SOME NEW TYPES OF PERFECT MAPPINGS

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ABSTRACT. *In this work, we introduce a new kind of perfect mappings, namely j-perfect mappings and j-ω-perfect mappings. Furthermore we devoted to study the relationship between j-perfect mappings and j-ω-perfect mappings. Finally, certain theorems and characterization concerning these concepts are studied; j = θ, δ, α, pre, b, β.*

Keywords: perfect mappings, *j*-perfect mappings, *j*-*ω*-perfect mappings.

1. INTRODUCTION

In 1966 N. Bourbaki [4] defined perfect mappings and he stated and proved several theorems concerning perfect mappings.Through out this work, (G , τ) and (H , σ) stands for topological spaces. Apoint g in G is said to be condensation point of $K \subseteq G$ if every S in τ with $g \in S$, the set $K \cap S$ is uncountable [8]. In 1982 the *ω*-closed set was first exhibiting by H. Z. Hdeib in[8], and he know it a sub set $K \subseteq G$ is called *ω*-closed if it incorporates each its condensation points and the *ω*-open set is the complement of the *ω*-closed set.The *ω*-interior of the set $K \subseteq G$ defined as the union of all *ω*-open sets content in K and is denoted by int$\omega(K)$. A point $g \in G$ is said to θ-cluster points of $K \subseteq G$ if cl$(S) \cap K \neq \varphi$ for each open set S of G containment g. The set of each θ-cluster points of K is called the θ-closure of K and is denoted by cl$\theta(K)$. A subset $K \subseteq G$ is said to be θ-closed[20] if K = cl$\theta(K)$. The complement of θ-closed set is said to be θ-open. A point $g \in G$ is said to θ-ω-cluster points of $K \subseteq G$ if ωcl$(S) \cap K \neq \varphi$ for each ω-open set S of$G$ containment g. The set of each θ-ω-cluster points of K is called the θ-ω-closure of K and is denoted by ωclθ (K). A subset $K \subseteq G$ is said to be θ-ω-closed [20] if $K = \omega$cl$\theta(K)$. The complement of θ-ω-closed set is said to be θ-ω-open. δ-closed [11] if K= cl$\delta(K)$ = {g $\in G$: int(cl $(S)) \cap K \neq \varphi$, $S \in \tau$ and g $\in S$ }. The complement of δ-closed is called δ-open set, δ- ω-closed if K = clδ (K) = {g $\in G$: intω(cl $(S)) \cap K \neq \varphi$, $S \in \tau$ and g $\in S$ }. The complement of δ-ω-closed is called δ-ω-open. A subset $K \subseteq G$ is said to be α-open [12] if $K \subseteq$ int (cl(int (K))), *pre*-open [11] if $K \subseteq$ int(c l (K)), *b*-open [2] if $K \subseteq$ cl(int$(K)) \cup$ int(cl(K)), the regular open [17] (resp. regular closed) if int(cl $(K)) = K$ (resp. cl(int $(K)) = K$, β-open [4] if $K \subseteq$ cl(int(cl (K))). A subset $K \subseteq G$ is said to be α-ω-open [13] if $K \subseteq$ intω(cl(int$\omega(K)$)), *pre*-ω-open [13] if $K \subseteq$ intω(cl (K)), *b*-ω-open[13] if $K \subseteq$ cl(int$\omega(K)) \cup$ intω(cl(K), β-ω-open [13] if $K \subseteq$ cl(intω(cl(K))). Several characterizations of ω-closed sets were provided in [1, 3, 10, 19].

Definition 1.1. A mapping $\lambda : (G , \tau) \rightarrow (H ,\sigma)$ is called continuous [6] (resp., θ-continuous [20], δ-continuous [14], α-continuous [13], *pre*-continuous [11], *b*-continuous [15], β-continuous [4]) if for every an open set T in H, $\lambda^{-1}(T)$ is an open (resp., θ-open , δ-open, α-open, *pre*-open, *b*-open, β-open) set in G.

2. *j*-Perfect Mappings

In this section we defined new types of *j*-perfect mappings and some theorems concerning of them.

Definition 2.1. A mapping $\lambda : G \rightarrow H$ is called supra perfect mapping (shortly *j*-perfect mapping), if it is closed, *j*-continuous, and for every $h \in H$, $\lambda^{-1}(h)$ compact, where j = θ, δ, α, *pre, b*, β.

Remark 2.2. The relation between *j*-perfect mappings are given by the following figure

θ-pm $\Rightarrow$ δ-pm $\Rightarrow$ pm $\Rightarrow$ α-pm $\Rightarrow$ *pre*-pm $\Rightarrow$ *b*-pm $\Rightarrow$ β-pm

Where *j*-pm = *j*-perfect mapping such that $j = \theta, \delta, \alpha$ *pre, b*, β.

In the higher figure the converses be not a right such that the shown by the following examples:

Example 2.3. Let $\lambda : (G , \tau_G) \rightarrow (H, \tau_H)$ be a mapping such that $G = H = \{ u ,v ,w, x \}$, and $\tau_G = \{G , \varphi , \{u\} , \{ v ,w \}, \{ u ,v ,w \}\}$,with τ_H = discrete topology, such that $\lambda (u) = u$, $\lambda (v) = v$, $\lambda (w) =\lambda (x) = w$, let $K= \{ v ,x \}$. Such that K is β-open set and is not *b*-open. Then λ is a β-perfect mapping but it is not *b*-perfect mapping.

Example 2.4. Let $\lambda : (G , \tau_G) \rightarrow (H, \tau_H)$ be a mapping such that $G = H = \{ u ,v ,w \}$, and $\tau_G = \{G , \varphi , \{u\} , \{v\}, \{u , v\} \}$, with $\tau_H = \{H , \varphi , \{u\} , \{v\}, \{v , w\} \ \{u , v \}\}$, such that $\lambda (u) = \lambda (w) = u$, $\lambda (v) = w$, let $K= \{ u, w \}$. Such that K is *b*-open set and is not *pre*-open. Then λ is a *b*-perfect mapping but it is not *pre*-perfect mapping.

Example 2.5. Let $\lambda : (G , \tau_G) \rightarrow (H, \tau_H)$ be a mapping such that $G = H = \{ u ,v \}$, and
$\tau_G = \{G , \varphi \}$,with $\tau_H = \{H , \varphi , \{u\} , \{v \}\}$, such that $\lambda (u) = u$, $\lambda (v) = v$, let $K= \{u \}$. Such that K is *pre*-open set and is not α-open. Then λ is *pre*-perfect mapping but it is not α-perfect mapping.

Example 2.6. Let $\lambda : (G , \tau_G) \rightarrow (H, \tau_H)$ be a mapping such that $G = H = \{ u ,v ,w \}$, and $\tau_G = \{G , \varphi \ \{u\}\}$, and $\tau_H = \{H , \varphi \ \{v\}\}$, such that $\lambda (u) = \lambda (v) = u$, $\lambda (w) =w$, let $K= \{ u, v \}$. Such that K is α-open set and is not open Then λ is α-perfect mapping but it is not perfect mapping.

Example 2.7. Let $\lambda : (G , \tau) \rightarrow (H , \sigma)$ be a mapping such that $G = \{a , b , c \}$, $H = \{1 , 2 \}$, $\tau = \{G , \varphi , \{a\} , \{b\} , \{a , b\}, \{b , c\} \}$, $\sigma = \{ H , \varphi , \{1\}\}$, such that λ (a) = λ (c) = 2 , λ (b) = 1 , let K= {b}. Such that K is open set and is not δ-open. Then λ is perfect mapping but it is not δ-perfect mapping

Example 2.8. Let $\lambda : (G , \tau) \rightarrow (G , \tau)$ be a mapping such that $G = \{ a ,b ,c \}$ $\tau = \{G , \varphi , \{a\} ,\{b\} , \{a ,b \} \}$, such that λ (a) = c , λ (b) = a , λ (c) = b , let K= {b} . Such that K is δ-open set and is not θ-open. Then λ is δ-perfect mapping but λ is not θ-perfect mapping

Theorem 2.9. Let (G , τ) be a regular space, The mapping $\lambda : (G , \tau) \rightarrow (H , \sigma)$ be a
is δ-perfect if and only if it is θ-perfect.

Proof: Let λ be a δ-perfect mapping. It suffices to demonstrated that λ is θ-continuous, let g $\in G$ and let T be an open set containment λ(g) in H. And because of λ is δ-continuous, there is an open set S containment g such that λ (S) ⊆ int(cl(T)). Because of int(cl(T)) ⊆ cl(T), then λ (S) ⊆ int(cl(T)) ⊆ cl(T), then λ (S) ⊆ cl(T), Since the space G is regular space, there is an open set S1 in G such that $g \in S1$ and cl(S1) ⊆ S, so λ (cl(S1)) ⊆ λ (S), It ensue thereupon λ (cl(S1)) ⊆ cl(T). So λ is θ-continuous. Hence λ is θ-perfect mapping.

Definition 2.10. A topological space is called a semi-regular [18] if for each point g of the space and each open set S containment g, there is an open set T such that g $\in T$ ⊆ int(cl(T)) ⊆ S.

Theorem 2.11. Let (G , τ) and (H , σ) be a semi-regular spaces. The mapping $\lambda : (G , \tau) \to (H , \sigma)$ is perfect if and only if it is δ-perfect.

Proof: Let λ be a perfect mapping. It suffices to demonstrated that λ is δ-continuous, let g $\in G$ and let T be an open set containment λ (g) in H. Because of λ is continuous, there is an open set S containment g, such that λ (S) ⊆ T. Because the space G is semi-regular space, there is an open set S1 in G such that $g \in S1$ and T ⊆ int(cl(T)) ⊆ S1, and H is semi-regular space, there is an open set T1 such that λ (g) $\in T1$ and S ⊆ int(cl(S)) ⊆ T1,then it follows that λ (int(cl(S))) ⊆ int(cl(T)). So λ is δ-continuous. Hence λ is δ-perfect mapping.

Theorem 2.12. Let (G , τ) be a regular space. The mapping $\lambda : (G , \tau) \to (H , \sigma)$ is perfect if and only if it is θ-perfect.

Proof: Let λ be a perfect mapping. It suffices to demonstrated that λ is θ-continuous, let g $\in G$ and let T be an open set containment λ (g) in H. Since λ is continuous, there is an open set S containment g, such that λ (S) ⊆ T. Because of T ⊆ cl(T), and G is regular space, there is an open set S1 in G such that $g \in S1$, and cl(S1) ⊆ S, so λ (cl(S1)) ⊆ λ (S), then λ(cl(S1)) ⊆ λ(S) ⊆ cl(T). So λ is θ-continuous. Hence consider λ is θ-perfect mapping.

Definition 2.13. let G be a space and $K \subseteq G$ is called :

1. t-set [16], if int(K) = int cl(K).
2. B-set [16], if $K = S \cap T$; where S is an open set and T is an t-set.
3. tα-set if int(K) = int(cl(int(K))).
4. $B\alpha$-set if $K = S \cap T$; where S is an open set and T is an tα-set.

Definition 2.14. The space (G , τ) is called B-condition (resp., $B\alpha$-condition) if every *pre*-open (resp., α- open) set is B-set (resp., $B\alpha$-set) .

Example 2.15. let G = { a ,b ,c } τ = {G , φ , {a} , {a ,b } }and $K \subseteq G$ such that K = {a ,b }is *pre*-open set ,the space (G , τ) is B-condition.

Example 2.16. let G = { a ,b ,c } τ = {G , φ , {a} } and $K \subseteq G$ such that K = {a }is α-open set, the space (G , τ) is $B\alpha$-condition.

Theorem 2.17. Let the space (G , τ) be Bα-condition. The mapping $\lambda : (G , \tau) \to (H , \sigma)$ is α-perfect if and only if it is perfect.

Proof: Let λ be a α-perfect mapping to prove it is perfect. It suffices to demonstrated that λ continuous, let g $\in G$ and let T be an open set containment λ (g) in H. Because λ is α-continuous, there is an open set S containment g, such that λ(S) ⊆ int(cl(int(T1))). Because of the space G have Bα-condition, there is a subset T1 α-open set in H such that λ(g) $\in T1$ is $B\alpha$-set then int(cl(int($T1$))) ⊆ int($T1$), also int($T1$) ⊆ $T1$. It follows that λ(S) ⊆ T, then λ is continuous. Hence consider λ is perfect mapping.

Definition 2.18. [7] The space (G , τ) is called a door space if each subset of G is open or closed.

Theorem 2.19. Let (G , τ) be a door spaces. The mapping $\lambda : (G , \tau) \to (H , \sigma)$ is b-perfect if and only if it is *pre*-perfect.

Proof: Assume that λ be a b-perfect mapping. It suffices to demonstrated that λ is *pre*-continuous, let g $\in G$ and let T be an open set containment λ (g) in H. λ is b-continuous there is an open set S containment g, such that λ(S) ⊆ int(cl(T1)) ∪ cl(int (T1)). Because G is a door space, there is a subset T1 an open in H, such that λ (g) $\in T1$ and T1 ⊆ int(cl(T1)) ∪ cl(int (T1)), then λ(S) ⊆ T1 also T1 ⊆ int(cl(T1)) Then λ(S) ⊆ int(cl(T1)). So λ is *pre*-continuous. Hence consider λ is *pre*-perfect mapping.

Theorem 2.20. Let (G , τ) be a door space. The mapping $\lambda : (G , \tau) \to (H , \sigma)$ is

(a) *pre*-perfect mapping if and only if it is perfect mapping.

(b) β-perfect mapping if and only if it is b-perfect mapping.

Proof: (a) suppose that λ be a *pre*-perfect mapping. It suffices to demonstrated that λ continuous, let g $\in G$ and let T be an open set containment λ(g) in H. because of λ is *pre*-continuous there is an open set S containment g, such that λ(S) ⊆ int(cl(T1)), and G is a door space, there is a subset T1 an open in H, such that λ(g) $\in T1$, and int(cl(T1)) ⊆ T, Then λ(S) ⊆ int(cl(T1)) ⊆ T. It follows that λ(S) ⊆ T. So λ is continuous. Hence consider λ is perfect mapping.

The same way to show (b).

Theorem 2.21. Let (G , τ) be $B\alpha$-condition. The mapping $\lambda : (G , \tau) \to (H , \sigma)$ is *pre*-perfect if and only if it is α-perfect.

Proof: Let $\lambda : (G , \tau) \to (H , \sigma)$ be *pre*-perfect mapping, to prove it is α-perfect to demonstrated that λ is α-continuous, let g $\in G$ and let T be an open set containment λ(g) in H, such that λ (g) $\in T$. Because of λ is *pre*-continuous, there is an open set S such that λ (S) ⊆ int(cl(T), and int(cl(T) ⊆ T then λ (S) ⊆ T, and G is $B\alpha$-condition there is α-open T1 such that T1 ⊆ int(cl(int (T1)),, then λ (S) ⊆ int(cl(int (T1)). So consider λ is α-perfect mapping.

Theorem 2.22. Let (G , τ) be B-condition. The mapping $\lambda : (G , \tau) \to (H , \sigma)$ is perfect if and only if it is *pre*-perfect.

Proof: (⇒) it is obvious

(⇐) Let λ be a *pre*-perfect mapping to demonstrated it is perfect mapping. It suffices to prove that λ continuous, let g $\in G$ and let T be an open set containment λ (g) in H. Because of λ is *pre*-continuous, there is an open set S containment g, such that λ (S) ⊆ int(cl(T1)), and G is B-condition there is a subset T1 *pre*-open set in H, such that λ (g) $\in T1$, then int(cl(T1)) ⊆ int(T1) and int(T1) ⊆ T1. Then int(cl(T1)) ⊆ T1. It follows that λ (S) ⊆ T1, so λ is continuous. Hence consider λ is perfect mapping.

Definition 2.23. Let $\lambda : (G , \tau) \rightarrow (H ,\sigma)$ be a mapping such that is called B-continuous [16] (resp., Bα-continuous [16]), if for each an open T in H, $\lambda^{-1}(T)$ is an B-set (resp. Bα-set) in G.

Definition 2.24. Let $\lambda : (G , \tau) \rightarrow (H ,\sigma)$ be a mapping such that is called B-perfect (resp. , Bα-perfect) if it is closed, B-continuous(resp., Bα-continuous), and for every $h \in H$ such that $\lambda^{-1}(h)$ compact.

Theorem 2.25. For a mapping $\lambda : (G , \tau) \rightarrow (H ,\sigma)$ the following properties are equipotent :

(a) λ is perfect

(b) λ is *pre*-perfect and B-perfect.

(c) λ is α-perfect and Bα-perfect.

3. Supra ω-Perfect mappings

In this section we defined some new types of j-ω-perfect mappings and we show the relation between them.

Definition 3.1. A mapping $\lambda : (G , \tau) \rightarrow (H ,\sigma)$ is called [7] ω-continuous (resp., j-ω-continuous) if for every $g \in G$ and every open set T of H containing $\lambda(g)$ there exists S an ω-open (resp., j-ω-open)set in H, where $j = \theta, \delta, \alpha, pre, b, \beta$.

Definition 3.2. A mapping $\lambda : G \rightarrow H$ is called ω-perfect, if it is closed, ω-continuous, and for every $h \in H$, $\lambda^{-1}(h)$ compact.

Definition 3.3. A mapping $\lambda : G \rightarrow H$ is called supra ω-perfect mappings (shortly j-ω-perfect mappings) if it is closed, j-ω-continuous, and for every h $\in H$, λ^{-1}(h) compact, where $j = \theta, \delta, \alpha, pre, b, \beta$.

Remark 3.4. The relation between ω-perfect mappings, j-perfect mappings and j- ω-perfect mappings are given by the following figure.

$$\begin{array}{ccc}
\theta\text{-pm} & \Rightarrow & \theta\text{-}\omega\text{-pm}\\
\Downarrow & & \Downarrow\\
\delta\text{-pm} & \Rightarrow & \delta\text{-}\omega\text{-pm}\\
\Downarrow & & \Downarrow\\
\text{pm} & \Rightarrow & \omega\text{-pm}\\
\Downarrow & & \Downarrow\\
\alpha\text{-pm} & \Rightarrow & \alpha\text{-}\omega\text{-pm}\\
\Downarrow & & \Downarrow\\
pre\text{-pm} & \Rightarrow & pre\text{-}\omega\text{-pm}\\
\Downarrow & & \Downarrow\\
b\text{-pm} & \Rightarrow & b\text{-}\omega\text{-pm}\\
\Downarrow & & \Downarrow\\
\beta\text{-pm} & \Rightarrow & \beta\text{-}\omega\text{-pm}
\end{array}$$

Where j-pm = j-perfect mapping, and j-ω-pm = j- ω-perfect mapping, such that $j = \theta, \delta, \alpha, pre, b, \beta$.

In the higher figure the converses be not a right such that the shown by the following examples:-

Example 3.5. Let $\lambda : (G , \tau_G) \rightarrow (H, \tau_H)$ be a mapping such that $G = H = \{ u ,v ,w, x \}$, and $\tau_G =\{G , \varphi , \{u\} , \{ v ,w \}, \{ u ,v ,w \}\}$,with τ_H = discrete topology, such that $\lambda (u) = u$, $\lambda (v) = v$, $\lambda (w) = \lambda (x) = w$, let $K= \{ v ,x \}$. Such that K is β-ω-open set and is not b- ω-open. Then λ is a β-ω-perfect mapping but it is not b-ω-perfect mapping.

Example 3.6. Let $\lambda : (G , \tau_G) \rightarrow (H, \tau_H)$ be a mapping such that $G = H = \{ u ,v ,w \}$, and $\tau_G =\{G , \varphi , \{u\} , \{v\}, \{u , v\} \}$, with $\tau_H =\{H , \varphi , \{u\} , \{v\}, \{v , w\} \ \{u , v \}\}$, such that $\lambda (u) = \lambda (w) = u$, $\lambda (v) = w$, let $K= \{ u, w \}$. Such that K is b-ω-open set and is not *pre*- ω-open. Then λ is a b-ω-perfect mapping but it is not *pre*-ω-perfect mapping.

Example 3.7. Let $\lambda : (G , \tau_G) \rightarrow (H, \tau_H)$ be a mapping such that $G = H = \{ u ,v \}$, and $\tau_G = \{G , \varphi \}$,with $\tau_H =\{H , \varphi , \{u\} , \{v \}\}$, such that $\lambda (u) = u$, $\lambda (v) = v$, let $K= \{u \}$. Such that K is *pre*- ω-open set and is not α- ω-open. Then λ is *pre*- ω-perfect mapping but it is not α-ω-perfect mapping.

Example 3.8. Let $\lambda : (G , \tau_G) \rightarrow (H, \tau_H)$ be a mapping such that $G = H = \{ u ,v ,w \}$, and $\tau_G = \{G , \varphi, \{u\}\}$, and $\tau_H = \{H , \varphi \ \{v\}\}$, such that $\lambda (u) = \lambda (v) = u, \lambda (w) =w$, let $K= \{ u, v \}$. Such that K is α-ω-open set and is not ω-open . Then λ is α-ω-perfect mapping but it is not ω-perfect mapping.

Example 3.9. Let $\lambda : (G , \tau) \rightarrow (H , \sigma)$ be a mapping such that $G = \{a , b , c \}$, $H = \{1 , 2 \}$,and $\tau = \{G , \varphi , \{a\} , \{b\} , \{a , b\}, \{b , c\} \}$, $\sigma = \{ H , \varphi , \{1\}\}$, such that λ (a) = λ (c) = 2 , λ (b) = 1 , let $K= \{ b \}$. Such that K is ω-open set and is not δ-ω-open . Then λ is ω-perfect mapping but it is not δ-ω-perfect mapping

Example 3.10. Let $\lambda : (G , \tau) \rightarrow (G , \tau)$ be a mapping such that $G = \{ a ,b ,c \}$and $\tau = \{G , \varphi , \{a\} ,\{b\} , \{a ,b \} \}$, such that λ (a) = c , λ (b) = a , λ (c) = b , let $K= \{b\}$. Such that K is δ-ω-open set and is not θ-ω-open . Then λ is δ-ω-perfect mapping but it is not θ-ω-perfect mapping.

Definition 3.11. A topological space G is called ω-regular [5] if for every ω-closd set F and every point $g \in$ G − F, there exists disjoint ω-open sets S and T such that $g \in S$ and F ⊆ T.

Theorem 3.12. Let (G , τ) be an ω-regular space. The mapping $\lambda : (G , \tau) \rightarrow (H , \sigma)$ is δ-ω-perfect mapping if and only if it is θ-ω-perfect mapping.

Proof: Let λ be a δ-ω-perfect mapping. It suffices to demonstrated that λ is θ-ω-continuous, let g $\in G$ and let T be an ω-open set containment λ (g) in H. Because of λ is δ-ω-continuous, there is an open set S containment g , such that $\lambda (S) \subseteq \text{int}\omega(\text{cl}(T))$. Because $\text{int}\omega (\text{cl}(T)) \subseteq \text{cl}(T)$, then λ (S) ⊆ intω (cl(T)) ⊆ cl(T), then λ (S) ⊆ cl(T), and G is ω-regular space, there is an ω-open set S1 in G, such that g $\in$ S1 and cl(S1) ⊆ S, so λ(cl(S1)) ⊆ λ(S), It follows that λ (cl(S1)) ⊆ cl(T). So λ is θ-ω-continuous. Hence consider λ is θ-ω-perfect mapping.

Definition 3.13. A topological space is called a semi-ω-regular, if for every point g of the space and every ω-open set S containment g , there is an ω-open set T such that g $\in T \subseteq \text{int}\omega(\text{cl}(T)) \subseteq S$.

Example 3.14. let $\lambda : (G , \tau) \rightarrow (G , \tau)$ be amapping, $G = \{ K, L, M , N \}$ and $\tau = \{G , \varphi , \{K\} ,\{L\} , \{K, L \},\{K, L, N\} \}$, such that $\lambda (K) = \lambda (L) = \lambda (M) =\lambda (N) = K$, and $\{K, L, M\}$ an ω-open but not open, then the space is semi-regular but not semi-ω-regular.

Theorem 3.15. Let (G , τ) and (H , σ) be a semi-ω-regular spaces. The mapping $\lambda : (G , \tau) \rightarrow (H , \sigma)$ is ω-perfect mapping if and only if it is δ-ω-perfect mapping

Proof: Let λ be an ω-perfect mapping. It suffices to demonstrated that λ is δ-ω-continuous, let g $\in G$ and let T be an ω-open set containment λ (g) in H. Because of λ is ω-continuous, there is an open set S containment g, such that $\lambda (S) \subseteq T$. Because of G is a semi-ω-regular space, there is an

ω-open set S1 in G such that $g \in S1$ and $T \subseteq \text{int}\omega(\text{cl}(T)) \subseteq S1$, and H is a a semi-ω-regular space such that $\lambda(\text{int}\omega\,(\text{cl}(S1))) \subseteq T$. Then $\lambda(\text{int}\omega\,(\text{cl}(S1))) \subseteq \text{int}\omega\,(\text{cl}(T))$. Hence λ is δ-ω-continuous. So consider λ is δ-ω-perfect mapping.

Theorem 3.16. Let (G , τ) be an ω-regular space. The mapping $\lambda : (G , \tau) \to (H , \sigma)$ is ω-perfect mapping if and only if it is θ-ω-perfect mapping.

Proof: Let λ be an ω-perfect mapping. It suffices to demonstrated that λ is θ-ω-continuous, let g $\in G$ and let T be an ω-open set containment λ(g) in H. Because of λ is ω-continuous, there is an open set S containment g, such that $\lambda(S) \subseteq T$. Because of $T \subseteq \text{cl}(T)$, and G is ω-regular space,there is an ω-open set $S1$ in G such that $g \in S1$ and $\text{cl}(S1) \subseteq S$, so $\lambda\,(\text{cl}(S1)) \subseteq \lambda(S)$, then $\lambda\,(\text{cl}(S1)) \subseteq \lambda\,(S) \subseteq \text{cl}(T)$. It follows that $\lambda\,(\text{cl}(S1)) \subseteq \text{cl}(T)$. Then λ is θ-ω-continuous. Hence λ is θ-ω-perfect mapping.

Definition 3.17. [13] let G be a space and $K \subseteq G$ is called

(a) An ω-set if $K = S \cap T$; where S is an open set and $\text{int}(T) = \text{int}\omega(T)$

(b) An ω-t-set, if $\text{int}(K) = \text{int}\omega(\text{cl}(K))$.

(c) An ω-B-set if $K = S \cap T$; where S is an open set and T is an ω-t-set.

(d) An ω-$t\alpha$-set, if $\text{int}(K) = \text{int}\omega(\text{cl}(\,\text{int}\omega\,(K))$.

(e) An ω-$B\alpha$-set if $K = S \cap T$; where S is an open set and T is an ω-$t\alpha$-set.

Definition 3.18. [9] Let (G , τ) be topological space ,we called G is ω-condition (resp., ω-B-condition, ω-$B\alpha$-condition) if every ω-open (resp. *pre*-ω-open, α-ω-open) set is ω-set (resp., B-ω-set, ω-$B\alpha$- set).

Theorem 3.19. Let a space (G , τ) be an ω-$B\alpha$-condition.The mapping $\lambda : (G , \tau) \to (H , \sigma)$ is α-ω-perfect if and only if it is ω-Perfect.

Proof: Let $\lambda : (G , \tau) \to (H , \sigma)$ be α-ω-perfect mapping, to prove it is ω-perfect to demonstrated that λ is ω-continuous, let g $\in G$ and let T be an ω-open set containment $\lambda(g)$ in H, such that $\lambda\,(g) \in T1$ and $\text{int}\omega(\text{cl}(\,\text{int}\omega\,(T1)) \subseteq T$, because λ is α-ω-continuous,

, there is an ω-open set S containment g, such that $\lambda(S) \subseteq \text{int}\omega\,(\text{cl}(\text{int}\omega\,(T1)))$. Because of the space G have ω-$B\alpha$-condition, there is a subset $T1$ α-ω-open set in H such that $\lambda(\text{g}) \in T1$ is $B\alpha$-ω-set then $\text{int}\omega\,(\text{cl}(\text{int}\omega\,(T1))) \subseteq \text{int}\omega\,(T1)$, also $\text{int}\omega\,(T1) \subseteq T1$. It follows that $\lambda(S) \subseteq T$, then λ is ω-continuous. Hence consider λ is ω-perfect mapping.

Lemma 3.20. [7] If a space (G , τ) is a door space then every *pre*-ω-open is ω-open.

Theorem 3.21. Let (G , τ) be a door space. The mapping $\lambda : (G , \tau) \to (H , \sigma)$ is

(a) *pre*-ω-perfect if and only if it is ω-perfect .

(b) β-ω-perfect if and only if it is b-ω-perfect .

Proof: (a) prove by lemma 3.20

the same way to show (b)

Theorem 3.22. Let a space (G , τ) be an ω-$B\alpha$-condition. The mapping $\lambda : (G , \tau) \to (H , \sigma)$ is *pre*-ω-perfectif and only if it is α-ω-Perfect.

Proof: Let $\lambda : (G , \tau) \to (H , \sigma)$ be *pre*-ω-perfect mapping, to prove it is α-ω-perfect to demonstrated that λ is α-ω-continuous, let g $\in G$ and let T be an ω-open set containment $\lambda\,(g)$ in H, such that $\lambda\,(g) \in T1$, and $\text{int}\omega(\text{cl}\,(T1)) \subseteq T$, because of λ is *pre*-ω-continuous, there is an ω-open set S containment g, and G is ω-$B\alpha$-condition, then $\lambda(S) \subseteq \text{int}\omega(\text{cl}(\,\text{int}\omega\,(T1))$. It follows that $\lambda(S) \subseteq T1$, so λ is α-ω-perfect mapping.

Theorem 3.23. Let (G , τ) be a door space. The mapping $\lambda : (G , \tau) \to (H , \sigma)$ is b-ω-perfect if and only if it is *pre*-ω-perfect.

Proof: suppose that λ be a b-ω-perfect mapping. It suffices to demonstrated that λ is *pre*-ω-continuous, let g $\in G$ and let T be an open set containment λ(g) in H. Because G is a door space, there is a subset $T1$ an ω-open in H, such that $\lambda(g) \in T1$, and $\text{int}\omega\,(\text{cl}(T1)) \cup \text{cl}(\,\text{int}\omega\,(T1)) \subseteq T$. Because of λ is b-ω-continuous, there is an open set S containment g, such that $\lambda\,(S) \subseteq \text{int}\omega\,(\text{cl}(T1)) \cup \text{cl}(\,\text{int}\omega\,(T1))$. And $\text{int}\omega\,(\text{cl}(T1)) \subseteq \text{int}\omega\,(\text{cl}(T1)) \cup \text{cl}(\,\text{int}\omega\,(T1))$. Then $\text{int}\omega\,(\text{cl}(T1)) \subseteq T$. It follows that $\lambda(S) \subseteq \text{int}\omega\,(\text{cl}(T1))$, so λ is *pre*-ω-continuous. Hence consider λ is *pre*-ω-perfect mapping.

Theorem 3.24. Let (G , τ) be an ω-condition then. The mapping $\lambda : (G , \tau) \to (H , \sigma)$ is ω-perfect if and only if it is perfect.

Proof: Let λ be a ω-perfect mapping to prove it is perfect mapping. It suffices to demonstrated that λ continuous, let g $\in G$ and let T be an open set containment $\lambda\,(g)$ in H. Because G satisfy ω-condition, yond is an ω-open $T1$ in H, such that λ(g) $\in T1$, because of λ is ω-continuous, there is an ω-open set S containment x with $\lambda\,(S) \subseteq T$, so λ is continuous. Hence λ is perfect mapping.

Remark 3.25. Theorem 3.24. is not true in general. It mean if $\lambda : (G , \tau) \to (H , \sigma)$ is ω-perfect mapping, then it is not necessarily perfect mapping there shown in the next example.

Example 3.26. Let $G = \{1 , 2 , 3 \}$, $\tau = \{ G , \varphi , \{3\} \}$, $H = \{4 , 5 , 6 \}$, $\sigma = \{ H , \varphi , \{5, 6\}\}$ and let $\lambda : (G , \tau) \to (H, \sigma)$ be a mapping and know by $\lambda\,(1) = \lambda\,(2) = 4$, $\lambda\,(3) = 5$ since G and H are countable then any subset of G and H are ω-open let S= G are ω-continuous but not continuous since $\lambda(G) = \{4, 5\} \not\subset \{ 5, 6\}$ that λ is ω-perfect mapping but not perfect mapping

Theorem 3.27. Let a space (G , τ) be an ω-B-condition.The mapping $\lambda : (G , \tau) \to (H , \sigma)$ is *pre*-ω-perfect if and only if it is ω-perfect.

Proof: Let λ be a *pre*-ω-perfect mapping to prove it is ω-perfect mapping. It suffices to demonstrated that λ ω-continuous, let g $\in G$ and let T be an open set containment λ (g) in H. , because λ is *pre*-ω-continuous, there is an ω-open set S containment g, such that $\lambda\,(S) \subseteq \text{int}\omega(\text{cl}(T1))$. Because G satisfy ω-B-condition, there is a subset $T1$ *pre*-open also is open in H; $\lambda(g) \in T1$ and $\text{int}\omega(\text{cl}(T1)) \subseteq \text{int}\omega(T1)$ and $\text{int}\omega(T1) \subseteq T1$. It follows that $\lambda(\text{S}) \subseteq T1$, so λ is ω-continuous. Hence λ is ω-perfect mapping.

Theorem 3.28. Let (G , τ) be a door topological space. The mapping $\lambda : (G , \tau) \to (H , \sigma)$ is.

(a) *pre*-ω-perfect if and only if it is ω-perfect .

(b) β-ω-perfect if and only if it is b- ω-perfect .

Proof: Let λ be a *pre*-ω-perfect mapping to prove it is ω-perfect to demonstrated that λ is ω-continuous, let g $\in G$

and let T be an ω-open set containment $\lambda(g)$ in H, and G is a door space, there is a subset $T1$ an ω-open in H, such that $\lambda(g) \in T1$ and intω(cl($T1$)) $\subseteq T$, since λ is *pre*- ω-continuous, there is an ω-open set S containment g , such that $\lambda(S) \subseteq$ intω(cl($T1$)). It follows that $\lambda(S) \subseteq T$, so λ is continuous. Hence consider λ is ω-perfect mapping

Similarly we can prove (b).

Definition 3.29. Let λ be a mapping $\lambda : (G , \tau) \rightarrow (H ,\sigma)$ is called ω-B-continuous (resp., ω- Bα-continuous [7]). If for each an open T in H , $\lambda^{-1}(T)$ is an ω-B-set (resp., ω- Bα-set) in G.

Definition 3.30. Let λ be a mapping $\lambda : (G , \tau) \rightarrow (H ,\sigma)$ is called ω-B-perfect mapping (resp. , ω-Bα-perfect mapping) if it is closed , ω-B-continuous (resp. , ω-Bα-continuous) , and for every $h \in H$, $\lambda^{-1}(h)$ compact.

Theorem 3.31. Let λ be a mapping $\lambda : (G , \tau) \rightarrow (H ,\sigma)$, the mapping of following properties are equipotent :

(a) λ is ω-perfect.

(b) λ is *pre*-ω-perfect and ω-B-perfect.

(c) λ is α-ω-perfect a nd ω-Bα-perfect.

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