CORRIGENDUM [*R*-DYNAMIC CHROMATIC NUMBER OF CYCLES, COMPLETE GRAPHS AND FORESTS, IJPAM, VOL. 110 NO. 4 2016, 609-622].

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ABSTRACT*: In this article we would like to present a corrected proof of Theorem 4.6 of the article entitled "r-Dynamic Chromatic Number of Cycles, Complete Graphs and Forests" published in IJPAM* [*Vol. 110 No. 4 2016, 609-622*]. The original proof of the Theorem uses Corollary 4.5 of the article, which was found to be false (as pointed out by Dr. Sergio R. Canoy Jr. of MSU-IIT who gave us a counterexample). It is surprising that Theorem 4.6 is still true, and we would like to present a corrected version of the proof [not using the Corollary].

Keywords: r-dynamic k-coloring, r-dynamic chromatic number, vertex gluing

1. INTRODUCTION

To give the readers a background of the concept, we would like to give a walk through. The concept dynamic coloring of graphs is a generalization of the concept proper vertex coloring.

The dynamic coloring of a graph *G* is a proper coloring such that every vertex of *G* with degree at least two has at least two neighbors that are colored differently. This concept was introduced by Montgomery in [3]. Since then, the concept was studied by many authors, see for example [1-7]. A generalization of the dynamic coloring was also introduced by Montgomery in [3].

The concept *r*-dynamic coloring is further generalized to the concept *r*-dynamic *k*-coloring. This general concept was studied in [8] and [9].

A vertex coloring of a graph *G* = (*V*, *E*) is a function *f* from *V* to a finite set *C*, whose elements are called color. A *k*-coloring is a vertex coloring with at most *k*-colors. We always assume that $C=\{1,2,\ldots,k\}$. A *k*-coloring may also be viewed as a vertex partition $(V_1,V_2,\ldots,V_k)$, where $V_i=\{v\in V: f(v)=i\}$ are called the color classes. A graph is *k*-colorable if it admits a proper vertex coloring with at most *k*-colors. The chromatic number of a graph *G*, denoted by $\chi_r(G)$, is the smallest *k* such that *G* is *k*-colorable. An *r*-dynamic *k*-coloring of *G* is a proper coloring *f* such that for all vertices *v* in *V*, $|f(N(v))|\geq \min\{r,\deg_G(v)\}$. The *r*-dynamic chromatic number of a *G*, denoted by $\chi_r(G)$, is the smallest *k* such that *f* is an *r*-dynamic *k*-coloring of *G*.

Let $G_1,G_2,\ldots,G_t$ be graphs, each containing a complete subgraph K_r ($r\geq 1$). Let *G* be a graph obtained from the union of *t* graphs G_i by identifying the complete graphs K_r (from each graph G_i) in an arbitrary way. We call *G* a K_r-gluing of $G_1,G_2,\ldots,G_t$. In particular, when $r=1$ we say that *G* is a vertex-gluing.

2. RESULTS

Lemma 4.1 is found in [13]. This lemma is used in Theorem 2.3.

Lemma 2.1. Let $G=(V,E)$ be graph. Let $f:V\to C$ with $C=\{1,2,\ldots,k\}$ be a *k*-coloring of *G* and let $\{V_1,V_2,\ldots,V_k\}$ be a partition of *V* such that for each $i\in C$, $V_i=\{v\in V: f(v)=i\}$. Then *f* is an *r*-dynamic *k*-coloring of *G* if and only if

1. V_i is an independent set for each $i=1,2,\ldots,k$; and,
2. for each $v\in V$, $|\{j\in C: N(v)\cap V_j\neq\varnothing\}| \geq \min\{r, \deg_G(v)\}$.

Proof : Assume that *f* is an *r*-dynamic *k*-coloring of *G*. Since *f* is a proper coloring, V_i is an independent set for each $i=1,2,\ldots,k$. Let $v\in V$. Then $|f(N(v))| \geq \min\{r,\deg_G(v)\}$. This implies that $|\{j\in C: N(v)\cap V_j\neq\varnothing\}| \geq \min\{r,\deg_G(v)\}$. Conversely, assume that conditions (1) and (2) hold. Let $uv\in E$. Then *f* is a proper coloring by (1). Next, let $v\in V$. Then $|f(N(v))|=|\{j\in C: N(v)\cap V_j\neq\varnothing\}| \geq \min\{r,\deg_G(v)\}$ by (2). Therefore *f* is an *r*-dynamic *k*-coloring.

Theorem 2.2 is also found in [13], and we present it here with the corrected proof.

gives the *r*-dynamic chromatic number of the vertex gluing of two graphs.

Theorem 2.3. Let $G_1=(V_1,E_1)$ and $G_2=(V_2,E_2)$ be graphs. Let *G* be a vertex gluing of G_1 and G_2. If $r\leq\min\{\delta(G_1),\delta(G_2)\}$, then $\chi_r(G)=\max\{\chi_r(G_1), \chi_r(G_2)\}$.

Proof : Let *G* be the graph obtained by identifying $u\in V_1$ and $v\in V_2$, that is *G* is a vertex gluing of G_1 and G_2. Let $f_1=\{V_1^{(1)},V_2^{(1)},\ldots,V_{k_1}^{(1)}\}$ and $f_2=\{V_1^{(2)},V_2^{(2)},\ldots,V_{k_2}^{(2)}\}$ be *r*-dynamic k_1-coloring of G_1 and *r*-dynamic k_2-coloring of G_2, respectively, such that $k_1=\chi_r(G_1)$ and $k_2=\chi_r(G_2)$. Without loss of generality, suppose that $u\in V_1^{(1)}$ and $v\in V_1^{(2)}$, and $k_1\geq k_2$. Let $f:V(G)\to\{1,2,\ldots,\max\{\chi_r(G_1),\chi_r(G_2)\}\}$ be given by

$$f(w)=\begin{cases} f_1(w), & \text{if } w\in G_1 \\ f_2(w), & \text{if } w\in G_2 \end{cases},$$

that is, $f_1=\{V_1^{(3)},V_2^{(3)},\ldots,V_{k_1}^{(3)}\}$ where $V_j^{(3)}=V_j^{(1)}\cup V_j^{(2)}$ for all $j=1,2,\ldots,k_2$ and $V_q^{(3)}=V_q^{(1)}$ for $q=k_2+1,k_2+1,\ldots,k_1$. Let $xy\in E(G)$. Then either $xy\in E_1$ or $xy\in E_2$. Without loss of generality, suppose that $xy\in E_1$. Suppose $x\in V_r^{(1)}$ and $y\in V_s^{(1)}$ for some $r,s\in\{1,2,\ldots,k_1\}$. Since $f_1=\{V_1^{(3)},V_2^{(3)},\ldots,V_{k_1}^{(3)}\}$ is an *r*-dynamic k_1-coloring, by Lemma 4.1 $r\neq s$. Hence, $x\in V_r^{(3)}$ and $y\in V_s^{(3)}$ where $r\neq s$. This shows that *f* is a proper coloring.

Next, let $w \in V(G)$. Then either $w \in V_1$ or $w \in V_2$. Without loss of generality, assume that $w \in V_1$. Since f_1 is an *r*-dynamic k_1-coloring, by Lemma 4.1, $\left|\left\{j \in \{1,2,\ldots,k_1\} : N_G(w) \cap V_1^{(1)} \neq \varnothing\right\}\right| \geq \min\{r, \deg(w)\} = r$. Thus, $\left|\left\{j \in \{1,2,\ldots,k_1\} : N_G(w) \cap V_1^{(1)} \neq \varnothing\right\}\right| \geq r$. Therefore, *f* is an *r*-dynamic k_1-coloring of *G*. Hence, $\chi_r(G) \leq k_1 = \max\{\chi_r(G_1), \chi_r(G_2)\}$.

Next, suppose that $\chi_r(G) = k$. Let $g : V(G) \to \{1,2,\ldots,k\}$ be an *r*-dynamic *k*-coloring of *G*. Let $\{U_1, U_2, \ldots, U_k\}$ be a partition of V such that for each $i = 1,2,\ldots,k$, $U_i = \{v \in V : f(v) = i\}$. For each $i = 1,2,\ldots,k$, let $W_i^{(1)} = U_i \cap V_1$ and $W_i^{(2)} = U_i \cap V_2$. Define $g_1 : V_1 \to \{1,2,\ldots,k\}$ by $g_1(v) = i$ whenever $v \in W_i^{(1)}$, and $g_2 : V_2 \to \{1,2,\ldots,k\}$ by $g_2(v) = i$ whenever $v \in W_i^{(2)}$. Clearly, g_1 and g_2 are *r*-dynamic *k*-coloring of G_1 and G_2, respectively. Thus, $k \geq \max\{k_1, k_1\} = \max\{\chi_r(G_1), \chi_r(G_2)\}$. QED

Theorem 2.3 may be extended to a vertex gluing of *n* number of graphs $G_1, G_2, \ldots, G_n$ using the *Principle of Mathematical Induction*.

Corollary 2.4. Let $n \in N$ and $G_1, G_2, \ldots, G_n$ be graphs. Let *G* be a vertex gluing of $G_1, G_2, \ldots, G_n$. If $r \leq \min\{\delta(G_1), \delta(G_2), \ldots, \delta(G_n)\}$, then $\chi_r(G) = \max\{\chi_r(G_1), \chi_r(G_2), \ldots, \chi_r(G_n)\}$.

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