AN APPRAISAL OF STATISTICALLY APPROACHES FOR ESTIMATION OF PROBABLE MAXIMUM PRECIPITATION IN HIGHLAND CLIMATIC ZONE OF PAKISTAN

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ABSTRACT: Probable maximum precipitation (PMP) is the conceptual approach for calculation the magnitude of extreme storms used in the design of mini & large dams, reservoirs and other water resources projects. The main objective of this study is to compute the probable maximum precipitation (PMP) in high altitude areas (Murree) for one day duration. Daily precipitation data of Murree for 30 years (1981-2010) were collected to estimate the PMP. Annual maximum 1-day rainfall was sort out to estimate the PMP. The Hershfield method was used to estimate the 1-day PMP. Gumble approaches of extreme values, Log Pearson Type -III were used for the estimation of maximum rainfall. In comparative analysis, Chi- Square goodness of fit test shows that Gumble Distribution gives better results as compared to Log Pearson Type –III Distribution. PMP and maximum 1-day rainfall at different return periods are very useful into highland climatic zone for the designing of soil and water conservation structures, construction of mini dams, storage reservoirs and hydraulic structures.

Key Words: Probable Maximum Precipitation, Hershfield method, Gumble method, Log Pearson Type - III method, Frequency factor

INTRODUCTION
The water scarcity problems are evident in Pakistan, it is expected that in future there might be severe water crisis in the country. The economy of Pakistan is highly dependent on agriculture, for which water is vital source for agriculture. Pakistan has recently experienced devastating floods with disastrous consequences. The main reason of floods is the climatic change and heavy precipitation. The construction of mini and large dams is very necessary to overcome the water scarcity problem, electricity shortage and to control the floods. Dams provide a variety of economic, environmental and social benefits.

The need for the development of storage reservoirs and dams has gained great importance in Pakistan with the aim to provide irrigation for food production and to control the flood. In the design of reservoir and flood control dams in upstream of populated areas, most important issues are to control the possible flood at a location to ensure the safety of local habitation as stated by Maiedment [1]. Probable maximum precipitation (PMP) study provides rational information in optimal design of dam, reservoir storage capacity and flood-carrying structures (spillway and flood carrying tunnel). Hydrologist used the Probable maximum precipitation magnitude to calculate the probable maximum flood (PMF) that is helpful in the design of hydraulic and water conservation structures.

Statistical approach is used to compute the probable maximum precipitation in those regions where at least 30 to 40 years daily rainfall data is available. This technique also used for much larger areas and mostly used for small areas of watershed up to 1000 km² [2]. No meteorologist is needed in statistically approach and this technique requires less time to apply. A variety of procedures are available for the estimation of PMP such as Local Storm Maximization Method, Storm Transposition Method, Inferential Method, Generalized Method and Statistical method. These methods are adopted with respect to location of the watershed and availability of data as stated by WMO and Collier [3, 4].

Ghahraman et.al, [7] estimated the 24 hours PMP over Attrak by using the Hershfield formula with frequency factor of 15. Desa et al., [8] carried out the PMP in Malaysia by using statistical approach. 1-day maximum annual precipitation data of 30-60 years for 33 stations in Malaysia were used for the estimation of PMP. Statistically technique is basically a convenient and frequency analysis method. Statistically technique is preferred in those areas where climatological data such as Hourly rainfall data, daily dew point, temperature and daily wind speed data are not available [3][9]. In present study, different statistically approaches were used to estimate probable maximum precipitation for one day interval by using Hershfield, Gumble theory of extreme value and Log Pearson Type -III for high altitudes areas of Pakistan.

STUDY AREA DESCRIPTIONS
Murree is tehsil of Rawalpindi District and it is the exurb of Islamabad, approximately 60 kilometers southeast of Islamabad. The elevation of the Murree is situated between latitudes 33 54’ N and longitudes 73 26’ E. Pakistan is divided into four climatic zones: highland climate zone, lowland climatic zone, coastal climatic zone and arid climatic zone. Annual mean precipitation of Murree is 1789 mm that falls in highland climatic zone [11]. Coldest month is January with minimum and maximum temperature of -6 °C and 16.9 °C respectively [11]. Snowfall occurs in the study area usually in months of November to February and occasionally in March. The soil texture of the study region is different; loess, mixed material, alluvial and colluvial in nature and it is derived from sandstone and shale [12].

DATA ANALYSIS
Daily rainfall data of Murree for thirty year (1981-2010) was collected from Pakistan Meteorological Department (PMD), Islamabad. Annual maximum daily rainfall was sorted out as presented in Table. 2. The Hershfield method [13][14] was
used to estimate the Probable maximum precipitation and Gumbel method was used to compute the one day maximum rainfall for different return period i.e. 50, 100, 500, 1000 years for Murree.

**METHODOLOGY**

Three techniques; Hershfield, Gumble theory of extreme values and Log Pearson Type-III Distribution were used to compute the PMP and maximum precipitation. Chi-Square Goodness of fit test ($X^2 = \sum (\text{Observed} - \text{Expected})^2 / \text{Expected}$) was used to evaluate the most suitable and reliable distribution for this region.

**HERSHFIELD TECHNIQUE**

The Hershfield procedure to estimate the PMP is based on Chow general frequency equation as

$$PMP = X_{n\bar{}} + K_m \times S_n$$

Where: $X_{n\bar{}}$, $S_n$ is the mean and standard deviation of maximum series of N years, and $K_m$ is frequency factor. The empirically derived coefficient $K_m$ is calculated by using formula given as:

$$K_m = \left(\frac{X_m - X_{n\bar{}}}{S_{n-1}}\right)$$

Where: $K_m$ is frequency factor, $X_m$ is the largest value of the annual series, $X_{n\bar{}}$ is the mean of the annual series omitting the largest value and $S_{n-1}$ is the standard deviation of annual series omitting the largest value.

**GUMBEL'S METHOD**

Chow derived the specific relationship between return period ($T$) and frequency factor ($K$) for different probability distribution. The theoretical value of annual maximum daily rainfall for different return periods were computed by Gumble method [12]. The following equations were used to estimate maximum precipitation:

$$X_T = \bar{X} + K \sigma_{n-1}$$

Where $\sigma_{n-1}$ = precipitation standard deviation of the sample and $K =$ symbol of frequency factor.

$$K = -\sqrt{6/\pi} \left(\gamma_e + \ln \left[\frac{T_X}{(T_X - 1)}\right]\right)$$

Where “$\gamma_e$” is the Euler number (0.577216) and “$T_X$” is the desired return period of the quantity.

**LOG PEARSON TYPE III METHOD**

Three-parameter gamma distribution is sometime called Pearson type III distribution and most frequently used distributions for hydrologic frequency analysis. The equations were used to estimate to maximum precipitation:

$$C_s = N^2 \mu_3 / (N-1)(N-2)(\delta_z^3)$$

$$Z_T = Z + K_z \delta_z$$

$$X_T = \text{Exp}(Z_T)$$

Where; $N$ is the total number of observation, $C_s$ is the Coefficient of skewness and $X_T$ is the maximum amount of precipitation.

**RESULTS AND DISCUSSIONS**

Daily maximum 1-day precipitation in Murree for 30 years period (1981-2010) is shown in Fig. 2. Results revealed that the maximum (255 mm) and minimum (66 mm) annual 1-day precipitation was observed in year of 1992 and 1987, respectively. Hershfield technique was used to estimate the probable maximum precipitation (PMP). Gumbel method and Log Pearson type –III distribution was used to compute one day maximum rainfall for different return period 2years, 5 years, 10 years, and 25 years for the study area. The frequency factor ($K$) for Murree was estimated to be 3.50 for 1-day duration and this frequency factor was used to estimate 1-day PMP. The observed one day maximum precipitation during the period of analysis is presented in Table 1. This table shows that average annual 1-day maximum precipitation.
Precipitation of the study area is 121.85 mm with standard deviation and coefficient of variation 47.65 mm and 0.39, respectively. The PMP for one day duration over the region was computed 302.10 mm and the ratio of the one day PMP to highest 1-day rainfall (PMP/HOR) was 1.18. The expected maximum one day rainfall and Reduced variate (Y₁) for different return periods are given in Table 2. Similarly, The expected maximum one day rainfall and frequency factor for different return periods by using Log Pearson Type –III Distribution is presented in Table 3.

The coefficient of determination (R²) is the statistic that gives some information about the goodness of fit of a model. Gumble distribution was used to carried out the trend analysis and it was seen that logarithm trend line gives the better coefficient of determination (R² =0.9988). The trend analysis was also carried out using Gumble and Log Pearson type –III distribution and it was seen that power trend line gives better coefficient of determination (R² = 0.9929) and logarithm trend line does not give the better coefficient of determination (R² = 0.9325). The Results shows that Gumble distribution gives much better “R²” value as compared to Log Pearson type–III “R²” value. When Chi-Square goodness of fit test ((X² = Σ [(Observed - Expected)² / Expected]) applied, the value of Chi-Square is 10.21 on Gumble distribution and 11.06 on Log Pearson type –III distribution. So, Gumble distribution gives better results as compared to Log Pearson type –III distribution at high return period.

The expected maximum one day rainfall using Gumble distribution for 50 and 100 years return period was 245 and 271 mm, respectively. A maximum of 114 mm rainfall is expected to occur at every 2 years using Gumble and Log Pearson type –III distribution. Singh et al., [5] recommended that 2 to 100 return period is enough for construction of irrigation and drainage works, soil and water conservation measures and in dams. The PMF is calculated from the PMP which is used in drainage basin and major dams in the world are based on PMF. The spillway design floods for most of the dams in world were calculated by using the statistical approach. If the dams were to fail then loss of human life are expected. PMP is used to calculate the extent of flood plain areas at risk at maximum flood conditions. If reliable techniques of PMP are used then there should not be any risk of overtopping of dams.

In design of hydrological and civil structures, the structures have to be designed to carry maximum runoff expected in a specified recurrence interval. Durbude et.al, [15] has recommended different return periods for design of various hydraulic structures. Accordingly, The PMP for various hydrological structures were calculated and presented in Table 3. The PMP with different return period for vegetated waterways and Terrace outlets, Field diversion and small permanent masonry gully control structures were found as 184,200 and 184-200 mm, respectively. Similarly, PMP values for stock water dams, Earth fill dams-storage and storage and diversion dams having spillways were 219,219-271 and 245-271 mm, respectively.

Figure 2: Estimated Maximum One Day Rainfall for Different

![Graph showing estimated maximum one day rainfall for different return periods using Gumble distribution.](image)

Return Period Using Gumble Distribution
Table 1: Probable Maximum One Day Rainfall in Study Area

<table>
<thead>
<tr>
<th>Sr. No.</th>
<th>Parameters</th>
<th>values</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Average One Day Maximum Rainfall (mm)</td>
<td>121.85</td>
</tr>
<tr>
<td>2</td>
<td>Standard Deviation (mm)</td>
<td>47.65</td>
</tr>
<tr>
<td>3</td>
<td>Coefficient of variation (Cv)</td>
<td>0.3910</td>
</tr>
<tr>
<td>4</td>
<td>Highest Observed rainfall (mm) - HOR</td>
<td>255</td>
</tr>
<tr>
<td>5</td>
<td>Probable Maximum Precipitation (mm) - PMP</td>
<td>302.10</td>
</tr>
<tr>
<td>6</td>
<td>Frequency factor (Km)</td>
<td>3.50</td>
</tr>
<tr>
<td>7</td>
<td>Ratio PMP/HOR</td>
<td>1.18</td>
</tr>
</tbody>
</table>

Table 2: Estimated Maximum One Day Rainfall and Reduce Variate at Different Return Period at Murree

<table>
<thead>
<tr>
<th>Sr. No.</th>
<th>Return Period (Years)</th>
<th>Reduced Variate ($Y_i$)</th>
<th>Estimated Maximum One Day Rainfall By Using Gumble (mm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>2</td>
<td>0.37</td>
<td>114</td>
</tr>
<tr>
<td>2</td>
<td>5</td>
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<td>5</td>
<td>25</td>
<td>3.20</td>
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<td>6</td>
<td>30</td>
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<td>7</td>
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<tr>
<td>8</td>
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<td>9</td>
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<td>11</td>
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<td>13</td>
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<td>4.60</td>
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<td>5.52</td>
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<td>15</td>
<td>500</td>
<td>6.21</td>
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<tr>
<td>16</td>
<td>1000</td>
<td>6.91</td>
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CONCLUSIONS
A reliable statistical approach for estimating the one day PMP in the study area has been developed. Gumble distribution of extreme values is preferred on Log Pearson type-III distribution to estimate the maximum precipitation at different return periods into the study area. Probable maximum precipitation was estimated in the high altitude area with the aim to plan the soil and water conservation structures in the watershed. It is suggested that precipitation intensity is higher in higher altitude as compared to lower altitudes; So PMP values are higher in high elevation areas as compared to lower elevations areas. The study of Probable maximum precipitation is very useful in the designing of storage reservoirs and dams. PMP is used to investigate the soil and water conservation structures in the watershed. It is suggested that precipitation intensity is higher in higher altitude as compared to lower altitudes; So PMP values are higher in high elevation areas as compared to lower elevations areas. The study of Probable maximum precipitation is very useful in the designing of storage reservoirs and dams. PMP is used to investigate the adequacy of spillways of dams and design a civil structure properly. Statistical method was used and recommended to estimate the PMP due to limitation of data availability. The maximum one day rainfall for different return periods can be used for design of overflow arrangement of conservation structure, highway bridges, field diversion, storage dams, check dams and small and medium hydraulic structures.

REFERENCES
