MODELLING AND FORECASTING MONTHLY CRUDE OIL PRICES OF PAKISTAN: A COMPARATIVE STUDY OF ARIMA, GARCH AND ARIMA-GARCH MODELS

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ABSTRACT : *Crude oil is the most important product in world and it has meanings for each individual. This study comprising of developing a more appropriate model for forecasting the monthly crude oil prices of Pakistan. In this study three time series models are used namely Box-Jenkins ARIMA (Auto-regressive Integrated Moving Average, GARCH (Generalized Auto-regressive Conditional Hetero-scedasticity) and ARIMA-GARCH in modelling and forecasting the monthly crude oil prices of Pakistan. The capabilities of ARIMA, GARCH and ARIMA-GARCH in modelling and forecasting the monthly crude oil prices are evaluated by using Akaike's Information Criterion (AIC), MAE (Mean Absolute Error) and RMSE (Root Mean Squared Error). It can be concluded that the hybrid model of ARIMA-GARCH perform well as compared to the Box-Jenkins ARIMA and GARCH model.*

Key Words: ARIMA, GARCH, Crude Oil and Forecasting

1. INTRODUCTION

Crude oil is the most important and non-renewable product in the world. Its applications are abundant in daily life, from making plastic bags and detergents to fuelling ships and cars. In spite of the fact that it is being non-renewable, but the world still consumes crude oil every single second as it is very challenging to find a substitute source that can comparable its unique performances. With such a unique nature it is vital for us to develop a better understanding of its price changing aspects, so that many industries which consume or supply oils can make more up-to-date decisions. Forecasting of crude oil prices be responsible for suitable statistical information which helps the policy makers and government agencies as well as the investors to design and accomplish their assets in a well-organized fashion. The crude oil prices are very sensitive because of the instability nature of the geo-political and global socio-economic events. The influence in the crude oil prices is not only due the supply, demand, consumption or inventory factors but it is also depends on so many other irregular factors which are unpredictable and stochastic in nature. The task of appropriate modelling and forecasting are very challenging and complex due to the unpredictable and stochastic pattern of crude oil prices. On the other hand, due to the complex nature of crude oil prices this is a widely opened research area and most of the researcher have used different techniques for the prediction of crude oil prices. However, mainly there are two different approaches used for forecasting of crude oil prices. The first approach is to make the forecast in a cause and effect framework while the predicting variable is assumed to be affected by some more other variables simply called covariates. Occasionally it is also called fundamental analysis. This approach seems naturally more attractive because of placing ahead logical reasons for the ups and downs of our predictions. At this point there have been so many studies such as [1] and [2] used this method to examine the effect of portfolios and the aspects that might have contributed to the crude oil price increase in accumulation to the supply and demand of crude oil, by expanding a model of crude oil prices to include a nonlinear influence of OPEC capability utilization, processing plant utilization charges, and environments in futures markets as explanatory variables. This method has many restrictions e.g. we may not be sure that which variables may be accounted for the changes in the crude oil prices. Theoretically if we know the explanatory variables which is accounted for the change in crude oil prices but it's still very difficult to manage the exact functional form of these explanatory variables with the crude oil prices. Additionally it is almost impossible to make measurements for the future values of these explanatory variables. As a result the prediction made on the estimated values of the explanatory variables extremely increased the forecast errors.

The second forecasting approach is the Time Series modelling. In this method we are no longer to forecast the time series future movements on a set of some other variables rather we made future predictions on the basis of past behaviour of the variable alone. For example, [3], [4], [5], and [6] used the well-known Box-Jenkins techniques for forecasting of crude oil prices, however, several studies [7-9] used GARCH type of models for crude oil predictions. Further, it is also noted that we may not be able to explain the changes in the behaviour of the crude oil prices such as ups and downs, which is based on the inventory levels or economic theory or by natural reasoning that's why this time series data acted the way it did. This time series moved ups and downs in response of political instability, financial crises or some socioeconomic reasons but more or less of its actions might be influenced due to the some factors which is simply may not be explain or in other words we are not able to explain that moments of these factors.

The current study based on the second approach which involves a modelling technique to the time series data to forecast the crude oil monthly average prices of Pakistan. Islamic Republic of Pakistan is a country currently having a 195 million population (more or less) and consisting of a 0.793 million km^2 of area. It has a crude oil production capacity of almost 80 thousand barrels per day and its consumption are 455 thousand barrels per day according to the EIA report 2014. So Pakistan is a totally net imported crude oil country and now a day's it has facing some serious energy crises and mostly depends on crude oil prices if the prices are increase than the government expenses are automatically increased. Therefore a consistent and suitable estimates of the crude oil prices are of enormous meaning for policy making and planning.

In this paper, using a time series approach, aiming at building ARIMA, GARCH and ARIMA-GARCH models, by using the Box-Jenkins methodology. At the end comparison of forecasting accuracy of these models will be done with the help of MAE (Mean Absolute Error) and RMSE (Root Mean Square Error). Section 2 contains the related work which follows the reviews of this introduction, section 3 consists on discussion of methodology and mainly focus on the formulation of mathematical work whereas section 4 comprise on analysis, findings and results discussion and the last section 5 will consist of conclusion and future recommendation.

2. Literature Review

Modelling and forecasting the changing aspects of crude oil prices is not an easy task because the prices may possibly be fluctuate unpredictably from time to time and also depends on so many factors. Liu [3] study the dynamic relations among United States crude oil prices, gasoline-prices and gasoline stock by using the Box-Jenkins procedure with transmitting function models of US, whereas, [10] incorporate the time series models for the purpose of investigation and comparison of the forecast accuracy of the crude oil prices for the future values the study fit an autoregressive moving average model of order (1,2). Likewise, [11] discussed that the advantages of ARIMA models are double. Primarily, ARIMA models are a set of distinctive linear models which are supposed to be the best for the linear time series data and captured the linear features in the time series data. Consequently, ARIMA models are ideal on theoretical basis. Similarly, [4] considered the predictive content of the energy futures and surveyed the relationship among the futures prices and spot prices for the different commodities of energy and ARIMA (1, 1, 1) model was used for the crude oil prices prediction. Ahmad [6] using the Box-Jenkins techniques to forecast the average monthly crude oil prices of Oman at the end he recommended that the seasonal multiplicative model ARIMA (1, 1, 5) x (1, 1, 1) are used in practice for estimating the crude oil prices. However, [12] considered the issue of crude oil forecasting prices and they concluded that the models which produced lower MSPE in the futures values will be the better models for estimating the crude oil prices for the future.

On the other hand, [7] indicated that out of sample predictions of a single GARCH model equation is better for Vector Auto Regression (VAR) and for bivariate GARCH models, and are more superior in forecasting the petroleum prices for the futures. Similarly [5] used various types of GARCH models and pointed out uncertain models to forecast the daily WTI future prices instability, however, the observed results were incompatible and exposed their performances with respect to statistical tests and diverse measures. Furthermore, [13] used various GARCH models to forecast the instability of futures prices of daily crude oil operated on NYMEX. The authors reached to a decision that no model works well on continuous basis using the various statistical tests like DM test, performance measures as adjusted hetero-scedasticity MSE, MSE and MAE and success Ratio. In addition [8] using a new approach including non-parametric technique to models and predict crude oil price return instability, the results determine that out of sample volatility forecasting of the GARCH nonparametric model shows better performance from a class of GARCH parametric models. Moreover, [9] used the GARCH models for forecasting the spot prices of daily crude oil. This technique was used to demonstrate the advantages and performances of non-linear models over the liner models. The study comprises of fitting the three different GARCH models such as GARCH-G, GARCH-N and GARCH-T to the spot prices of daily crude oil. The different models produced different results over the different data sets the GARCH-G model considered as a best model for WTI while the GARCH-N model was the best candidate model for forecasting the spot prices of Brent daily crude oil. Lastly, Ahmed and Shabri [14] used the ARIMA, GARCH and SVM (Support Vector Mechanic) techniques and concluded that on the basis of forecast accuracy measurement error of RMSE and MAE that the performance of the proposed vector mechanic technique is better than all other usual methods.

3. METHODOLOGY

In this Section, the discussion on models which will be used for the forecasting of future values of the Pakistan monthly crude oil prices. The models are ARIMA, GARCH and hybrid ARIMA-GARCH.

3.1 ARIMA Modelling

One of the assumption on using the ARIMA, GARCH and hybrid ARIMA-GARCH models that the data should be stationary. Stationary series doesn't comprise of seasonality and trend factors and the data is smooth because its mean, variance and autocorrelation structure are remains constant over the interval of time. Stationarity of a time series data is very important to elaborate the future performance of the data series. However, if the data is not stationary first we transform the data into a stationary series by taking the differences. Once the stationarity is achieved of the series than the Model fitting can be carried out to fit the best model. By exploring the ARIMA Box-Jenkins methodology, suggested by [15] to generate a univariate time series forecasting model. It is one of the simple and common way for forecasting the univariate time series data such as the monthly average crude oil prices. To understand this methodology, first define the ARMA (ARIMA) model. An Autoregressive Moving Average (ARMA) (r, m) model of a univariate time series y_t has the following form:

$$y_t = \alpha_0 + \sum_{i=1}^{r} \alpha_i y_{t-i} + \sum_{j=1}^{m} \beta_j \, \varepsilon_{t-j} + \varepsilon_t$$

Where r is the number of autoregressive terms, m is the number of lagged error terms, α is the coefficient of autoregressive and constant terms while β is the coefficient of moving average terms. The ε_t in above expression is a white noise process with zero mean and σ^2 variance and no correlation across the time and they are also independently and identically distributed. The extension of an ARMA class is the ARIMA model. A time series y_t is called to be of the Autoregressive Integrated Moving Average (ARIMA) format if $\nabla^d y_t$ is an ARMA stationary process, where d is the number of differences to be taken from the original time series data. Using the Box Jenkins method, first confirm that the series is stationary and this can be done by differencing the time series many times (but at most two times usually). The test suggested by [16] or simply called ADF test will be used to check whether the transformed series is stationary or not. This methodology provides an alternative check for stationarity without plotting the observed series. It is a test to check whether the given time series has a unit root. If it does, than the given series is considered to be non-stationary. Next, step is to plot the autocorrelation and partial autocorrelation functions to determine the order of r and m for selected model. Sometimes, the situation of facing the problem that there are more than one potential candidate models for the final model. In these kind of situations the alternative approach is used to decide about the selected model and that approach is suggested by [17] to select an appropriate model by using the AIC (Akaike Information Criterion). The AIC criteria tells us that how well the estimated model fits the data comparatively with other models, and AIC is calculated by the following formula:

$$AIC(r,m) = -2\log(M) + 2(r+m)$$

Where M is the maximum likelihood function value of the ARMA (r,m) model and $(r+m)$ is the total parameters to be estimated. The best model in this approach is a model having lower AIC value. Once the model is specified, the next step is the estimation of the corresponding coefficients of autoregressive and moving average terms of the model and MLE method is used for this purpose. At the end, model diagnostics test is perform to check whether the estimated model is consistent with the specification of a univariate time series process with stationarity. For checking stationarity, one of the method is the plot of the quantiles of the fitted model residuals against the normal distribution quantiles which is called Normal QQ plot. The plot consist of a line such as QQ which shows a perfect match between the fitted model residuals and the Normal distribution. McLeod and Li [18] test can be perform to check whether the sample residuals are independent of the time factor similarly, Doornik-Hansen test suggested by [19] also used for checking the residuals normality. Moreover, if the assumptions are violated due to model residuals, than the alternative choice is the Generalised Autoregressive Conditional Hetero-scedasticity (GARCH) methodology to model the residuals. GARCH models were proposed by [20], and now are widely used to specify model innovations such as the differences between the observed values and fitted values of the selected model. GARCH models assume that the conditional variances of innovations follows an ARMA model. So GARCH (r,m) model, where r and m are the orders of GARCH and ARCH terms respectively, refers to σ_t of the following form: $\varepsilon_t = \sigma_t y_t$, y_t white noise,

$$\sigma_t^2 = \omega + \sum_{i=1}^{m} \theta_i \varepsilon_{t-i}^2 + \sum_{i=1}^{r} \varphi_i \sigma_{t-i}^2$$

Also note that if $r = 0$ the GARCH model will be equal to ARCH model.

3.2 Hybrid ARIMA-GARCH Modelling

In ARIMA-GARCH hybrid modelling there are two phase procedure. In first phase, the best ARIMA model is used for linear time series data and only residuals contain the non-linear part of the data. In second phase, the GARCH model is used to cover the non-linearity of the residuals. Now this hybrid ARIMA-GARCH model is used to analyse the time series and forecast the approximate series values see [21-25]. In this technique, the error term ε_t of ARIMA model is supposed to follow the GARCH process of orders r & m. For forecast accuracy the two major measures are used one is RMSE (Root Mean Square Error) and the other one is MAE (Mean Absolute Error). The smaller the values of the MAE and RMSE the better is the model. The formulas of these methods are as follows:

$$RMSE = \sqrt{\frac{\sum_{t=1}^{n}(y_t - \hat{y}_t)^2}{n}} \quad and$$

$$MAE = \frac{\sum_{t=1}^{n}|y_t - \hat{y}_t|}{n}$$

4. Analysis and Modelling of Pakistan Crude Oil Price

4.1 Application

In this study, the Pakistan average monthly crude oil prices (in Pakistani Rupees per barrel) series, was selected as the experimental sample. The data used in this study is the monthly data from Feb, 1986 to Mar, 2015, with a total of 350 observations. The data from Feb, 1986 to May, 2009 are used for the training set (280 observations), and the rest is used as the testing set (70 observations).

4.2 Modelling and Analysis

Starting from the plotting of the whole data set against the time. Figure 1 shows the original plot of the whole time series data. The first 280 training set observations are taken for modelling. The next step is the checking of stationarity among these observations. For stationarity using the ADF (Augmented Dickey-Fuller) test. The ADF test suggest a p-value of 0.7379, means that we can't reject the null hypothesis of non stationarity at 5% level of significance. To get the stationarity of the series take the first difference of the series which provides the p-value of 0.0001 for the ADF test which means the series is stationary after the first difference. Which is also clears from the Figure 2. After achieving the stationarity of the series, the next step is to decide about the orders of the autoregressive (AR) and moving average (MA) terms respectively. The ACF and PACF of the difference series are shown in Figure 3.

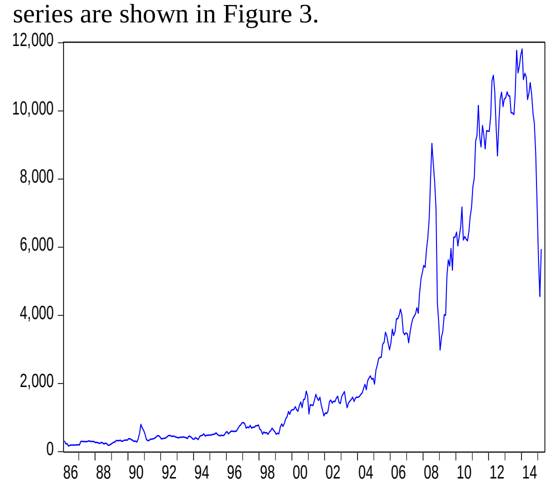


Figure 1. Crude Oil Prices from Feb,1986 to Mar, 2015

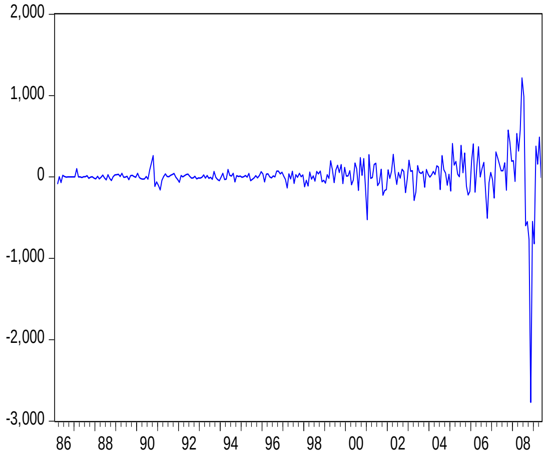


Figure 2: Graph of the First Difference of the sample Crude Oil Prices

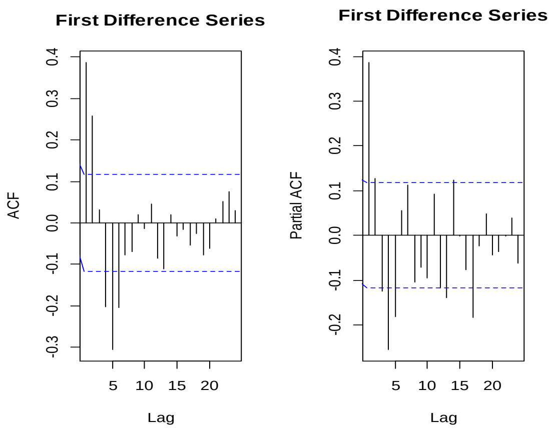


Figure 3: ACF and PACF Plots of the Difference Series

By looking the ACF there are 5 spikes which are significant at 1% and in PACF there are so many significant spikes but at 1% only 4 spikes are significant thus the selected model is ARMA of order (4, 5). The other method which is AIC criterion also suggest the same model by producing the minimum value of AIC i.e. 3784.41. Thus the final selected model is ARIMA (4, 1, 5) and is the best candidate model. Next step is the estimating of coefficients, Table 1 contains the coefficients of the selected model and their p-values.

Model: ARIMA, using observations 1986:03-2009:05 (T = 279) and Standard errors based on Hessian

Table 1

Co-eff.	Estimat e	Std. Error	t-value	p-value
Const.	16.191	8.331	1.943	0.052 *
AR(1)	1.0042	0.135	7.423	0.00 ***
AR(2)	−1.004	0.112	−8.91	0.00***
AR(3)	1.0445	0.111	9.39	0.00***
AR(4)	−0.430	0.097	−4.40	0.00 ***
MA(1)	−0.749	0.128	−5.83	0.00 ***
MA(2)	1.1528	0.135	8.49	0.00 ***
MA(3)	−1.071	0.157	−6.81	0.00 ***
MA(4)	0.2737	0.154	1.77	0.07 *
MA(5)	−0.351	0.131	−2.67	0.007**

By looking the p-values all the coefficients are highly significant except only one which is significant at 10% level of significance.

4.3 GARCH Fit for Fitted Model Residuals

First of all plotted the residuals of the fitted model. By looking the graph it seems reasonably stationary and gives the impression to evolve around a zero mean. There are some spikes or irregular variations in the graph which shows some variation in a specific time period. These variations comes to wars and other financial crises but the series adjusted our self by the end of that crises. The plot of the fitted model residuals are shown in Figure 4.

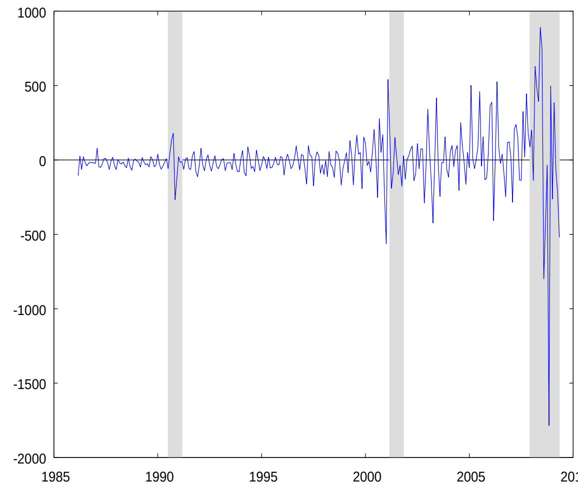


Figure 4: Plot of the Fitted Model Residual

Following is the plot of ACFs and PACFs of the residuals and squared of the residuals:

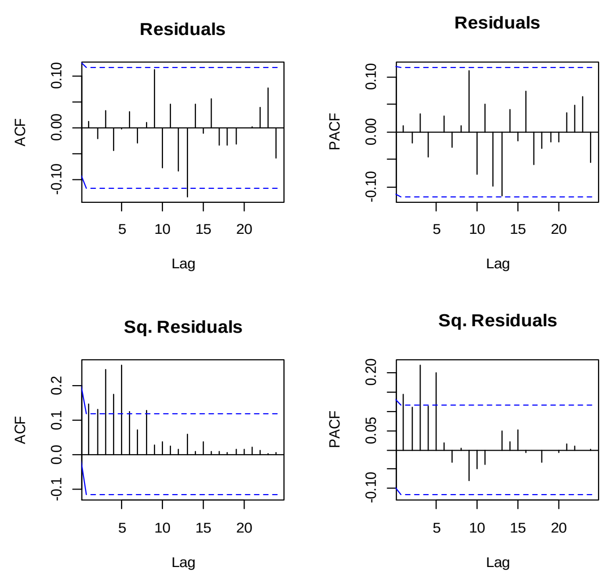


Figure 5: ACF and PACF Plots of the Residulas and Squared of Residuals

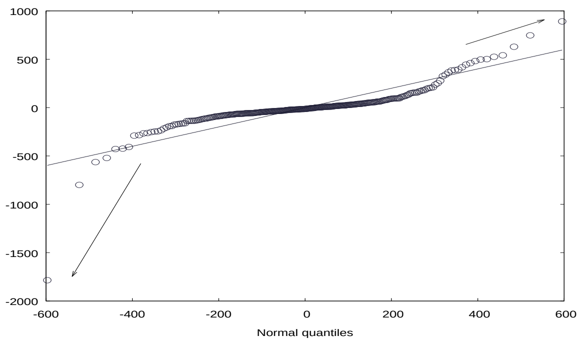


Figure 6: Q-Q Plot of the Fitted Model Residuals

In Figure 5 there is no significant spikes or autocorrelation for the fitted model residuals as expected. But in the ACF and PACF of squared of residuals there are so many spikes which are significant at 1% level. The normality of the residuals are also tested.

The QQ plot suggests that the fitted model residuals are not normal at the end of the tails of the distribution showing by the arrows and also from the p-value (i.e. 0.000) of Doornik-Hansen test. Hence conclude that the residuals are not normally distributed.

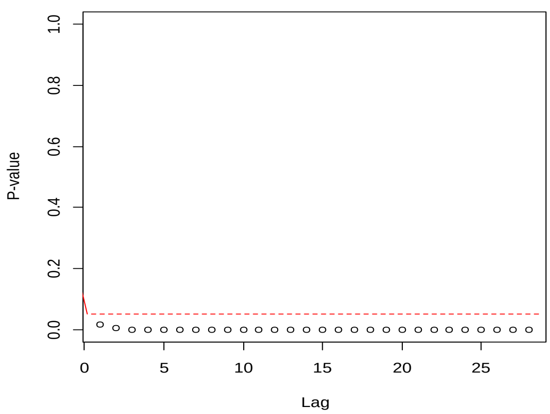


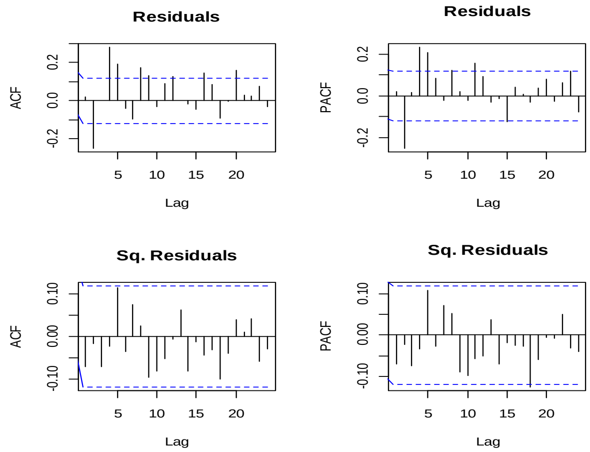
Figure 7: Mcleod.Li Test Results Plotted Residuals

On the other hand, the Mcleod.Li test suggests strong evidence that the residuals are auto-correlated. The plots of the p-values against the different lags indicates that all p-values lie below the 0.05 threshold. Therefore a GARCH model may be appropriate to fit the data. To decide about the order of GARCH model, AIC values for various orders of GARCH model are calculated. GARCH (1, 1) model provides the lowest value of AIC (i.e. 3414.75). The summary of the fitted model are as follows:

Table 2

Co.eff	Estimate	Std.	t-value	P-Value
a0	87.113	102.50	0.85	0.395
a1	0.3462	0.0764	4.529	5.93e-6 ***
b1	0.7435	0.0432	17.21	< 2e-16 ***

For the purpose of adequacy, the plot of ACF and PACF of the fitted GARCH model residuals and squared of residuals are shown in figure 8. From the graph it is clears that all of



the significant spikes from the squared of residuals are removed.

Figure 8: ACF and PACF Plots of the GARCH (1, 1) Model Residuals

4.4 GARCH Modelling

In this section the GARCH model will be fit for the time series training set data and will forecast the future values for the test set. For GARCH model the data is converted to the log return instead. The formula for the log returns is $z_t = \ln(\frac{y_t}{y_{t-1}})$. AIC values is calculated for the different order of GARCH models but the lowest value of AIC (i.e. -469.71) is observed for the GARCH (1,1) model so the selected model is GARCH(1,1). The summary output of the fitted GARCH model are presented in table 3.

Model: GARCH, using observations 1986:03-2009:05 (T = 279) and Standard errors based on Hessian

Table 3

Co-eff	Estimate	Std. Error	t-value	p-value
a(0)	0.00534	0.00107	5.008	5.5e-7 ***
a(1)	0.45306	0.16956	2.672	0.007 ***
b(1)	0.17186	0.11287	1.523	0.1278

4.5 Modelling of ARIMA-GARCH

In this study, the relative performance of the ARIMA, GARCH and ARIMA-GARCH models for the monthly crude oil prices of Pakistan will be compare. The models forecasting accuracy will be measure through the well-known methods such as MAE and RMSE. In the above sections ARIMA and GARCH best models are fitted for the training data set. Here is the combination of best ARIMA and GARCH model and develop a new hybrid model. The new hybrid model is ARIMA (4,1,5) + GARCH(1,1). The summary statistics of the fitted model are presenting in Table 4.

Table 4

Co-eff	Estimate	Std. Error	t-value	P-value
const	4.9394	2.69057	1.836	0.0664 *
AR(1)	0.8562	0.04335	19.75	2e-16 ***
AR(2)	-0.8952	0.04553	-	2e-16 ***
AR(3)	0.7804	0.03571	21.855	2e-16 ***
AR(4)	-0.8518	0.04952	-17.2	2e-16 ***
MA(1)	-0.8042	0.08362	-9.617	2e-16 ***
MA(2)	0.8669	0.06642	13.052	2e-16 ***
MA(3)	-0.7295	0.07363	-9.908	2e-16 ***
MA(4)	0.8497	0.07428	11.439	2e-16 ***
MA(5)	-0.0439	0.02278	-1.930	0.0543 *
a0	51.644	9.61566	5.371	7.8e-8
a1	0.3540	0.05333	6.6378	3e-11***
b1	0.6449	0.06169	10.453	2e-16 ***

From the above table concluded that all of the coefficients of the selected model is highly significant except only one coefficient which is significant at 10% level of significance, now also confirms the fitness of the model from the standardized residuals test. The results of the standardized residuals are as given in Table 5.

Table 5

Test			statistic	P-value
Jarque-Bera Test	R	Chi^2	16.04794	0.00032
Shapiro-Wilk Test	R	W	0.980068	0.00061
Ljung-Box Test	R	Q(10)	3.112459	0.97865
Ljung-Box Test	R	Q(15)	19.40306	0.19604
Ljung-Box Test	R	Q(20)	22.23395	0.32795
Ljung-Box Test	R^2	Q(10)	9.540896	0.48165
Ljung-Box Test	R^2	Q(15)	24.0484	0.06427
Ljung-Box Test	R^2	Q(20)	27.68517	0.11706
LM Arch Test	R	TR^2	12.99083	0.36970

4.6 Forecasting Accuracy Comparison

The aim of this paper is to develop different feasible models and analyse the forecast accuracy of the fitted models for the test set which is from June, 2009 to Mar, 2015 consisting of 70 observations. The following table shows the different measure of forecast accuracy for the different fitted models and used the one-step ahead forecast. For each selected model the values of RMSE and MAE are generated and get a better picture that how well these models predict the future values. The lowest value in each case is highlighted indicating the best model in terms of forecasting power.

Table 6

Model	MAE	RMSE
ARIMA (4,1,5)	209.73	390.05
GARCH (1,1)	187.18	371.95
ARIMA (4,1,5)+GARCH(1,1)	180.420	233.55

So on the basis of MAE and RMSE the hybrid model ARIMA-GARCH is selected for the future forecasting of monthly crude oil prices of Pakistan. In figure 9 shows predicted values from of the fitted model and the actual values are presented.

5. DISCUSSION AND CONCLUSION

Average monthly crude oil prices of Pakistan are studied in this paper for modelling and forecasting. Primarily the data divided into two parts, the first part taking as a training set and the other part as a testing set. There are some fluctuations in the prices but the series is stationary after the first difference and fit an ARIMA (4, 1, 5) model to the training set and calculate its residuals. Due to the fluctuations in oil prices especially during the Iraq war (1990), afghan war (2002) and world financial crisis (2008) also shown these fluctuations in Figure 4. During these periods crude oil prices have some significant effect. In particular, this volatile period responsible for the larger variance in the historical prices of the crude oil. Accordingly, this significant variance provides the spontaneous reasoning on the use of ARCH/GARCH model. The empirical study of 280-months crude oil data series indicates that the hybrid ARIMA (4,1,5)-GARCH(1,1) model provide the optimal results and improved the estimating power and forecasting accuracy compared to the ARIMA and GARCH model. Thus the final selected model is the hybrid ARIMA-GARCH model. Which is recommended for the forecasting of average monthly crude oil prices of Pakistan. Inculcating the whole combination of powerful and flexibility of ARIMA and the strength of GARCH models in handling volatility and risk in the data series as well as to overcome the linear and data limitation see [26] in the ARIMA models made the combination of ARIMA-GARCH as a new potential approach in analyzing and forecasting the

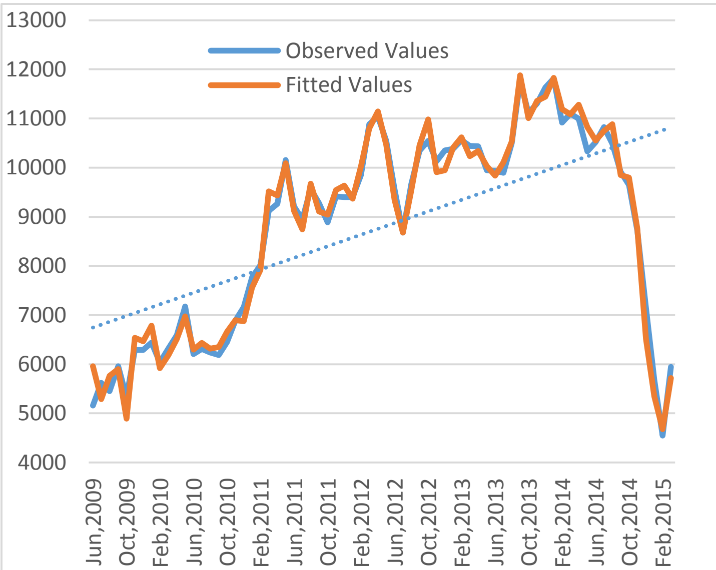


Figure 9: Test Set Observed and Predicted Values

average monthly crude oil prices.

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