STUDENTS EXPERIENCE ON COGNITIVE-DEMAND MATHEMATICAL TASKS: A DESCRIPTIVE CASE STUDY

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Abstract: *This study investigated twenty freshman students on their experiences with the utilization of cognitive-demand mathematical tasks as an instructional strategy. The cohort was exposed to mathematical tasks using non-routine problems in Algebra. After all the topics and mathematical tasks were engaged in, students' post opinion and experience on the mathematical tasks they undertook was qualitatively analyzed. The data gathered were analyzed using the 6-phase trustworthy thematic analysis. The results of the analysis revealed similarities in the students' experiences and opinions on the instructional tasks they undertook. Factors that should be considered and looked into for better implementation of cognitive-demand mathematical tasks are the appropriateness of time to do the tasks, the mental abilities and academic background of the students, and the support of the faculty and their classmates in building and completing the tasks.*

Keywords*: Cognitive-Demand Mathematical Tasks, Conceptual Understanding, Mathematical Fluency*

1. INTRODUCTION

Mathematics does not come from the mathematical tasks alone, but they are also used to test out and confirm that the learner is developing. Hence, instructional plans in mathematics must have a marked influence on what is taught in mathematics classrooms. One key idea in designing instructional tasks is to give cognitive-demand mathematical tasks that can cause the students to be reflective, think deeply, and construct the desired ideas. It may not be the use of these mathematical tasks that makes teaching effective, but to bring out the students to be reflective which is the most important consideration for teachers. With this premise, the teacher, may not only infuse cognitive-demand mathematical tasks to develop mathematical fluency but also blended the instructional design with activities like reflective exercises to achieve a conceptual understanding.

Cognitive demands of problem-solving tasks are so important because as stated in the Professional Standards for Teaching Mathematics [1], opportunities for student learning are not created simply by putting students into groups, by placing manipulatives in front of them, or by handing them a calculator. Rather, it is the level and kind of thinking where students are engaged that determines what they learn [2]. According to Doer [3], there is no decision that teachers make that has a greater impact on students' opportunities to learn, and on their perceptions about what mathematics is, than the selection or creation of the tasks with which the teacher engages the students in studying mathematics.

As Stein, Smith, Henningsen, and Silver [4] said, "Not all tasks are created equal and different tasks will provoke different levels and kinds of student thinking". They have developed a Task Analysis Guide (TAG), a research-based guide that provides a helpful four-tiered classification of the nature of mathematical problems. The first two are lower-level cognitive demands: 1) Memorization - which involves either producing previously learned information, involving exact reproduction of previously-seen material, have no connection to the concepts or meaning that underlie the information being learned; and 2) Procedures Without Connections- which are algorithmic based on prior work, has an obvious indicator of

needs to be done or how to do it, have no connection to the concepts or meaning that underlie the procedure being used, are focused on producing correct answers rather than developing mathematical understanding, and require only

"how" explanations, no "why" explanations; The other two tasks are higher-level cognitive demands, namely: 3) Procedures with Connections to Concepts requires student deeper understanding of concepts and ideas, suggest pathways that are broad and general procedures that have close connections to underlying conceptual ideas, can be represented in multiple ways, and cannot be followed without thinking; and 4) Doing Mathematics- this requires complex and non-algorithmic procedure which allows students to access relevant experiences, helps them to analyze tasks and examines task constraints, encourages them to explore and understand relationships, demands self-monitoring, and considerable cognitive effort. These last two tasks are the ones that teachers need to develop high cognitive thinking among students.

In the United States of America, a team of researchers was commissioned to evaluate the levels of cognitive-demand mathematical problems used in textbooks. The Quantitative Understanding: Amplifying Student Achievement and Reasoning (QUASAR) project researchers analyzed the gathered data and noted that students need opportunities on a regular basis to engage with mathematics problem-solving that leads to deeper, more generative understandings about the nature of mathematical concepts, processes, and relationships. They also found that teachers implementing mathematics problems with high levels of cognitive demand rarely select the tasks from commercial textbook series [5].

In the context of Filipino tertiary students, Mariquit and Luna [6] investigated the use of cognitive-demand mathematical tasks to probe students' conceptual understanding, mathematical fluency, and mathematics anxiety as influenced by their mental ability and the types of mathematical tasks they engage in. They found out that mental ability influences the participants' conceptual understanding and mathematical fluency. Though their study revealed that the instructional tasks that students undertook positively influenced their cognitive development but they also observed that these cognitive-demand algebra tasks using non-routine problems built the participants' anxiety towards mathematics. Hence, this descriptive case study was undertaken to explore students' experiences with the utilization of cognitive-demand tasks as an instructional strategy in teaching tertiary mathematics. While the students were trained to develop fluency and conceptual understanding through cognitive-demand mathematical tasks, it is important to validate qualitatively what works best for a successful implementation of the instructional strategy as students engage in more challenging mathematical tasks.

2. METHODS

The study employed a qualitative descriptive case study. A purposive sampling was adopted in this study. Twenty first-year college students of St. Peter's College- Iligan City, Philippines were participants in the study. They were the students taught cognitive-demand mathematical tasks using non-routine problems for one semester. The study used a coding system that indicates the category of mental ability, sex, and the student's assigned number. The category of ability is coded as follows: C_1- Category 1 (with difficulty or low ability); C_2- Category 2 (without difficulty or high ability). Lowercase *m* is the code for male respondents while *f* is for female respondents. Thus, the code C_1f_{16} means "Category 1, female, student 16". The researcher observed the classes and conducted the interview after the semester. Then, the recorded videos were transcribed. The videos provided data that could reveal how the learner-participant responded to a class delivered through a cognitive-demand mathematical tasks strategy. For the data analysis, the researcher used the 6-phase trustworthy thematic analysis. A trustworthy thematic analysis was employed to ensure the validity and reliability of the research findings [7].

3. RESULTS AND DISCUSSION

Based on the data gathered, the observation during task activities, and the post-opinion interview, all information was classified in order to find the effects of the task implementation. The idea here is to obtain data that could help in a better analysis of the effects of the cognitive-demand mathematical task on the student's performance. The themes that emerged from the students' experiences with the instructional tasks they undertook are delineated below:

Students' demand for tasks to be manageable within the allotted time

Students expressed that the allotted time is not suitable for the majority of the tasks. As a common feedback from the cohort, Code C2m14 said in the vernacular: *" Di gyud enough ang oras sa pag-solve a problem. Nahadlok ko basin di nako ka-solve ana."* which means: "Time is not enough to wrestle with the task. I'm afraid I cannot solve the problems." For many of the tasks, the researcher-instructor gave timed exercises because, in the previous meetings, too much time was allowed. However, it created a negative effect because students drift off into off-task behavior. In the first implementation of the cognitive-demand mathematical task for the cohort, Task 28B for their seatwork, about 60 minutes was consumed to wrestle with the task. The students were assisted as they struggle to do the task. They became complacent to complete the task and were not sure how to continue. Giving too much time would pave the way to the decline of the level of cognitive demand of the task during implementation because the students would tend to ask for hints and the focus shifted away from conceptual understanding toward the solution rather than the thinking processes entailed in reaching the solution. With this observation, students perceived that the majority of the tasks cannot be accomplished within a short time.

Mental abilities and academic background is imperative

Most of the student participants blamed their poor academic experiences in high school. As they perceived, this is the most influential aspect that hinders them to do the task completely. Many of them were used to doing routine tasks that they said do not build upon their previous learning. Code C2m15 said:

" Kulang akong knowledge para ma-solve nako ang task. Ma-mental block ko permi." Which means "My knowledge is not enough to do the task. I'm always mentally blocked." Code C2m20 also responded:

"Basahon man sa gyud nako ang problem then usbon sad nakog basa hangtud masabtan nako. Ang naandan nga method akong gamiton hangtod maka-visualize ko para maka-himog equations nga akong ma-solve. Pero maglisod gyud ko permi kay ang pattern di man mu-work. Mawala gyud ko ug di na nako mahinumduman ang previous contents sa lessons para u nta ma-apply nako sa problem.

It means that he read the tasks and re-read them for comprehension. He used the usual procedure and steps that he learned then made some visual presentations to see if he could make some equations to solve. But most of the time, he found the tasks difficult because the pattern did not work. He was lost and could not recall the contents that he studied in previous topics that he could have applied to the tasks.

"Further, student Code C2m17 also replied: *"Wa man ko maanad aning mga lisod nga problems. Naanad ko atong step by step nga among gisunod. Muundang nako pag mag-modeling na. Di na dayon ko kalahutay then usahay managna na lang ko sa akong answer."*

Code C2m17 said he is not used to solving difficult problems. He is more used to solving exercises that have steps to follow. He stopped when he reached the mathematical modeling part. He could no longer move after it, so he just guessed the final answer. The difficulties encountered by the students who had confessed to having a poor background in Algebra in high school mainly had to do with cognitive demand mathematical tasks.

However, students who came from Science Curriculum in high school responded in favor of the cognitive demand tasks. Code C2m18 mentioned: *"What I noticed, I struggled for a long time to work through the tasks but once I understand it and have a clearer view of the whole picture, my mind becomes firm. I notice that this makes me more active to think higher with the problems."*

Code C2m19 supported: *"The given mathematical tasks were difficult and complicated but challenging because they are different from the usual and habitual tasks that we do. In working with the tasks, we could work in groups and share each other's ideas and clarify understandings. We learn from each other. But we are also afraid because our grades might be affected if we can't solve difficult tasks correctly.*

However, since we know the grading system, only the quiz and term exams are given big factors to pass the subject, not the tasks or activities done after lecture time. We are positive that we will get good grades in the exam."

Some students who came from the science curriculum in high school responded in favor of the high cognitive-demand mathematical tasks. The majority of the students blamed their poor academic experiences in high school which they thought was the most influential aspect that hindered them to do the task completely.

Faculty and peer support in building and completing cognitive-demand mathematical tasks is vital

Another factor in the successful implementation of the cognitive-demand mathematical task was the support of the instructor and co-mentoring with their classmates. As verified from the same student- Code C2m15, *"Mag-share man mig ideas to solve in group then mangutana mi sa ka-grupo ug makat-on sad mi sa anang paagi."* Code C2m15 meant that in working with the tasks, they could work in groups and share each other's ideas and clarify understandings. They learn from each other. Code C2m19 said: *"I was able to do at least an independent thinking and many times, I ask myself if I'm on the right track in solving the problem. The problems caused us the feeling of anxiety, however, you (the instructor) always told us that we all can do it, we just need to think about it."* This reaction is one of the indications of a student's feelings of competence and skill. With this, they are motivated to remain engaged with a task at a high level.

Another task experience (board work) was cited by Code C2m18: *"At first, the task gets in our minds but as we move on, we feel lost in the details. But as we both (student-teacher) push each other to complete the task and justify reasoning, we realized that we already learn at a higher order."* As an example, in Task 34B, the requirement in the problem was twisted by asking the students to give and solve 3x3 linear equations and make generalizations in classifying the system of equations as dependent (infinitely many solutions) or inconsistent (no solution). The student's prior knowledge was only about the classification of two given linear systems. Code C2f20 said, in creating her equations, she just triplicated the first equation by multiplying by 2 to have a second equation, then multiply the second equation by 2 again to have a third equation. With the 3x3 linear equations that she created, she proceeded with using Cramer's Rule to get the solution. She was able to solve the solutions with the equations that he made. She almost gave up when asked to make a generalization as to how he would classify the system. However, when she was encouraged to make another example of the same process she created so she can further notice how to generalize, she finally observed that the determinant of the denominator D and the numerators Dx. Dy, and Dz for $x = \frac{D_x}{D}$, $y = \frac{D_y}{D}$, and are always 0. So she made a sophisticated assumption which she never expected would turn out to be the exact answer: If D = 0 and all the determinants in the numerator are 0, then the equations in the system are dependent, meaning the system has infinitely many solutions.

After Code C2f20 has shown his generalization, Code C2f14 got an idea from her and made her second attempt to complete the task. She then changed all constants in the equations of Code C2f20 and solved the system. She was told to repeat the same process with another set of equations and there she observed that D=0, and if at least one of Dx, Dy, or Dz is zero, the solution set is empty. This means that the system is inconsistent or has no solutions as generalized.

The key to being able to do cognitive-demand mathematical tasks is for teachers to motivate the students that everyone has the basic ability to do the task and that everyone with average or above-average mental ability has the so-called "math genes". To wrestle with the task, the students must have sufficient want and desire to solve the problem. Figures 1 and 2 show a concrete example from Codes C1m3 and C1f12 who belong to the cohort with just average mental abilities. However, Codes C1m3 and C1f12 outperformed the above-average group in deriving the quadratic formula.

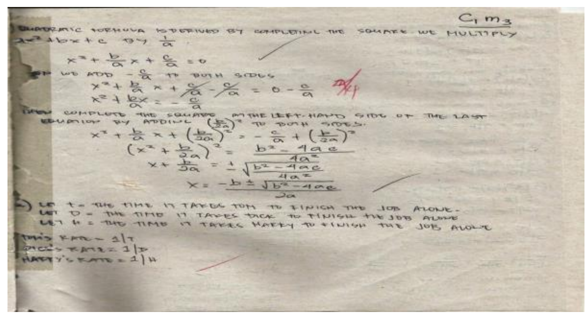

Figure 1. Derivation of Code C1m3 of the Quadratic Formula

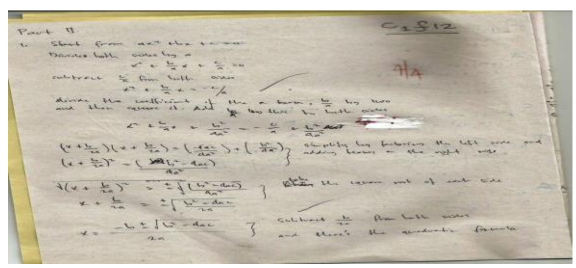

Figure 2. Derivation of Code C1f12 of the Quadratic Formula

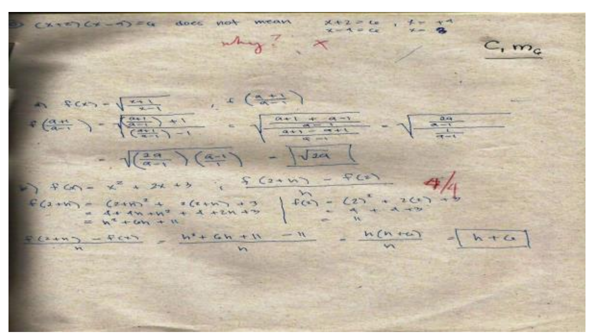

Figure 3. Solution of Code C1m6 on Evaluating a Function

Figure 3 is another example from Code C1m6. He has an average mental ability but he outperformed the above-average students on evaluating a function at the indicated element of the domain. The secret to being able to perform the task was his desire and interest to do so in the particular topic that he had knowledge about. While the rest of the students left this item unsolved, he never stopped thinking until he arrived at the correct answer.

Students successfully completed the task and are motivated to remain engaged with the task at a high level due to the support of the instructor and co-mentoring with their classmates.

4. CONCLUSION AND RECOMMENDATION

Based on the observations, a better analysis of the effects of the cognitive demand mathematical task on the student's performance is found. Factors that should be considered and looked into by instructors for better implementation of cognitive-demand mathematical tasks are the appropriateness of time to do the tasks, the mental abilities and academic background of the students, and the support of the faculty and their classmates in building and completing the tasks.

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