1 z + (p_2 - \frac{p_1^2}{2})z^2 + \dots\right] \quad \text{and}$$

$$v(z) = \left(\frac{q(z)-1}{q(z)+1}\right) = \frac{1}{2}\left[q_1 z + (q_2 - \frac{q_1^2}{2})z^2 + \dots\right]$$

Then p and q are analytic in U with $p(0) = q(0) = 1$, $Re(p(z), q(z)) > 0$ for $z \in U$ then $|p_i| \le 2$ and $|q_i| \le 2$. Also,

$$\varphi(u(z)) = \varphi\left(\frac{p(z)-1}{p(z)+1}\right) = 1 + \frac{1}{2}\varphi_1 p_1 z$$

and

$$+\frac{1}{2}\left(\varphi_1\left((p_2 - \frac{p_1^2}{2}\right) + \frac{1}{2}p_1^2\varphi_2\right)z^2 + \dots \quad (2.1)$$

$$\varphi(u(w)) = \varphi\left(\frac{q(w)-1}{q(w)+1}\right) = 1 + \frac{1}{2}\varphi_1 q_1 w$$

$$+\frac{1}{2}\left(\varphi_1\left((q_2 - \frac{q_1^2}{2}\right) + \frac{1}{2}q_1^2\varphi_2\right)w^2 + \dots \quad (2.2)$$

Definition 2.1 *A function* $f \in A$ *given by (1.1) is said to be in the subclass* $SC_\Sigma(\lambda,\varphi)$ *for* $0 \le \lambda \le 1$ *if it satisfies*

$$\frac{z(D_\lambda f(z))}{(1-\lambda)K_0(f(z)) + \lambda z K_1(f(z))} \prec \varphi(z),$$

for $z \in U$ (2.3) □□I

and

$$\frac{w(D_\lambda g(w))}{(1-\lambda)K_0(g(w)) + \lambda w K_1(g(w))} \prec \varphi(w),$$

for $w \in U$ (2.4)

where $g(w) = f^{-1}(w) = w - a_2 w^2 + (2a_2^2 - a_3)w^3 - \dots,$

The coefficient bounds for a function f in the class $SC_\Sigma(\lambda,\varphi)$ which is analytic in the open disc U are estimated.

Theorem 2.2 *If a function* $f \in SC_\Sigma(\lambda,\varphi)$ *for* $0 \le \lambda \le 1$, *then*

$$|a_2| \le \sqrt{\frac{\varphi'(0)}{(2+7\lambda) + 2(2\varphi'(0) - \varphi''(0))}} \quad (2.5)$$

and

$$|a_3| \le \frac{\varphi'(0)}{(2+7\lambda)} + \frac{\varphi'(0)}{(2+7\lambda) + 2(2\varphi'(0) - \varphi''(0))} \quad (2.6)$$

Proof. There are three functions p, q *and* $\varphi \in P$ such that

$$\frac{z(D_\lambda f(z))}{(1-\lambda)K_0(f(z)) + \lambda z K_1(f(z))} = (\varphi(p(z)))^\alpha$$

and

$$\frac{w(D_\lambda g(w))}{(1-\lambda)K_0(g(w))+\lambda w K_1(g(w))}=(\varphi(q(w)))^\alpha$$

It follows from (2.1) and (2.2) that

$$2a_2=\frac{1}{2}\varphi_1 p_1 \qquad (2.7)$$

$$(2+7\lambda)a_3=\frac{1}{2}\left(\varphi_1\left(p_2-\frac{p_1^2}{2}\right)+\frac{1}{2}p_1^2\varphi_2\right) \quad (2.8)$$

$$-2a_2=\frac{1}{2}\varphi_1 q_1 \qquad (2.9)$$

and

$$(2+7\lambda)(2a_2^2-a_3)=\frac{1}{2}\left(\varphi_1\left(q_2-\frac{q_1^2}{2}\right)+\frac{1}{2}q_1^2\varphi_2\right) \qquad (2.10)$$

From (2.7) and (2.9) we have

$$p_1=-q_1 \text{ and } 32a_2^2=\varphi_1^2(p_1^2+q_1^2) \qquad (2.11)$$

From (2.8) and (2.10) we get

$$2(2+7\lambda)a_2^2=\frac{1}{2}\left(\varphi_1\left(p_2-\frac{p_1^2}{2}\right)+\frac{1}{2}p_1^2\varphi_2\right)$$

$$=\frac{1}{2}\left(\varphi_1\left(q_2-\frac{q_1^2}{2}\right)+\frac{1}{2}q_1^2\varphi_2\right)$$

And

$$2(2+7\lambda)a_2^2=\frac{1}{2}\left(\varphi_1\left((q_2+p_2)-16a_2^2\right)+16a_2^2\varphi_2\right) \text{Then}$$

$$a_2^2=\frac{\varphi_1(q_2+p_2)}{4(2+7\lambda)+16(\varphi_1-\varphi_2)}$$

Since $|P_2|\le 2$ and $|q_2|\le 2$ By Lemma 1.1, then

$$|a_2|\le\sqrt{\frac{\varphi'(0)}{(2+7\lambda)+2(2\varphi'(0)-\varphi''(0))}}$$

in order to estemiate $|a_3|$, we use the equations (2.8) and (2.9) to have

$$2(2+7\lambda)a_3-2(2+7\lambda)a_2^2$$
$$=\frac{1}{2}\left(\varphi_1\left(p_2-\frac{p_1^2}{2}\right)+\frac{1}{2}p_1^2\varphi_2\right)$$
$$-\frac{1}{2}\left(\varphi_1\left(q_2-\frac{q_1^2}{2}\right)+\frac{1}{2}q_1^2\varphi_2\right)$$

and $p_1^2=q_1^2$, then

$$a_3=\frac{\varphi_1(p_2-q_2)}{4(2+7\lambda)}+a_2^2$$

And

$$|a_3|\le\frac{\varphi'(0)}{(2+7\lambda)}+\frac{\varphi'(0)}{(2+7\lambda)+2(2\varphi'(0)-\varphi''(0))}$$

This complete the proof of the theorem. W

The Fekete-Szego inequality resulte for a univalent normalized functions $f\in SC_\Sigma(\lambda,\varphi)$ is obtained .

Theorem2.3 *Let* $\varphi(w)=1+\varphi_1 z+\varphi_2 z^2+\cdots$ *such that* $\varphi'(0)>0$ *and* $f\in SC_\Sigma(\lambda,\varphi)$ *for* $0\le\lambda\le 1$. *Then*

$$|a_3-\mu a_2^2|\le\begin{cases}\dfrac{\varphi''(0)}{(2+7\lambda)}-\mu\dfrac{(\varphi'(0))^2}{4(2+7\lambda)}, & if \quad \mu\le\varepsilon_1\\ \dfrac{\varphi''(0)}{(2+7\lambda)}, & if \quad \varepsilon_1\le\mu\le\varepsilon_2\\ -\dfrac{\varphi''(0)}{(2+7\lambda)}+\mu\dfrac{(\varphi'(0))^2}{4(2+7\lambda)}, & if \quad \mu\ge\varepsilon_2\end{cases} \text{ where}$$

$$\varepsilon_1=\frac{(\varphi'(0)-\varphi''(0))(2+7\lambda)}{(\varphi'(0))^2} \text{ and}$$

$$\varepsilon_2=\frac{(\varphi'(0)+\varphi''(0))(2+7\lambda)}{(\varphi'(0))^2}$$

Proof:Let $\varphi(u(z))=\xi(z)=1+\xi_1 z+\xi_2 z^2+\cdots$. Then by (2.1)

$$1+\xi_1 z+\xi_2 z^2+\cdots=1+\frac{1}{2}\varphi_1 p_1 z$$
$$+\frac{1}{2}\left(\varphi_1\left((p_2-\frac{p_1^2}{2}\right)+\frac{1}{2}p_1^2\varphi_2\right)z^2+\cdots$$

Therefore $\xi_1=\varphi_1 p_1$ and

$$\xi_2=\frac{1}{2}\left(\varphi_1\left((p_2-\frac{p_1^2}{2}\right)+\frac{1}{2}p_1^2\varphi_2\right).$$

From (2.8) and (2.10) we get

$$a_2=\frac{\varphi_1 p_1}{4(2+7\lambda)} \text{ and}$$

$$a_3=\frac{\varphi_1(p_2-\frac{p_1^2}{2})+\frac{1}{2}p_1^2\varphi_2)}{2(2+7\lambda)}$$

Thus

$$a_3 - \mu a_2^2 = \frac{\varphi_1}{2(2+7\lambda)}(p_2 - \frac{p_1^2}{2})$$
$$+ \frac{\varphi_2 p_1^2}{4(2+7\lambda)} - \mu \frac{p_1^2 \varphi_1^2}{16(2+7\lambda)^2}$$

$$= \frac{\varphi_1}{2(2+7\lambda)}[p_2 - p_1^2(\frac{1}{2}(1 - \frac{\varphi_2}{\varphi_1} - \mu \frac{\varphi_1}{4(2+7\lambda)}))]$$

Therefore

$$a_3 - \mu a_2^2 = \frac{\varphi'(0)}{2(2+7\lambda)}(p_2 - p_1^2 \upsilon)$$

where $\upsilon = \frac{1}{2}(1 - \frac{\varphi''(0)}{\varphi'(0)} - \mu \frac{\varphi'(0)}{4(2+7\lambda)})$.

Assume that $\varepsilon_1 = \frac{(\varphi'(0) - \varphi''(0))(2+7\lambda)}{(\varphi'(0))^2}$

and $\varepsilon_2 = \frac{(\varphi'(0) + \varphi''(0))(2+7\lambda)}{(\varphi'(0))^2}$.

So that, if $\mu \le \varepsilon_1$ then

$$|a_3 - \mu a_2^2| \le \frac{\varphi''(0)}{(2+7\lambda)} - \mu \frac{(\varphi'(0))^2}{4(2+7\lambda)}$$

and if $\varepsilon_1 \le \mu \le \varepsilon_2$ then

$$|a_3 - \mu a_2^2| \le \frac{\varphi''(0)}{(2+7\lambda)}$$

also, if $\mu \ge \varepsilon_2$ then

$$|a_3 - \mu a_2^2| \le -\frac{\varphi''(0)}{(2+7\lambda)} + \mu \frac{(\varphi'(0))^2}{4(2+7\lambda)}$$

Discussing the equality in the above bounds needs to define the following operators:

(i) $\gamma_\psi = \frac{zM'_{\psi_n}(z)}{K_0(M_{\psi_n}(z))} = \psi(z^{n-1}),$

$$M'_{\psi_n}(0) - 1 = 0 = M_{\psi_n}(0).$$

(ii) $\gamma_\sigma = \frac{zJ'_\sigma(z)}{K_0(J_\sigma(z))} = \psi(\frac{z(z+\sigma)}{1+\sigma z}),$

$$J'_\sigma(0) - 1 = 0 = J_\sigma(0) \ \ for \ \ (0 \le \psi \le 1).$$

(iii) $\gamma_{-\sigma} = \frac{zH'_\sigma(z)}{K_0(H_\sigma(z))} = \psi(-\frac{z(z+\sigma)}{1+\sigma z}),$

$$H'_\sigma(0) - 1 = 0 = H_\sigma(0) \ \ for \ \ (0 \le \psi \le 1).$$

where the operators γ_ψ, γ_σ and $\gamma_{-\sigma}$ are in $SC_\Sigma(\lambda, \varphi)$.

By Lemma 1.2, if $\mu = \varepsilon_1$ then the equality holds if and only if $\gamma_\sigma = f$ or one of its rotations. If $\mu = \varepsilon_2$ then the equality holds if and only if $H_\sigma = f$ or one of its rotations. If $\varepsilon_1 < \mu < \varepsilon_2$ then the equality holds if and only if $M_{\psi_3} = f$ or one of its rotations. And if $\mu > \varepsilon_2$ or $\mu < \varepsilon_1$ then the equality holds if and only if $M_{\psi_2} = f$ or one of its rotations. W

In particular, the function φ in the class of strongly starlike functions of order $\alpha (0 < \alpha \le 1)$ can be written

$$\varphi(z) = \left(\frac{1+z}{1-z}\right)^\alpha = 1 + 2\alpha z + 2\alpha^2 z^2 + \ldots$$

Then $\varphi_1 = 2\alpha$ and $\varphi_2 = 2\alpha^2$.

Corollary2.4. *If a function* $f \in SC(\lambda, \left(\frac{1+z}{1-z}\right)^\alpha)$ *for* $0 \le \lambda \le 1$ *and* $0 < \alpha \le 1$ *then*

$$|a_2| \le \sqrt{\frac{2\alpha}{(2+7\lambda) + 8\alpha(1-\alpha^2)}}$$

and $|a_3| \le \frac{2\alpha}{(2+7\lambda)} + \frac{2\alpha}{(2+7\lambda) + 8\alpha(1-\alpha)}$

And for the value of $0 \le \beta < 1$ in the function

$$\varphi(z) = \frac{1 + (1-2\beta)z}{1-z}$$
$$= 1 + 2(1-\beta)z + 2(1-\beta)z^2 + \ldots$$

we have $\varphi_1 = \varphi_2 = 2(1-\beta)$.

Corollary2.5.*If a function* $f \in SC(\lambda, \frac{1+(1-2\beta)z}{1-z})$ *for* $0 \le \lambda \le 1$ *and* $0 \le \beta < 1$ *then*

$$|a_2| \le \sqrt{\frac{2(1-\beta)}{(2+7\lambda)}} \ \ and \ \ |a_3| \le \frac{4(1-\beta)}{(2+7\lambda)}.$$

The last resulte is the Hanklel determinante $H_1(2) = |a_3 - a_2^2|$, Feket-Szegö functional for $\mu = 1$, for $q = 2$ and $n = 1$.

Corollary2.6 *Let* $f \in SC_\Sigma(\lambda, \varphi)$ *for* $0 \le \lambda \le 1$ *and* $\varphi(z) = 1 + \varphi_1 z + \varphi_2 z^2 + \cdots$. *Then*

$$|a_3 - a_2^2| \le \frac{\varphi''(0)}{2+7\lambda}$$

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