ENHANCING MATHEMATICAL PROFICIENCY ASSESSMENT: INSIGHTS FROM MATHEMATICS TEACHERS

Mary Cris J Go
Northern Bukidnon State College, Bukidnon, Philippines
mjgo@nbsc.edu.ph

ABSTRACT: *Mathematical proficiency is a crucial aspect of education, particularly for elementary and secondary-level learners. It is essential for their future success in the field and beyond. Developing appropriate assessment tools is a significant endeavor in education to measure and foster this proficiency. This study explores the experiences of mathematics teachers in assessing learners' mathematical proficiency, addressing the gap in understanding how educators evaluate and promote mathematical proficiency in the classroom. This study employs a phenomenological approach, and qualitative data was gathered through focus group discussions (FGDs) with six (6) mathematics teachers, consisting of three (3) high school teachers and three (3) higher education mathematics instructors. This study searches into four strands: Conceptual Understanding, Procedural Fluency, Strategic Competence, and Adaptive Reasoning. Two key themes emerged in each strand: in Conceptual Understanding, the importance of connecting mathematical concepts to real-life applications and the ability to analyze problems; in Procedural Fluency, the clarity of explanations and accurate procedure performance; in Strategic Competence, the selection and application of strategies and analyzing complex problems; and in Adaptive Reasoning, the application of mathematical concepts in real-life scenarios and the ability to make conjectures and draw conclusions. This study contributes to the development of a comprehensive tool that bridges the gap in mathematical assessment by exploring the criteria employed by experienced mathematics teachers to assess learners' mathematical proficiency. Ultimately, this research will improve mathematics education, empowering learners with the mathematical skills they need to thrive in the modern world*.

Keywords: mathematical proficiency; assessment; phenomenology; conceptual understanding; procedural fluency; strategic competence; adaptive reasoning

I. INTRODUCTION

The landscape of education is rapidly evolving, driven by the demands of the Fourth Industrial Revolution (4IR). In this era of technological advancements, automation, artificial intelligence, and data-driven decision-making, mathematics education has never been more critical, Butler-Adam [1]. Learners must develop mathematical proficiency, not only to meet the demands of the job market but also to become informed and empowered global citizens. In the 21st century, mathematical proficiency is an essential skill for navigating complex problems, making informed decisions, and contributing to the development of society [2].

The urgency of developing mathematical proficiency is underscored by the disruptive impact of the 4IR on the job market. This revolution has led to a paradigm shift in employment trends, with a growing demand for individuals who possess advanced problem-solving skills, critical thinking abilities, and proficiency in science, technology, engineering, and mathematics (STEM) fields. To equip learners with the necessary skills to thrive in this environment, it is crucial to develop their mathematical proficiency from an early age [3].

One of the cornerstones of fostering mathematical proficiency is the development of appropriate assessment tools. Effective assessment is a key driver of learning, providing feedback to both teachers and students about progress and areas in need of improvement, Burkhardt [4]. With well-designed assessments, educators can tailor their teaching methods to better meet the unique needs of each learner. Developing an appropriate and comprehensive assessment tool for mathematical proficiency is essential for nurturing mathematical talent and ensuring that students are well-prepared for future challenges. However, the development of such an assessment tool requires a profound understanding of how mathematics teachers assess learners' mathematical proficiency in the classroom. It is the teachers who play a pivotal role in shaping students' mathematical abilities, guiding their understanding of key mathematical concepts, and honing their problem-solving skills. These educators are the frontline agents in the endeavor to equip students with mathematical proficiency.

Understanding the process and criteria that mathematics teachers employ to assess learners' mathematical proficiency is the foundation of this study. This research aims to address the existing gap in knowledge regarding how mathematics teachers evaluate students' mathematical abilities, with a specific focus on the four critical strands of Conceptual Understanding, Procedural Fluency, Strategic Competence, and Adaptive Reasoning.

Researchers such as Leung [5] have highlighted the significance of aligning classroom assessments with instructional goals. They argue that effective classroom assessments when designed and executed thoughtfully, can promote students' mathematical understanding. Furthermore, Shepard [6] emphasizes that assessment plays a substantial role in the mathematics classroom, affecting instruction, curriculum design, and student learning. Their study found that mathematics teachers integrate assessment practices into their daily teaching to understand their students' progress better. Additionally, Hattie and Timperley [7] contend that feedback from assessment is essential for enhancing learning. Their research demonstrates that teachers' understanding of how learners perceive their assessments can lead to more effective teaching strategies. These scholars collectively affirm the critical role of assessment in shaping the mathematics classroom environment and fostering mathematical proficiency.

The need to study how mathematics teachers assess learners' mathematical proficiency in the classroom becomes even more apparent when considering the empirical work of Suurtamm [8]. Their research explores the relationship between assessment and curriculum. They emphasize that how teachers assess can significantly influence what they teach. As such, understanding teachers' assessment practices can provide insights into the teaching and learning processes that promote mathematical proficiency. To comprehensively address the research gap and bridge the divide between assessment and mathematical proficiency, the current study employs a phenomenological approach. Phenomenology is well-suited to this project as it enables an in-depth exploration of teachers' lived experiences and perceptions regarding learners' mathematical proficiency. This method delves into the subjective world of mathematics teachers, allowing them to express their perspectives, understandings, and practices related to assessing students' mathematical abilities [9]. Phenomenology aligns with the research objectives by offering an effective means to explore the lived experiences of mathematics teachers who assess learners' mathematical proficiency in the classroom. By engaging teachers in reflective discussions about their assessment practices, this research aims to uncover the nuanced criteria and indicators they employ, thereby providing valuable insights for the development of the Mathematical Proficiency Assessment tool. This tool aims to address the research gap and contribute to the ongoing efforts to enhance mathematical proficiency among basic and secondary-level learners.

II. METHODS

Research Design

This research employs a phenomenological research design to investigate how mathematics teachers assess learners' mathematical proficiency in the classroom across four mathematical proficiency strands: Conceptual Understanding, Procedural Fluency, Strategic Competence, and Adaptive Reasoning. Phenomenology is a qualitative research approach that seeks to understand and describe the lived experiences of individuals concerning a particular phenomenon, Creswell & Poth [10]. Phenomenology is appropriate for this research because it enables an in-depth exploration of how mathematics teachers assess learners' mathematical proficiency in the classroom, providing insights into the phenomena and meaning-making processes within their teaching practices, Creswell & Poth [10]. Phenomenology aligns with the research's primary goal of uncovering the rich and nuanced perspectives of mathematics teachers regarding the assessment of mathematical proficiency.

By adopting this design, the study focuses on capturing the essence of these experiences, highlighting the complexities and intricacies of the assessment process. As Creswell & Poth [10] emphasize, phenomenology allows researchers to delve into the core of human experiences, revealing the significance, nuances, and variations that might be overlooked by other research approaches. Furthermore, the choice of phenomenology as the research design provides a framework for understanding the processes through which mathematics teachers assess learners' mathematical proficiency in the classroom. It allows the study to explore the criteria, indicators, and contextual factors that influence teachers' assessments. This aligns with the overarching research goal of developing a comprehensive understanding of mathematical proficiency assessment.

Research Participants

Six (6) participants were selected for two separate FGDs, consisting of three (3) secondary-level mathematics teachers and three (3) higher education mathematics teachers. The selection of these participants was based on specific criteria. First, the participants were required to be currently serving as mathematics teachers, ensuring that they possessed practical and recent experiences in assessing learners' mathematical proficiency in real classroom settings. Second, participants were required to have a minimum of five years of teaching experience, ensuring that they had accumulated a wealth of experience and insights that could contribute to the study. Third, all selected participants were actively teaching mathematics subjects. The rationale for these criteria was to ensure that the participants were well-versed in the context of mathematical education and capable of providing meaningful insights. Furthermore, the selection of six (6) participants aligns with the phenomenological research approach, which typically involves a small number of participants to allow for a detailed examination of their experiences, Creswell [11]. Phenomenological research is not focused on generalizability, but rather on understanding the unique experiences of individuals. Therefore, the findings of this study may not be generalized to all mathematics teachers but will provide valuable insights into the lived experiences of the selected participants, which can be valuable for in-depth understanding and further research in the field.

Data and Collection

The research employed focus group discussions (FGDs) as the primary data collection method. FGDs are particularly effective in a qualitative research context, as they allow participants to engage in interactive and open discussions about their experiences and perspectives. FGDs foster dialogue and encourage participants to build on each other's responses, leading to rich and varied insights [12].

In this study, FGDs were conducted following ethical considerations and a structured process. Prior to the conduct of the FGDs, the researcher sent a consent letter to all selected participants, outlining the study's purpose and assuring confidentiality. The participants were required to sign the consent letter, demonstrating their voluntary participation in the discussions. The FGDs were divided into two separate sessions, one for the secondary-level teachers and another for the higher education teachers. The division aimed to maintain the homogeneity of the groups and ensure that discussions were centered on participants' shared experiences. The sessions occurred over two days, with an hour dedicated to orientation and an additional hour for the FGD discussions. During the orientation sessions, participants were provided with a clear understanding of the research objectives and the FGD process. They were encouraged to share their experiences candidly and express their opinions openly. Ethical considerations were emphasized to ensure that participants felt comfortable and respected throughout the discussions. They were also provided with an hour-long explanation of the four mathematical proficiency strands. Examples were shared to help participants understand how learners' mathematical proficiency could be assessed in each strand. This introductory session aimed to ensure that all participants had a common understanding of the research focus.

The FGD questionnaire (Table 1) used in this research was rigorously developed to align with the study's objectives. Prior to the FGDs, the questionnaire was validated by three (3) mathematics experts. The experts reviewed the questions to ensure their relevance, clarity, and appropriateness for exploring how mathematics teachers assess learners' mathematical proficiency in the classroom. The questionnaire was structured around four strands of mathematical proficiency: Conceptual Understanding, Procedural Fluency, Strategic Competence, and Adaptive Reasoning. Furthermore, the questions were framed to facilitate discussions that revolved around the theme of how mathematics teachers assess learners' mathematical proficiency within each of the four strands. The questions aimed to capture the diverse experiences and practices of the participating teachers in assessing mathematical proficiency, from their perspectives. The FGD questions were designed to be open-ended, allowing participants to express their thoughts and experiences freely. The questions were structured to encourage in-depth discussions, providing ample space for participants to elaborate on their assessment methods and the challenges they encountered.

Table 1. FGD Questionnaire

Research Questions
1. How do you assess students' mathematical proficiency in terms of a) Conceptual Understanding? b) Procedural Fluency? c) Strategic Competence? d) Adaptive Reasoning?
2. When rating students written answers, what indicators do you consider when assessing your students' mathematical proficiency in terms of a) Conceptual Understanding? b) Procedural Fluency? c) Strategic Competence? d) Adaptive Reasoning?
3. How do you describe a student who has a strong, average or weak mathematical proficiency in terms of a) Conceptual Understanding? b) Procedural Fluency? c) Strategic Competence? d) Adaptive Reasoning?.

Data Analysis

The process of thematic analysis followed established procedures to ensure the credibility and trustworthiness of the qualitative data and data analysis. The thematic analysis process was employed to extract the emerging themes from the FGD data. The process followed established procedures in qualitative research, Braun & Clarke [13]. It began with a careful review of the voice recordings of the FGDs. These transcriptions were then translated from vernacular to English, preserving the participants' original responses. Next, the coding process was conducted. The process involved tagging similar codes related to the themes across the different strands of mathematical proficiency. These tags were then grouped based on shared characteristics and content. This process helped in refining and finalizing the themes.

The credibility and trustworthiness of the qualitative data were ensured by employing peer debriefing and member checking. In peer debriefing, an independent researcher reviewed the themes and their interpretation to validate the findings, Johnson, Adkins & Chauvin [14]. Member checking involved sharing the emerging themes with the participants to confirm their accuracy and alignment with their responses [15].

The themes were named based on their core content and essence, ensuring that they accurately represented the participants' experiences and perspectives. Thematic analysis, with these measures in place, provides a robust and comprehensive way to explore how mathematics teachers assess learners' mathematical proficiency in the classroom, making the findings trustworthy and insightful [13].

The participants were coded for anonymity and identification purposes, using labels such as T1 for Teacher 1, T2 for Teacher 2, and so forth. This coding system allowed for a systematic organization of the data, facilitating the analysis and presentation of the findings.

III. RESULTS AND DISCUSSION

Conceptual Understanding

Conceptual Understanding is the foundation of mathematical proficiency, encompassing students' comprehension of mathematical concepts and their ability to apply these concepts. The two emerging themes within this strand are "Connection to Real Life" and "Ability to Analyze Problems."

Connection to Real Life

The first emerging theme under Conceptual Understanding is the "Connection to Real Life." Mathematics teachers repeatedly emphasized the importance of ensuring that learners can relate mathematical concepts to real-life situations. This theme aligns with the existing literature, which highlights the significance of connecting mathematical concepts to practical applications. It is particularly crucial in the context of the 4th industrial revolution, where mathematics plays a pivotal role in addressing real-world challenges [16].

In the FGDs, one teacher (T1) highlighted, "In conceptual understanding, students must be conceptive and always engage in the process. Especially when a teacher introduces a new topic, they must understand why the lesson is important in our daily learning." This sentiment echoes findings from Boaler [17], who argues that learners need to comprehend the practicality and significance of mathematical concepts in order to cultivate a robust conceptual understanding. Another teacher (T4) noted, "The students clearly understand the given mathematical problem without the teacher explaining what it is all about." This statement emphasizes the need for learners to independently recognize the connections between mathematical concepts and real-life scenarios. Aligning with Boaler's work, it underscores that learners with strong conceptual understanding can identify and apply mathematical concepts in their daily lives [17].

Furthermore, the "Connection to Real Life" theme has direct implications for the development of learners' mathematical proficiency. Learners who can relate mathematical concepts to real-life situations are more likely to retain and apply their mathematical knowledge effectively, Benson-O'Connor, McDaniel, and Carr [18]. Therefore, this theme is a crucial indicator for developing mathematical proficiency assessment tools that measure learners' ability to establish connections between mathematics and real-world problems.

Ability to Analyze Problems

The second emerging theme within Conceptual Understanding, "Ability to Analyze Problems," underscores the importance of equipping learners with the skills to critically analyze mathematical problems. Mathematics teachers in the FGDs highlighted the need for learners to engage in analytical thinking when approaching mathematical challenges. This theme is consistent with the notion that developing learners' analytical skills is fundamental to their mathematical proficiency [19].

Teacher 1 noted, "In conceptual understanding, students must be conceptive and always engage in the process. Especially when a teacher introduces a new topic, they must understand why the lesson is important in our daily learning." This statement suggests that learners with strong conceptual understanding are expected to engage in critical thinking and analytical processes when confronted with new mathematical concepts. This aligns with the findings of Battista [19], who emphasizes the importance of learners' ability to analyze mathematical problems. Moreover, T1 stated, "If a student has a strong conceptual understanding in the lesson, he or she will analyze and make an equation." This assertion indicates that learners with strong conceptual understanding can critically evaluate problems and apply analytical thinking to formulate mathematical equations. Similarly, Battista [19] highlights the significance of learners' capacity to break down problems into manageable components, a hallmark of analytical thinking.

The "Ability to Analyze Problems" theme is pivotal for developing mathematical proficiency assessment tools. Assessments that measure learners' analytical skills can provide valuable insights into their conceptual understanding. By gauging how well learners can critically assess and solve mathematical problems, assessment tools can identify areas where learners excel and areas that require improvement. This information is crucial for tailored instruction and intervention.

Procedural Fluency

Procedural Fluency, the second strand of mathematical proficiency, is centered around learners' ability to execute mathematical procedures with accuracy and efficiency. This strand plays a crucial role in students' overall mathematical proficiency and is essential for their success in various mathematical contexts.

Clarity of Explanation

The first emerging theme within Procedural Fluency is "Clarity of Explanation." Mathematics teachers highlighted the importance of learners being able to clearly explain their problem-solving processes and the mathematical steps they undertake. This theme is consistent with the literature emphasizing the significance of articulating mathematical procedures [20].

Teacher T2 mentioned, "Students can estimate answers to problem-solving and can explain their procedures or strategy in Layman's terms." This indicates that students should not only solve problems but also articulate their procedures effectively. The ability to provide a clear and understandable explanation of their mathematical reasoning is fundamental to demonstrating procedural fluency. This observation aligns with Yezbick [21], which emphasizes the importance of students' ability to communicate their problem-solving processes. Moreover, another teacher, T5, noted, "To assess students' mathematical proficiency in terms of procedural fluency is by solving a math problem while articulating what they are doing." This approach to assessing procedural fluency involves having students verbalize their problem-solving steps. In the study conducted by Jonassen and Hung [22], she highlighted that articulating the problem-solving process is a valuable indicator of procedural fluency.

Accurate Procedure Performance

The second emerging theme under Procedural Fluency is "Accurate Procedure Performance." This theme emphasizes the necessity for learners to perform mathematical procedures accurately. While articulation is important, the ability to execute procedures with precision is equally crucial for procedural fluency. This theme resonates with previous research indicating that procedural fluency involves performing mathematical procedures efficiently [23].

Teacher T3 noted, "If the students can identify the given and then can determine what is asked in the problem, then I can say that he/she has a good conceptual understanding." This statement underlines the importance of students' ability to accurately identify problem components and requirements. In a similar vein, National Research Council [23] argues that procedural fluency entails applying procedures accurately and efficiently. Another teacher, T4, emphasized the importance of providing "a step-by-step, complete, and correct procedure in solving mathematical problems." This observation underscores the significance of learners' ability to execute procedures accurately and methodically. According to Tikhomirova, Misozhnikova, Malykh, Gaydamashko, & Malyk [24], fluency in mathematical procedures involves not only getting the correct answer but also following precise and efficient procedures.

The "Accurate Procedure Performance" theme is critical in the development of mathematical proficiency assessment tools. These tools should assess students' ability to execute mathematical procedures with precision, focusing on both the process and the correctness of their procedures. This theme highlights the importance of accuracy in procedural fluency, as students who can perform procedures accurately demonstrate a higher level of mathematical proficiency.

Strategic Competence

Strategic Competence represents the third strand of mathematical proficiency and centers on students' ability to select and apply appropriate problem-solving strategies. This strand is pivotal in assessing learners' mathematical proficiency and plays a significant role in students' overall success in mathematics [23].

Selection and Application of Strategies

One of the emerging themes within the Strategic Competence strand is "Selection and Application of Strategies." Mathematics teachers, both at the secondary and higher education levels, underscored the significance of students' capacity to select and apply appropriate problem-solving strategies. This theme aligns with the work of Liljedahl, Santos-Trigo, Malaspina, & Bruder [25], which emphasizes the importance of selecting and applying effective problem-solving strategies.

Teacher T4 highlighted, "The student is able to show different methods when presenting solutions in solving the mathematical problem." This indicates that students should possess the ability to choose and apply various methods and strategies to solve mathematical problems effectively. In a similar vein, the study conducted by Guzman [26] emphasized the value of students' ability to employ multiple problem-solving strategies. Furthermore, teacher T3 mentioned, "I consider of when and how to use different problem-solving techniques, their ability to monitor and evaluate their own problem-solving process and their capacity to transfer mathematical knowledge and skills to new situations." This observation underscores the importance of students' metacognitive skills in choosing and applying strategies. Metacognition, as identified by Kramarski and Gutman [27], plays a pivotal role in students' strategic competence development.

Analyzing Complex Problems

The second emerging theme under Strategic Competence is "Analyzing Complex Problems." This theme emphasizes the importance of students' ability to analyze intricate mathematical problems and devise effective solutions. Analyzing complex problems is essential for students to tackle real-world mathematical challenges [28].

Teacher T2 noted, "Ask the student to create new problem-solving (problem posing) related to the given problem and provide another way of solving the problem." This observation underscores the significance of students' creativity and analytical skills in dealing with complex problems. The ability to create new problems and devise alternative solutions is an indicator of students' strategic competence, as highlighted by Jonassen [29]. Moreover, teacher T6 emphasized the need for students to "internalize the problem that it actually happens in real life." This statement underscores the importance of students' ability to relate mathematical problems to real-life scenarios and make meaningful connections. Students who can bridge the gap between theoretical mathematical problems and practical applications demonstrate a high level of strategic competence. This aligns with the findings of Leung, Graf, and Lopez-Real [30], who highlighted the importance of real-life problem-solving in students' mathematical proficiency development.

The "Analyzing Complex Problems" theme is fundamental for the development of mathematical proficiency assessment tools. These tools should evaluate students' capacity to analyze and solve complex mathematical problems, focusing on their creativity and adaptability in problem-solving. This theme emphasizes the importance of students' analytical and problem-posing skills as crucial components of strategic competence.

Adaptive Reasoning

The Adaptive Reasoning strand of mathematical proficiency focuses on students' ability to apply mathematical concepts and procedures to new and unfamiliar situations. It entails making conjectures, testing hypotheses, and drawing conclusions by using mathematical reasoning. The strand emphasizes the practicality and real-world application of mathematical skills, aligning with the demands of the 4th industrial revolution [31].

Application in Real-Life Scenarios

One of the emerging themes within the Adaptive Reasoning strand is "Application in Real-Life Scenarios." Mathematics teachers underscored the importance of students' capacity to apply mathematical concepts and procedures to authentic, everyday situations. The ability to recognize and utilize mathematics in practical contexts is a cornerstone of adaptive reasoning [32].

Teacher T1 emphasized, "A student has a strong conceptual understanding if he demonstrates a deep understanding of key mathematical concepts and can apply them to solve complex problems." This statement aligns with the notion that adaptive reasoning extends beyond theoretical comprehension to practical problem-solving, which resonates with the 4th industrial revolution's emphasis on applied knowledge World Economic Forum [33]. Additionally, teacher T4 noted, "Let the student explain the mathematical problem solving and give his/her possible solution for the given problem." This emphasizes the importance of students' ability to articulate and present mathematical solutions, a skill that has real-world applications. Communicating and explaining mathematical reasoning is integral to adaptive reasoning [18].

Making Conjectures and Drawing Conclusions

The second emerging theme within the Adaptive Reasoning strand is "Making Conjectures and Drawing Conclusions." This theme underscores the necessity for learners to formulate hypotheses, conduct experiments, and derive meaningful conclusions using mathematical reasoning. Adaptive reasoning involves the ability to think critically and infer from mathematical data [34].

Teacher T5 mentioned, "Ask the student to provide real-life situations related to the mathematical skills being learned. Then ask the students to predict the possible outcome of the situation." This observation highlights the importance of students' capacity to make educated guesses and anticipate outcomes in practical situations. It aligns with the 4th industrial revolution's emphasis on predictive analytics and data-driven decision-making Schwab [35]. Furthermore, teacher T6 pointed out, "Let the student internalize the problem so that it actually happens in real life." This implies that students should be capable of connecting mathematical scenarios with real-life experiences, thus enriching their understanding of mathematical concepts. The ability to internalize mathematical scenarios contributes to adaptive reasoning [36].

The "Making Conjectures and Drawing Conclusions" theme plays a pivotal role in the development of mathematical proficiency assessment tools. These tools should assess students' capacity to formulate hypotheses, perform analyses, and draw meaningful conclusions using mathematical reasoning. This theme underscores the importance of nurturing students' critical thinking skills and their capacity to apply mathematics to real-life situations.

IV. CONCLUSION AND RECOMMENDATION

In conclusion, this research searches into the assessment practices of mathematics teachers in evaluating and promoting learners' mathematical proficiency. Employing a phenomenological approach, the study explored four strands of mathematical proficiency: Conceptual Understanding, Procedural Fluency, Strategic Competence, and Adaptive Reasoning. Two key themes emerged within each strand, offering valuable insights into the criteria employed by experienced mathematics teachers to assess learners' mathematical proficiency.

In the realm of Conceptual Understanding, the themes of "Connection to Real Life" and "Ability to Analyze Problems" were identified. The "Connection to Real Life" theme emphasizes the importance of learners relating mathematical concepts to practical, real-world situations. This aligns with the contemporary demand for mathematical skills in addressing the challenges of the 4th industrial revolution. The ability to establish such connections has implications for the development of assessment tools that measure the practical application of mathematical knowledge, crucial for the modern era.

Simultaneously, the theme of "Ability to Analyze Problems" underscores the necessity of nurturing learners' analytical skills. Strong conceptual understanding, as highlighted by teachers, involves engaging in critical thinking and analytical processes when encountering new mathematical concepts. This theme has implications for the development of assessment tools targeting learners' analytical skills, offering a comprehensive measure of their conceptual understanding.

Moving to Procedural Fluency, the identified themes are "Clarity of Explanation" and "Accurate Procedure Performance." "Clarity of Explanation" highlights the importance of learners not only solving problems accurately but also articulating their problem-solving processes clearly. This theme emphasizes the role of communication in demonstrating procedural fluency, indicating that assessment tools should consider both problem-solving accuracy and the ability to articulate procedures effectively.

In parallel, the theme of "Accurate Procedure Performance" underscores the need for learners to execute mathematical procedures with precision. Beyond articulation, the ability to perform procedures accurately is deemed essential for procedural fluency. This theme has implications for the development of assessment tools focusing on the correctness and precision of learners' procedural execution, providing a nuanced measure of procedural fluency.

Transitioning to Strategic Competence, the themes identified are "Selection and Application of Strategies" and "Analyzing Complex Problems." "Selection and Application of Strategies" underscores the significance of students' capacity to choose and apply effective problem-solving strategies. This theme has implications for assessment tools that aim to measure strategic competence, emphasizing the need for students to exhibit a diverse range of problem-solving approaches.

Simultaneously, the theme of "Analyzing Complex Problems" highlights the importance of students' ability to dissect intricate mathematical problems. This analytical skill is crucial for tackling real-world mathematical challenges. Assessment tools focusing on these themes can effectively measure learners' strategic competence, providing insights into their problem-solving capabilities.

Lastly, in the strand of Adaptive Reasoning, the themes are "Application in Real-Life Scenarios" and "Making Conjectures and Drawing Conclusions." "Application in Real-Life Scenarios" emphasizes the practicality of mathematical skills, aligning with the demands of the 4th industrial revolution. Assessment tools can be designed to evaluate students' ability to apply mathematical concepts in practical, everyday situations.

Simultaneously, the theme of "Making Conjectures and Drawing Conclusions" underscores the importance of learners' critical thinking skills. The ability to formulate hypotheses, conduct analyses, and draw meaningful conclusions is integral to adaptive reasoning. Assessment tools focusing on these themes offer a comprehensive measure of learners' ability to apply mathematical reasoning in unfamiliar situations.

In essence, this research contributes to the development of a comprehensive tool for assessing mathematical proficiency. The identified themes within each strand offer nuanced insights into the criteria employed by mathematics teachers, providing a foundation for the design of effective assessment tools. Ultimately, such tools can bridge the existing gap in mathematical assessment, paving the way for enhanced mathematics education and equipping learners with the skills necessary to thrive in the modern world. The study not only sheds light on the intricacies of mathematical assessment but also aligns with the broader discourse on education's role in preparing individuals for the challenges of the 4th industrial revolution.

Acknowledgment
The researcher would like to thank the Department of Science and Technology of the Philippines for funding this study.

REFERENCE

[1] Butler-Adam, J. (05 2018). The Fourth Industrial Revolution and education. South African Journal of Science, 114. doi:10.17159/sajs.2018/a0271

[2] Australian Curriculum, Assessment and Reporting Authority. (n.d.). Mathematics. Retrieved November 9, 2023, from The Australian Mathematics Teacher vol. 68 no. 1 (ed.gov)

[3] Australian Association of Mathematics Teachers. (2019). Standards for excellence in teaching mathematics in Australian schools

[4] Burkhardt, H. (2007). Mathematical Proficiency: What Is Important? How Can It Be Measured? In Mathematical Sciences Research Institute (Author) & A. Schoenfeld (Ed.), Assessing Mathematical Proficiency (Mathematical Sciences Research Institute Publications, pp. 77-98). Cambridge: Cambridge University Press. doi:10.1017/CBO9780511755378.010

[5] Leung, A. (01 2017). Variation in Tool-Based Mathematics Pedagogy. doi:10.1007/978-94-6300-782-5_4

[6] Shepard, L.A. (2000). The Role of Classroom Assessment in Teaching and Learning. CSE Technical Report.

[7] Hattie, J., & Timperley, H. (2007). The Power of Feedback. Review of Educational Research, 77(1), 81-112. https://doi.org/10.3102/003465430298487

[8] Suurtamm, C. et al. (2016). Assessment in Mathematics Education. In: Assessment in Mathematics Education. ICME-13 Topical Surveys. Springer, Cham. https://doi.org/10.1007/978-3-319-32394-7_1

[9] Abakpa, B., Agbo-Egwu, A., & Abah, J. (08 2017). EMPHASIZING PHENOMENOLOGY AS A RESEARCH PARADIGM FOR INTERPRETING GROWTH AND DEVELOPMENT IN MATHEMATICS EDUCATION. ABACUS: The Journal of the Mathematical Association of Nigeria, 42, 391–405.

[10] Creswell, J. and Poth, C. (2017) Qualitative Inquiry and Research Design: Choosing among Five Approaches. Sage, London.

[11] Creswell, J. W. (2007). Qualitative inquiry and research design: Choosing among five approaches (2nd ed.). Sage Publications, Inc.

[12] Krueger, R.A. and Casey, M.A. (2015) Focus Groups: A Practical Guide for Applied Research. 5th Edition, SAGE, Thousand Oaks, 252 p.

[13] Braun, V., & Clarke, V. (01 2006). Using thematic analysis in psychology. Qualitative Research in Psychology, 3, 77–101. doi:10.1191/1478088706qp063oa

[14] Johnson, J. L., Adkins, D., & Chauvin, S. (2020). A Review of the Quality Indicators of Rigor in Qualitative Research. American journal of pharmaceutical education, 84(1), 7120. https://doi.org/10.5688/ajpe7120

[15] Birt, L., Scott, S., Cavers, D., Campbell, C., & Walter, F. (2016). Member Checking: A Tool to Enhance Trustworthiness or Merely a Nod to Validation?. Qualitative health research, 26(13), 1802–1811. https://doi.org/10.1177/1049732316654870

[16] Arthur, Y. D., Owusu, E. K., Asiedu-Addo, S. K., & Arhin, A. K. (2018). Connecting Mathematics To Real Life Problems : A Teaching Quality That Improves Students ' Mathematics Interest. Retrieved from https://api.semanticscholar.org/CorpusID:201639252

[17] Boaler, J. (2008). What's math got to do with it? Helping children learn from their most hated subject – and why it's important for America. New York, NY: Viking.

[18] Benson-O'Connor, Carol D.; McDaniel, Christina; and Carr, Jason (2019) "Bringing Math to Life: Provide Students Opportunities to Connect their Lives to Math," Networks: An Online Journal for Teacher Research: Vol. 21: Iss. 2. https://doi.org/10.4148/ 2470-6353.1299

[19] Battista, M. T. (2007). The development of geometric and spatial thinking. In F. K. Lester Jr. (Ed.), Second handbook of research on mathematics teaching and learning (pp. 843-908). Information Age Publishing.

[20] Hoover, M., & Lai, Y. (2019). Pedagogical context and its role in articulating mathematical knowledge for teaching. In Proceedings of the 13th International Congress on Mathematical Education (pp. 1-8).

[21] Yezbick, E. (2020). Fostering student engagement through a real-world, collaborative project across disciplines and institutions. Higher Education Pedagogies, 5(1), 30–51. doi:10.1080/23752696.2020.1750306

[22] Jonassen, D.H., Hung, W. (2012). Problem Solving. In: Seel, N.M. (eds) Encyclopedia of the Sciences of Learning. Springer, Boston, MA. https://doi.org/10.1007/978-1-4419-1428-6_208

[23] National Research Council. (2001). Adding it up: Helping children learn mathematics. National Academies Press.

[24] Tikhomirova, T. N., Misozhnikova, E. B., Malykh, A. S., Gaydamashko, I. V., & Malykh, S. B. (2017). Mathematical fluency in high school students. Psychology in Russia: State of the Art, 10(1), 95–104. https://doi.org/10.11621/pir.2017.0107

[25] Liljedahl, P., Santos-Trigo, M., Malaspina, U., Bruder, R. (2016). Problem Solving in Mathematics Education. In: Problem Solving in Mathematics Education. ICME-13 Topical Surveys. Springer, Cham. https://doi.org/10.1007/978-3-319-40730-2_1

[26] Guzman Gurat M. (2018) "Mathematical problem-solving strategies among student teachers", Journal on Eficiency and Responsibility in Education and Science, Vol. 11, No. 3, pp. 53-64, online ISSN 1803-1617, printed ISSN 2336-2375, doi: 10.7160/ eriesj.2018.110302.

[27] Kramarski, B., & Gutman, M. (2006). How Can Self-Regulated Learning Be Supported In Mathematical E-Learning Environments? Journal of Computer Assisted Learning, 22, 24-33

[28] Schoenfeld, A. H. (1985). Mathematical problem solving. New York: Academic Press.

[29] Jonassen, D. H. (2003). Designing research-based instruction for story problems. Educational Psychology Review, 15(3), 267-296

[30] Leung, F. K. S., Graf, K. D., & Lopez-Real, F. J. (2018). Real-life problem-solving in mathematics: A multi-perspective analysis. ZDM Mathematics Education, 50(3), 307-318.

[31] Peng, Y. (2016). Mathematics education in the fourth industrial revolution. In K. Makar, S. Dole, J. Visnovska, M. Goos, A. Bennison, & K. Fry (Eds.), Research in Mathematics Education in Australasia 2012-2015 (pp. 3-20). Springer Singapore

[32] Lesh, R., & Doerr, H. M. (Eds.). (2003). Beyond constructivism: Models and modeling perspectives on mathematics problem solving, learning, and teaching. Lawrence Erlbaum Associates Publishers

[33] World Economic Forum. (2016, January 14). The Fourth Industrial Revolution: what it means, how to respond

[34] Lee, J. (2000). The Development of Adaptive Reasoning in the Mathematics Classroom. Learning to Teach, 7(1), 1-23

[35] Schwab, K. (2016) The Fourth Industrial Revolution. World Economic Forum.

[36] Sfard, A. (2007). When the rules of discourse change, but nobody tells you: Making sense of mathematics learning from a commognitive standpoint. Journal of the Learning Sciences, 16(4), 565-613.