F-Separated Sets and $F-$Connected Spaces

M. Baloush[1*], T. Noiri[2]
[1]National University College of Technology, Amman, Jordan
[2]22949-1 Shiokita-cho, Hinagu, Yatsushiro-shi, Kumamoto-ken, 869-5142, Japan
*For Correspondence; Tel. +962799614970, Email: malohblosh@yahoo.com

ABSTRACT – In this paper, we introduce the definitions of $F-$separated sets, $F-$connected sets and $F-$compact sets and study their properties.

Keywords: F-open set, F-separated set, $F-$connected space, $F-$compact sets

1 INTRODUCTION

In 2023, Alqahtani has introduced the notion of F-open sets in a topological space. Every F-open set is an open set but not conversely. The family of F-open sets is not necessarily topology but has similar properties with open sets. In [1] , the author obtained many properties of F-open sets. In this paper, we introduce F-separated sets and F-connected sets. In Section 2, we obtain several properties of F-separated sets. In Section 3, we investigate the further properties of F-connected and F-compact relative sets. Finally, it is shown that F-connected sets are preserved under F-continuous surjections.

Definition 1.1. [1] *An open subset A of a topological space (X,τ) is called an $F-$open set if $Cl(A)\backslash A$ is a finite set. That is, A is an open set and the frontier of A is a finite set.*

Definition 1.2. [1] *A closed subset A of a topological space(X,τ) is called an F-closed set if $A\backslash Int(A)$ is a finite set. That is, A is a closed set and the frontier of A is a finite set.*

Definition 1.3. [1] *Let U be a subset of a topological space (X,τ). Then, the $F-$closure of U is defined as the intersection of all $F-$closed sets containing U, and is denoted by $Cl^F(U)$.*

2 *F*-SEPARATED SETS AND THIER PROPERTIES

In this section we will define the $F-$separated sets and discuss their properties.

Definition 2.1. *Let (X,τ) be a topological space and A,B be nonempty subsets of X. Then A and B are said to be $F-$ separated if $A\cap Cl^F(B)=\emptyset$ and $Cl^F(A)\cap B=\emptyset$.*

Theorem 2.2. *Let A and B be $F-$ separated sets in a space X, and let D and K be nonempty subsets of A and B, respectively. Then D and K are also $F-$ separated in X.*

Proof. Let D and K be nonempty subsets of the $F-$ separated sets A and B, respectively. Since $D\subseteq A$, then $Cl^F(D)\subseteq Cl^F(A)$. But $Cl^F(A)\cap B=\emptyset$ which implies that $Cl^F(D)\cap B=\emptyset$, and since $K\subseteq B$, then

$$Cl^F(D)\cap K=\emptyset\cdots\cdots(1).$$

Similarly, since $K\subseteq B$, then $Cl^F(K)\subseteq Cl^F(B)$. Now $A\cap Cl^F(B)=\emptyset$ which implies that $A\cap Cl^F(K)=\emptyset$, and since $D\subseteq A$, then

$$D\cap Cl^F(K)=\emptyset\cdots\cdots(2)$$

By (1) and (2), we get $Cl^F(D)\cap K=\emptyset$ and $D\cap Cl^F(K)=\emptyset$. Therefore, D and K are $F-$ separated in X.

Theorem 2.3. *Let A,B be nonempty disjoint subsets of a space X such that A and B are either both $F-$ open or both $F-$ closed. Then A and B are $F-$ separated.*

Proof. Let A,B be nonempty subsets of X.

(1) Suppose that A and B are both $F-$ closed. Since $A\cap B=\emptyset$,then $A\cap B=A\cap Cl^F(B)=Cl^F(A)\cap B=\emptyset$. Therefore, A and B are $F-$ separated.

(2) Suppose that A and B are both $F-$ open. Since $A\cap B=\emptyset$. Then $A\subseteq X-B$ which implies that $Cl^F(A)\subseteq Cl^F(X-B)=X-B$. Hence $Cl^F(A)\subseteq X-B$, and $Cl^F(A)\cap B=\emptyset$. Similarly, we have $Cl^F(B)\cap A=\emptyset$. Therefore A and B are $F-$ separated.

Theorem 2.4. *Let A,B be nonempty subsets of X such that A,B are either both $F-$ open or both $F-$ closed. If $C=A\cap(X-B)$ and $D=B\cap(X-A)$, then C,D are $F-$ separated.*

Proof. Let A,B be nonempty subsets of X.

(1) Suppose that A and B are both $F-$ closed. Since $C=A\cap(X-B)$, then $C\subseteq A$, which implies that $Cl^F(C)\subseteq Cl^F(A)=A$. Hence $Cl^F(C)\cap D=\emptyset$. Similarly, since $D=B\cap(X-A)$, then $D\subseteq B$, which implies that $Cl^F(D)\subseteq Cl^F(B)=B$.Hence $Cl^F(D)\cap C=\emptyset$. Therefore C and D are $F-$separated.

(2) Suppose that A and B are both $F-$ open, and since $C=A\cap(X-B)$, then $C\subseteq X-B$, which implies that $Cl^F(C)\subseteq Cl^F(X-B)=X-B$. Hence $Cl^F(C)\cap D=\emptyset$. Similarly, we have $Cl^F(D)\cap C=\emptyset$. Therefore C and D are $F-$ separated.

Theorem 2.5. *Let A,B be nonempty subsets of X. Then A,B are $F-$ separated if and only if there exist $F-$ open sets U and V such that $A\subseteq U,B\subseteq V,A\cap V=\emptyset$, and $B\cap U=\emptyset$.*

Proof. $Necessity$. Let A,B be $F-$ separated sets. Since $A\cap Cl^F(B)=\emptyset$ and $Cl^F(A)\cap B=\emptyset$, then $A\subseteq X-Cl^F(B)$ and $B\subseteq X-Cl^F(A)$. Since $Cl^F(A),Cl^F(B)$ are $F-$ closed, then $U=X-Cl^F(B)$, $V=X-Cl^F(A)$ are $F-$ open sets and $A\subseteq U$ and $B\subseteq V$. Now $A\subseteq Cl^F(A)=X-V$, then $A\cap V=\emptyset$. Similarly, Since $B\subseteq Cl^F(B)=X-U$, then $B\cap U=\emptyset$.

$Sufficiency$. let U and V be $F-$ open sets such that $A\subseteq U,B\subseteq V,A\cap V=\emptyset$, and $B\cap U=\emptyset$. Since U and V are $F-$ open sets, then $X-U$ and $X-V$ are $F-$ closed sets. But $A\cap B=\emptyset$, then $A\subseteq X-V$ and $B\subseteq X-U$. Now, $Cl^F(A)\subseteq Cl^F(X-V)=X-V$ which implies that $Cl^F(A)\cap V=\emptyset$ and then $Cl^F(A)\cap B=\emptyset$. Similarly, since $B\subseteq X-U$, then $Cl^F(B)\subseteq Cl^F(X-U)=X-U$ which implies that $Cl^F(B)\cap U=\emptyset$ and then $Cl^F(B)\cap A=\emptyset$. Therefore, A,B are $F-$ separated.

3 *F*-CONNECTED SETS AND THIER PROPERTIES

In this section we will define the $F-$connected sets and discuss their properties.

Definition 3.1. *A subset A of X is said to be $F-$ connected if it can not be represented as the union of two nonempty $F-$ separated sets. If X is $F-$connected, then X is called an $F-$*

connected space.

Theorem 3.2. *A non-empty subset C of X is $F-$ connected if and only if for every pair of $F-$ separated sets A and B in X with $C \subseteq A \cup B$, one of the following possibilities holds:*

1. $C \subseteq A$ *and* $C \cap B = \emptyset$,
2. $C \subseteq B$ *and* $C \cap A = \emptyset$.

Proof. Necessity. Let C be an $F-$ connected subset of X. Let A and B be F-separated sets in X such that $C \subseteq A \cup B$, then $C \cap B = \emptyset$ and $C \cap A = \emptyset$ can not hold at the same time. If $C \cap B = \emptyset$, then $C \subseteq A$, and if $C \cap A = \emptyset$, then $C \subseteq B$. Finally, if $C \cap B \neq \emptyset$ and $C \cap A \neq \emptyset$, then by Theorem 2.2, both $C \cap B$ and $C \cap A$ are $F-$ separated and $C = (C \cap B) \cup (C \cap A)$ which is a contradiction since C is an $F-$ connected subset of X.
Sufficiency. Suppose C is not an $F-$ connected set of X, then there exists two nonempty $F-$ separated sets A and B in X such that $C = A \cup B$. By conditions (1) and (2) we have either $C \cap B = \emptyset$ or $C \cap A = \emptyset$ which implies that either $A = \emptyset$ or $B = \emptyset$ which is a contradiction since A and B are nonempty sets in X. Therefore, C is an $F-$ connected set of X.

Theorem 3.3. *Let U be a subset of of X. Then the following are equivalent;*

1. *U is $F-$ connected,*
2. *There exist no two $F-$ closed sets A and B such that $A \cap U \neq \emptyset$, $B \cap U \neq \emptyset$, $U \subseteq A \cup B$ and $A \cap B \cap U = \emptyset$,*
3. *There exist no two $F-$ closed sets A and B such that $U \nsubseteq A, U \nsubseteq B, U \subseteq A \cup B$ and $A \cap B \cap U = \emptyset$.*

Proof.

1. $(1 \Rightarrow 2)$: Suppose that there exist two $F-$ closed sets A and B such that $A \cap U \neq \emptyset$, $B \cap U \neq \emptyset$, $U \subseteq A \cup B$ and $A \cap B \cap U = \emptyset$. Then
   $(A \cap U) \cup (B \cap U) = (A \cup B) \cap U = U$,
   and
   $Cl^F(A \cap U) \cap (B \cap U) \subseteq Cl^F(A) \cap (B \cap U) = A \cap B \cap U = \emptyset$
   Hence, $Cl^F(A \cap U) \cap (B \cap U) = \emptyset$. By the same argument we get $(A \cap U) \cap Cl^F(B \cap U) = \emptyset$. Therefore, U is not $F-$ connected. This shows that (1) implies (2).
2. $(2 \Rightarrow 3)$: Let (2) hold and suppose that there exist two $F-$ closed sets A and B such that $U \nsubseteq A, U \nsubseteq B, U \subseteq A \cup B$ and $A \cap B \cap U = \emptyset$. Then $A \cap U \neq \emptyset$ and $B \cap U \neq \emptyset$ which is a contradiction.
3. $(3) \Rightarrow (1)$: Suppose that U is not $F-$ connected. Then there exist two non-empty $F-$ separated sets C and D such that $U = C \cup D$. Now $Cl^F(C) \cap D = C \cap Cl^F(D) = \emptyset$. Let $A = Cl^F(C)$ and $B = Cl^F(D)$, then $U \subseteq A \cup B$ and $Cl^F(C) \cap Cl^F(D) \cap (C \cup D) = \left(Cl^F(C) \cap Cl^F(D) \cap (C)\right) \cup \left(Cl^F(C) \cap Cl^F(D) \cap (D)\right) \subset (Cl^F(D) \cap C) \cup (D \cap Cl^F(C) = \emptyset$. Now if $U \subseteq A$, then $Cl^F(D) \cap U = B \cap U = B \cap (U \cap A) = \emptyset$, which is a contradiction, then $U \nsubseteq A$. Similarly, $U \nsubseteq B$. Thus U is a $F-$ connected.

Theorem 3.4. *Let U be an $F-$ connected subset of X. If $U \subseteq V \subseteq Cl^F(U)$, then V is also $F-$ connected.*

Proof. Let U be an $F-$ connected subset of X such that $U \subseteq V \subseteq Cl^F(U)$. Suppose that V is not $F-$ connected. Then by Theorem 3.3, there exists two $F-$ closed sets A and B such that $V \nsubseteq A, V \nsubseteq B, V \subseteq A \cup B$ and $A \cap B \cap V = \emptyset$. Since $U \subseteq V$, then $U \subseteq A \cup B$ and $A \cap B \cap U = \emptyset$. Now if $U \subseteq A$, then $Cl^F(U) \subseteq Cl^F(A) = A$. Therefore, $V \subseteq A$ which is a contradiction. Thus $U \nsubseteq A$. By the same argument, $U \nsubseteq B$. Which contradicts that U is a $F-$ connected.

Corollary 3.5. *If U is an $F-$ connected subset of X, then $Cl(U)$ is F-connected.*

Prof. Since every F-open set is open, $Cl(U) \subset Cl^F(U)$ and by Theorem 3.4, $Cl(U)$ is F-connected.

Theorem 3.6. *If A and B are $F-$ connected subsets of a space X and A, B are not F-separated, then $A \cup B$ is F-connected.*

Proof. Let A and B be $F-$ connected subsets of a space X. Suppose that $A \cup B$ is not F-connected. Then, there exist two non-empty F-separated sets G and H such that $A \cup B = G \cup H$. Hence, $Cl^F(G) \cap H = \emptyset$ and $G \cap Cl^F(H) = \emptyset$. Since A and B are F-connected, $A \subset G$ or $A \subset H$, and $B \subset G$ or $B \subset H$. Therefore, (i) $A \subset G$ and $B \subset H$ or (ii) $A \subset H$ and $B \subset G$.

(i) Suppose that $A \subset G$ and $B \subset H$.Then, $A \cap H \subset G \cap H = \emptyset$ and $B \cap G \subset H \cap G = \emptyset$. Therefore, $(A \cup B) \cap G = (A \cap G) \cup (B \cap G) = (A \cap G) = A$ and $(A \cup B) \cap H = (A \cap H) \cup (B \cap H) = (B \cap H) = B$. Hence $Cl^F(A) \cap B = Cl^F([(A \cup B) \cap G]) \cap [(A \cup B) \cap H] \subset Cl^F(G) \cap H = \emptyset$ and $Cl^F(A) \cap B = \emptyset$. Similarly, we obtain $A \cap Cl^F(B) = \emptyset$. This shows that A, B are F-separated. This is a contradiction.

(ii) Suppose that $B \subset G$ and $A \subset H$. Then, $B \cap H \subset G \cap H = \emptyset$ and $A \cap G \subset H \cap G = \emptyset$. Therefore, $(A \cup B) \cap H = (A \cap H) \cup (B \cap H) = A \cap H = A$ and $(A \cup B) \cap G = (A \cap G) \cup (B \cap G) = B \cap G = B$. Hence $Cl^F(A) \cap B = Cl^F([(A \cup B) \cap H]) \cap B \subset Cl^F(H) \cap G = \emptyset$ and $A \cap Cl^F(B) = A \cap Cl^F[(A \cup B) \cap G] \subset H \cap Cl^F(G) = \emptyset$. This shows that A, B are F-separated. This is a contradiction.

Therefore, $A \cup B$ is F-connected.

Corollary 3.7. *If A and B are $F-$ connected subsets of a space X and A, B are disjoint, then $A \cup B$ is F-connected.*

Proof. If A, B are disjoint, then A, B are not F-separated. By Theorem 3.1, $A \cup B$ is F-connected.

Theorem 3.8. *If $\{M_\alpha : \alpha \in \Delta\}$ is a nonempty family of F-connected subsets of a space (X, τ) and $\cap_{\alpha \in \Delta} M_\alpha \neq \emptyset$, then $\cup_{\alpha \in \Delta} M_\alpha$ is F-connected.*

Proof. Suppose that $\cup_{\alpha \in \Delta} M_\alpha$ is not F-connected. Then there exist nonempty F-separated sets H, G such that $\cup_{\alpha \in \Delta} M_\alpha = H \cup G$. Since $\cap_{\alpha \in \Delta} M_\alpha \neq \emptyset$, there exists a point $x \in \cap_{\alpha \in \Delta} M_\alpha$ and $x \in \cup_{\alpha \in \Delta} M_\alpha$.Therefore, $x \in H$ or $x \in G$.

(i) Let $x \in H$. Since $x \in M_\alpha$ for every $\alpha \in \Delta$ and $M_\alpha \subset H \cup G$, by Theorem 3.2, $M_\alpha \subset H$ or $M_\alpha \subset G$. Since $H \cap G = \emptyset$, we have the following: (a) if $M_\alpha \subset H$ for every $\alpha \in \Delta$, then $\cup_{\alpha \in \Delta} M_\alpha \subset H$ and $G = \emptyset$. This is a contradiction. (b) if $M_\alpha \subset G$ for every $\alpha \in \Delta$, then $\cup_{\alpha \in \Delta} M_\alpha \subset G$ and $H = \emptyset$. This is a contradiction.

(ii) Let $x \in G$. Since $x \in M_\alpha$ for every $\alpha \in \Delta$ and $M_\alpha \subset H \cup G$, by Theorem 3.2, $M_\alpha \subset H$ or $M_\alpha \subset G$. Since $H \cap G = \emptyset$, we have the following: (a) if $M_\alpha \subset G$ for every $\alpha \in \Delta$, then $\cup_{\alpha \in \Delta} M_\alpha \subset G$ and $H = \emptyset$. This is a contradiction. (b) if

$M_\alpha \subset H$ for every $\alpha \in \Delta$, $\cup_{\alpha\in\Delta} M_\alpha \subset H$ and $G = \emptyset$. This is a contradiction.Therefore, x is not contained in $H \cup G = \cup_{\alpha\in\Delta} M_\alpha$. This is a contradiction. Consequently, $\cup_{\alpha\in\Delta} M_\alpha$ is F-connected.

Corollary 3.9. Let (X,τ) be a topological space. Then:

1) *If each pair of points x, y in a space (X,τ) lies in some F-connected subset $E_{x,y}$ of X, then X is F-connected.*
2) *If $X = \cup_{n=1}^{\infty} X_n$, where each X_n is F-connected and $X_{n-1} \cap X_n \neq \emptyset$ for each $n \geq 2$, then X is F-connected.*

Proof.

1) Choose a point $a \in X$ and fix it. Then, for each point $x \in X$, there exists an F-conneced set E_x such that $x, a \in E_x$ and hence $X = \cup_{x\in X} E_x$. By Theorem 3.8, X is F-connected.

2) X_1 is F-connected. If $X_1 \cup \ldots \cup X_{n-1}$ is F-connected, by Theorem 3.1 $A_n = X_1 \cup \ldots \cup X_n$ is F-connected for $n = 1,2,\ldots$, where $\cap A_n = X_1 \neq \emptyset$ and by Theorem 3.8 $X = \cup_{n=1}^{\infty} A_n$ is F-connected.

Theorem 3.10. *Let $\{M_\alpha : \alpha \in \Delta\}$ be a nonempty family of F-connected subsets of a space (X,τ) and A be a nonempty F-connected set. If $A \cap M_\alpha \neq \emptyset$ for each $\alpha \in \Delta$, then $A \cup (\cup_{\alpha\in\Delta} M_\alpha)$ is F-connected.*

Proof. Since $A \cap M_\alpha \neq \emptyset$ for each $\alpha \in \Delta$, by Theorem 3.6, $A \cup M_\alpha$ is F-connected for each $\alpha \in \Delta$. Moreover, $A \cup (\cup_{\alpha\in\Delta} M_\alpha) = \cup (A \cup_{\alpha\in\Delta} M_\alpha)$ and $\cap (A \cup M_\alpha) \supset A \neq \emptyset$. Therefore, by Theorem 3.3 $A \cup (\cup_{\alpha\in\Delta} M_\alpha)$ is F-connected.

Definition 3.11. *A function $f : (X,\tau) \to (Y,\sigma)$ is said to be F-continuous if for each open set $V \in \sigma$, $f^{-1}(V)$ is F-open in (X,τ).*

Theorem 3.12. *If $f : (X,\tau) \to (Y,\sigma)$ is an F-continuous surjection and (X,τ) is F-connected, then (Y,σ) is F-connected.*

Proof. Suppose that (Y,σ) is not F-connected. Then, there exist F-separated sets A and B such that $A \neq \emptyset$, $B \neq \emptyset$, $Y = A \cup B$. Hence $Cl^F(A) \cap B = \emptyset = A \cap Cl^F(B)$. Since f is F-continuous, $f^{-1}(Cl(A))$ is F-closed and $f^{-1}(A) \subset f^{-1}(Cl(A))$. Therefore, $Cl^F(f^{-1}(A)) \subset f^{-1}(Cl(A)) \subset f^{-1}(Cl^F(A))$ and hence $Cl^F(f^{-1}(A)) \cap f^{-1}(B) \subset f^{-1}(Cl^F(A)) \cap f^{-1}(B) = f^{-1}(Cl^F(A) \cap B) = \emptyset$. Similarly, we obtain $f^{-1}(A) \cap Cl^F(f^{-1}(B)) = \emptyset$. Hence, $f^{-1}(A)$ and $f^{-1}(B)$ are F-separated. Since f is surjective, $f^{-1}(A)$ and $f^{-1}(B)$ are nonempty. Moreover, $X = f^{-1}(Y) = f^{-1}(A \cup B) = f^{-1}(A) \cup f^{-1}(B)$. This shows that (X,τ) is not F-connected.

After we studied the properties of F-connected and $F-$ separated sets we can state the relation between F-connected and $F-$ separated with connected and separated sets in the following proposition:

Proposition 21.

1. *Let A, B be nonempty subsets of X. If A, B are $F-$separated, then A, B are separated.*
2. *Let U be a subset of X. If U is connected, then it is $F-$connected.*

Proof.

1. If A, B are $F-$separated, then $A \cap Cl^F(B) = empty = Cl^F(A) \cap B$. Since $\tau F \subset \tau, Cl(A) \subset Cl^F(A)$ and $Cl(B) \subset Cl^F(B)$. Hence $A \cap Cl(B) = empty = Cl(A) \cap B$. and A, B are separated.

2. Suppose that U is not $F-$connected. Then, there exist nonempty $F-$separated sets A and B such that $U = A \cup B$. By (1), A, B are separated and U is not connected.

Definition 3.13. *A function $f : (X,\tau) \to (Y,\sigma)$ is $F-$ open [1] (resp. $F-$preserving) if $f(U)$ is $F-$open in Y for each open (resp. $F-$open) set U in X.*

It is obvious that every F-open function is F-preserving.

Definition 3.14. Let (X,τ) be a topological space and A be a subset of X. A is said to be $F-$compact relative to X if for every cover $\{V\alpha : \alpha \in \Delta\}$ of A by open sets of X, there exists a finite subset Δ_0 of Δ such that V_α is F-open for each $\alpha \in \Delta_0$ and $A \subset \cup \{V\alpha : \alpha \in \Delta\_0\}$.

If X is $F-$compact relative to X, then X is said to be $F-$compact [1].

Theorem 3.15. Let $f : (X,\tau) \to (Y,\sigma)$ be a continuous and $F-$preserving sur jection. If A is F-compact relative to X, then $f(A)$ is $F-$compact relative to Y.

Proof. Let $\{V\alpha : \alpha \in \Delta\}$ be any cover of $f(A)$ by open sets of Y. Then $A \subset f^{-1}(f(A)) \subset \cup \{f^{-1}(V_\alpha) : \alpha \in \Delta\}$. Since f is continuous, $\{f^{-1}(V_\alpha) : \alpha \in \Delta\}$ is an open cover of A. Since A is $F-$compact, there exists a finite subset Δ_0 of Δ such that $f^{-1}(V_\alpha)$ is $F-$open for each $\alpha \in \Delta_0$ and $A \subset \cup \{f^{-1}(V_\alpha) : \alpha \in \Delta_0\}$. Therefore, $f(A) \subset \cup f(\{f^{-1}(V\alpha) : \alpha \in \Delta_0\}) = \cup \{f(f^{-1}(V\alpha)) : \alpha \in \Delta_0\}) = \cup \{V_\alpha : \alpha \in \Delta_0\}$. Since $f^{-1}(V_\alpha)$ is $F-$open and f is $F-$preserving, then, $f(f^{-1}(V\alpha)) = V_\alpha$ is $F-$open for each $\alpha \in \Delta_0$. Therefore, $f(A)$ is $F-$compact relative to Y.

Corollary 3.16. Let $f : (X,\tau) \to (Y,\sigma)$ be an $F-$continuous and $F-$open surjection. If X is $F-$compact, then Y is $F-$compact.

Proof. Every $F-$continuous (resp. $F-$open) function is continuous (resp. $F-$preserving). Hence this follows from immediately from Theorem 3.15.

Finally, and after we studied the properties of F-connected and $F-$ separated sets we can state the relation between F-connected and $F-$ separated with connected and separated sets in the following proposition:

Proposition 3.17. *Let A, B and U be nonempty subsets of X:*

1. *If A, B are $F-$separated, then A, B are separated.*
2 *If U is connected, then it is $F-$connected.*

Proof.

1. If A, B are $F-$separated, then $A \cap Cl^F(B) = empty = Cl^F(A) \cap B$. Since $\tau F \subset \tau, Cl(A) \subset Cl^F(A)$ and $Cl(B) \subset Cl^F(B)$. Hence $A \cap Cl(B) = empty = Cl(A) \cap B$. and A, B are separated.

2. Suppose that U is not $F-$connected. Then, there exist nonempty $F-$separated sets A and B such that $U = A \cup B$. By (1), A, B are separated and U is not connected.

4 REFERENCES

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