RECONSTRUCTION OF CONTINUOUS SIGNALS USING MODIFIED ZOH TECHNIQUE

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ABSTRACT: *The common methods for reconstruction of the continuous signals are Zero Order Hold (ZOH) and First Order Hold (FOH). In these techniques, the primary signal is approximated using the sampling points. In FOH method, there is no suitable amplitude and phase for the filter. Furthermore, the approximated points are not exactly produced from the sampling points. In comparison to FOH, the ZOH technique improves the amplitude and the phase of the filter; however, the signal constructed using this technique is not a suitable approximation of the main signal. In this paper, a similar approach has been proposed that in addition to maintaining ZOH filter amplitude, improves the filter phase. Furthermore, using the proposed technique, the reconstructed signal would be a good approximation of the main signal*. The proposed technique is implemented using MATLAB and the results are presented.

Key words**:** *Reconstruction of signals; Zero Order Hold; First Order Hold; ZOH; FOH.*

1. INTRODUCTION

Missing the signal samples during signal processing has been always of the main concerns to the researchers and users. Consequently, reconstruction of the signal and extracting its information is the main issue in this case [1,2]. As the number of the missed sampling points and their degree of importance is not known, the mentioned sampling is not reliable [3]. The conventional method for reconstruction of a continuous signal in the form of a discrete signal is periodic sampling in a way that a continuous time signal is sampled with the sampling period equal to T [4,5]. So the samples of this signal are given by $x(nT)$ [6]. So the reconstructed form of the continuous signal namely $x[n]$ is expressed as following:

$$x[n] = x(nT) \tag{1}$$

In the sampling theory, the sampling rate is important and the approximation of the main signal will be more accurate when the sampling rate is higher [7]. In this case, if the signal variations rate is considerable, the sampling period should be decreased in order to consider the intermediate variations in the sampling process [8]. There are several methods for sampling of continuous time signals, among them, using ZOH and FOH techniques for signal approximation are more interested [9]. In these methods, the input signal is first approximated by ZOH or FOH technique, and then the reconstruction is done. Reconstruction accuracy depends on ZOH and FOH approximation precision [10,11]. To improve the accuracy of the reconstructed signal, the considered sampling period is very large [12]. In this paper, a modified ZOH approximation method for signal reconstruction is proposed and the comparative results are presented.

2. ZERO ORDER HOLD (ZOH) ECONSTRUCTION METHOD

ZOH technique is the most common method for reconstructing a discrete signal. In this method, two adjacent sampling points are connected with a line with zero slope. Namely, when the sampler until reaches to the next sampling point, it retains its previous value. The relationship that expresses the reconstructed signal between any two consecutive sampling points is given by:

$$\begin{aligned} h(t) &= x(kT) \\ kT &< t < (k+1)T \\ 0 &< \tau < T \end{aligned} \tag{2}$$

The filter equation in Laplace domain is expressed as following:

$$H(s) = \frac{(1 - e^{-Ts})}{s} \tag{3}$$

The amplitude and phase diagrams of the ZOH filter are shown in Figures 1,2.

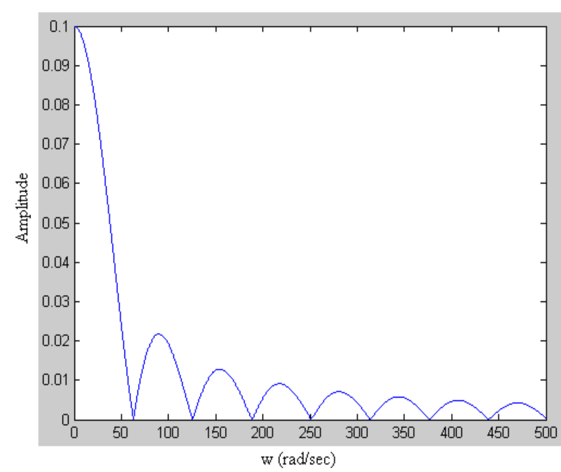

Fig. 1. ZOH filter diagram; Amplitude

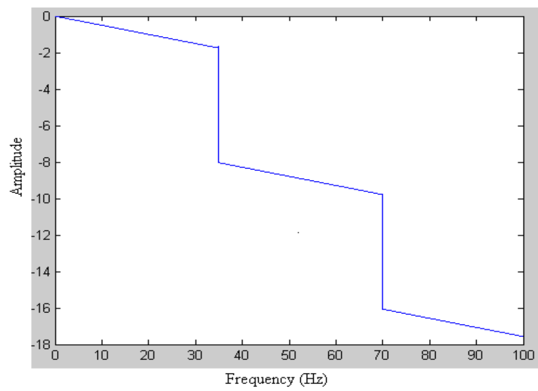

Fig. 2. ZOH filter diagram; Phase

3. FIRST ORDER HOLD (FOH) ECONSTRUCTION METHOD

In this way, any two consecutive sampling points are connected by line segments and where to draw a line from one sampling point, the next sampling point is required, appeared delay in system, accordingly the negative phase will be increase and may make the system unstable. The following expression shows the reconstructed signal between any two consecutive sampling points:

$$h(t) = x(kT) + \frac{x(kT) - x((k+1)T)}{T}\tau \qquad (4)$$

The filter equation in Laplace domain is given by:

$$H(s) = \left(\frac{1+Ts}{T}\right)\left(\frac{1-e^{-Ts}}{s}\right)^2 \qquad (5)$$

The impulse response of this system (H(S)) is not in the expected manner, namely, the sampling points are not connected through a line and this process is performed by a significant error. Figure 3 shows an example of this sampling method where the yellow diagram displays the reconstructed signal.

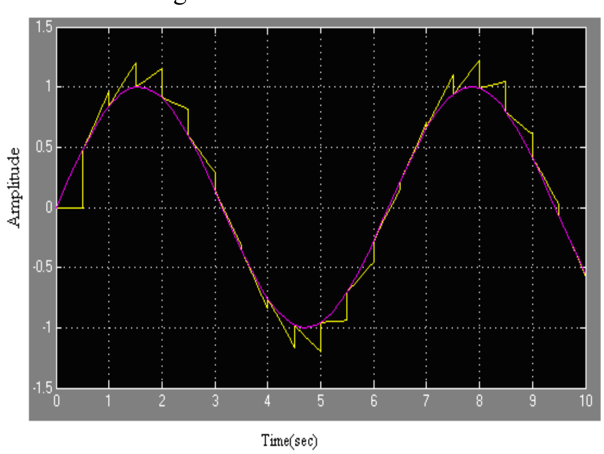

Fig. 3. Signal reconstruction using First Order Hold (FOH) method

The amplitude and phase diagrams of the FOH filter are esented in figures 4 and 5.

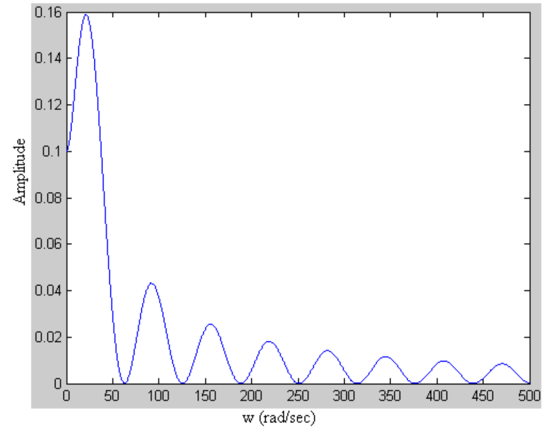

Fig. 4. FOH filter diagram; Amplitude

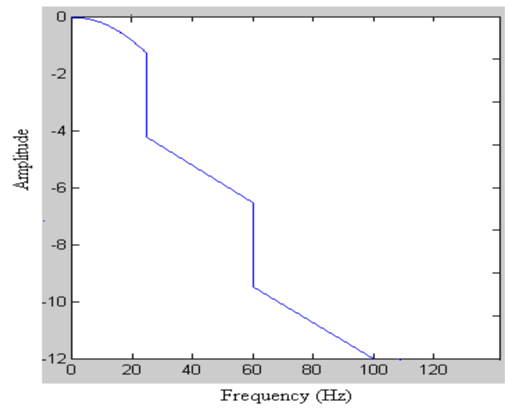

Fig. 5. FOH filter diagram; Phase

4. MODIFIED ZOH METHOD

This technique uses line segments with constant value for signal reconstruction like the ZOH method, Except that, h(t) is equal to x(kT) in a half sampling period and x((k+1)T) in the next half period. The performance of ZOH filter and the proposed modified ZOH filter are shown for a sinusoidal signal in Figures 6 and 7.

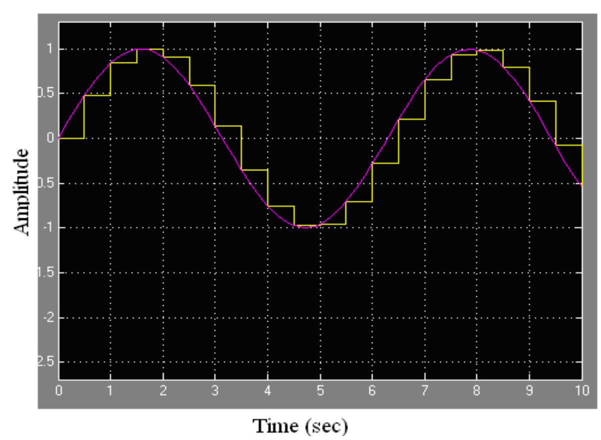

Fig. 6. Signal reconstruction with ZOH method

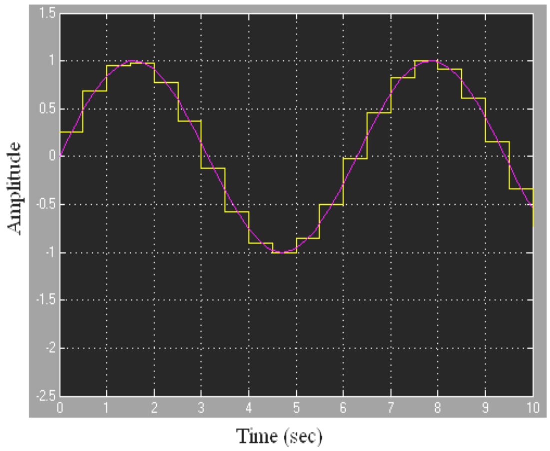

Fig. 7. Signal reconstruction with Modified ZOH method

As seen, using the proposed technique, the reconstructed signal is much closer to the main signal in comparison to ZOH technique where their difference is considerable.The relationship expressing the reconstructed signal between any two consecutive points is given by:

$$H(s) = x(kT)[u(t-(k-\tfrac{1}{2})T) - u(t-(k+\tfrac{1}{2})T)] + x((k+1)T)[u(t-(k+\tfrac{1}{2})T) - u(t-(k+\tfrac{3}{2})T)] \quad (6)$$

The filter equation in Laplace domain is given by:

$$H(s) = \frac{e^{Ts/2} - e^{-Ts/2}}{s} \quad (7)$$

The amplitude and phase diagrams of this filter are shown in Figures 8, 9.

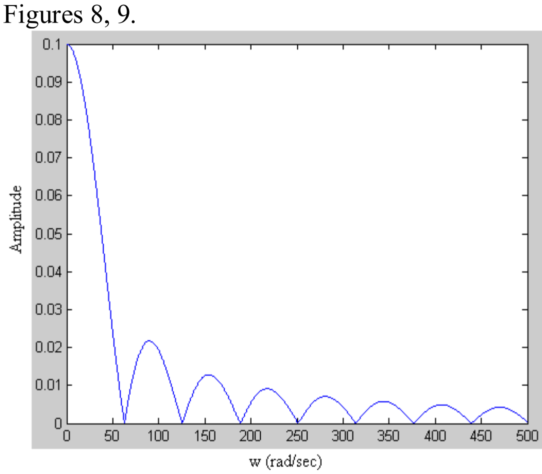

Fig. 8. Modified ZOH filter diagram; Amplitude

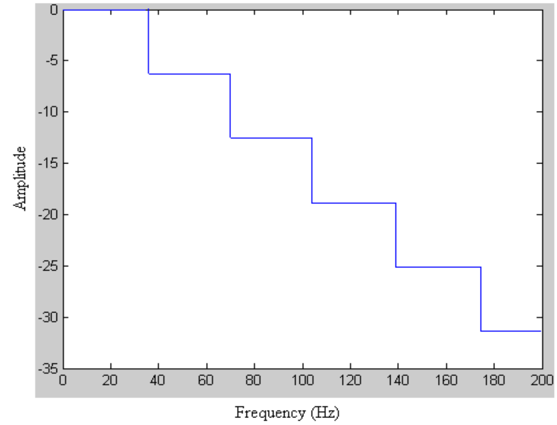

Fig. 9. Modified ZOH filter diagram; Phase

$$|G_{zoh(m)}| = |G_{zoh}| \quad (8)$$

$$\angle G_{zoh} = -\varpi T/2 + \angle \sin(\varpi T/2) \quad (9)$$

$$\angle G_{zoh(m)} = \angle \sin(\varpi T/2) \quad (10)$$

Obviously, by using this method, in addition to keeping the amplitude, the phase is also improved.

The Figures 10, 11, 12, 13, 14 and 15 are presented in order to compare the amplitude and phase characteristics of ZOH, FOH and the proposed technique.

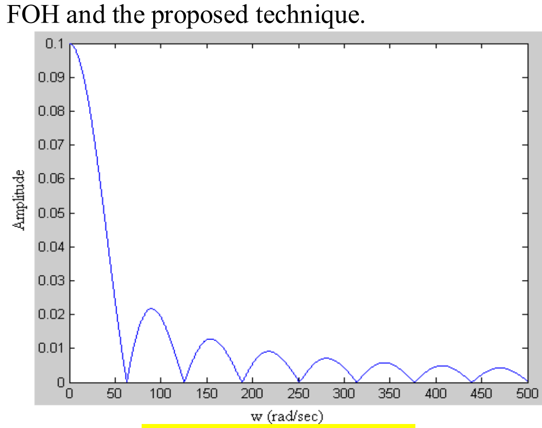

Fig. 10. ZOH filter diagram; Amplitude

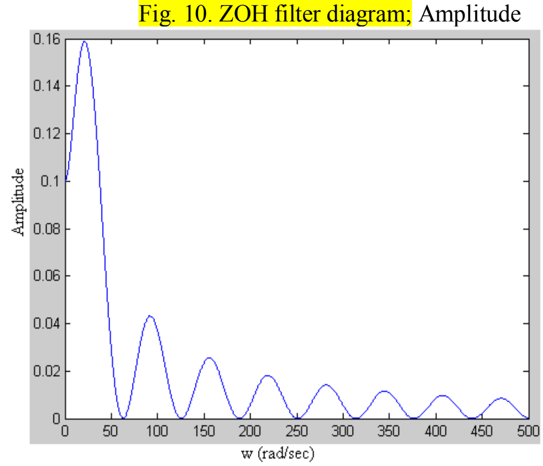

Fig. 11. FOH filter diagram; Amplitude

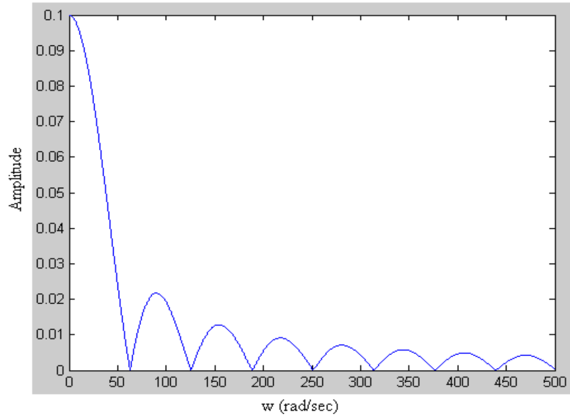

Fig. 12. Modified ZOH filter diagram; Amplitude

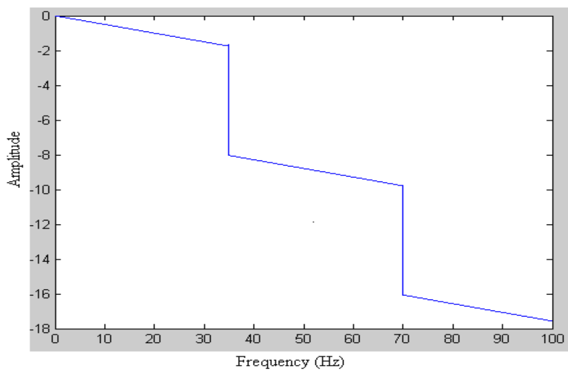

Fig. 13. ZOH filter diagram; Phase

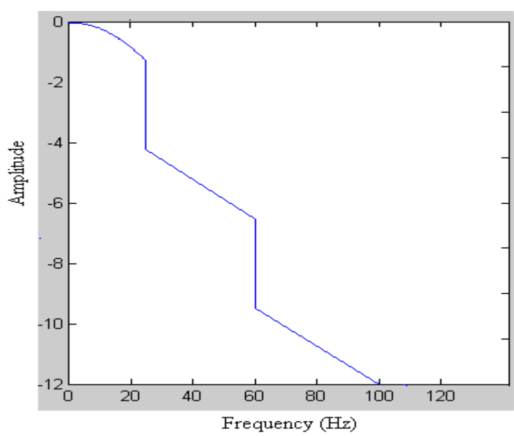

Fig. 14. FOH filter diagram; Phase

5. CONCLUSION

This paper deals with the reconstruction of continuous time signals using a modified ZOH technique. The proposed method uses a line with a constant value for reconstruction of the signal between each of the two consecutive sampling points, namely, h(t) in a half sampling period is equal to x(kT) and in the next half period is equal to x((k+1)T). As a result, In comparison to ZOH method where the difference between the main signal and its reconstructed form is evident, using the proposed innovative technique, the reconstructed signal is much closer to the main signal. The proposed method was implemented in MATLAB software and the results show the validity of the above claims.

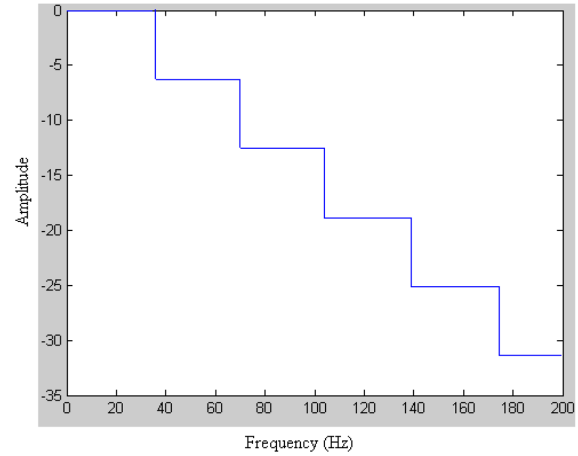

Fig. 15. Modified ZOH filter diagram; Phase

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