MODELING AND SIMULATION OF AN HETEROGENEOUS LIQUID IN A PARTIALLY FILLED 3-D RECTANGULAR CONTAINER

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ABSTRACT: *The main focus of this study is to investigate the effects of heterogeneity of liquid on the sloshing of a partially filled 3-D rectangular tank under horizontal excitation. A theoretical model in the heterogeneous liquid state is developed using the separation of variables method to solve the equations generated by this model. A three-dimensional numerical method is implemented using Comsol-Multiphysics software to study the effect of the heterogeneity of a liquid whose density changes as a function of liquid depth on the free surface of the liquid. The influence of varying liquid densities on sloshing is examined and discussed. The numerical method is validated by comparing simulation results with existing numerical data, and the comparison reveals reasonable accuracy. Dynamic pressure is presented and analyzed. As shown in the results, the free surface profiles increase as the heterogeneity parameter increases. The heterogeneity of the inviscid liquid has a significant influence on the sloshing.*

Keywords: *Sloshing, free surface, numerical simulation, FEM*

1. INTRODUCTION

Heterogeneous liquid sloshing refers to the movement of fluids whose density changes as a function of height in a partially filled container, which is a well-known phenomenon in many industrial and environmental fields [1,2]. Such as waste storage tanks and transport of crude oil tanks. After prolonged storage, waste materials and crude oil are gradually deposited at the bottom of the tank. Sludge may be deposited at the base of the tank [3]. So, the difference in density can be significant from top to bottom of the tank. The liquid can be classified as a heterogeneous liquid. The sloshing of the continuously heterogeneous liquid has its characteristics.

There are relatively few studies about the sloshing of heterogeneous liquids compared to studies on homogeneous uniform densities. Understanding the behavior of non-uniform density on the dynamic response of the contained liquids is necessary. Formerly studied by Rayleigh [4] and Love [5], then was the topic of a limited number of searches by Capodanno [6], Essaouini et al. [7, 8], and El Bahaoui et al. [9].

In this work, we propose to study the sloshing problem of an incompressible and inviscid homogeneous-heterogeneous liquid in a partially filled 3-D rectangular tank, taking into consideration the effects of the heterogeneous density, which has usually been neglected in practice. A theoretical model in the form of a heterogeneous liquid is developed using the separation of variables method for solving the equations related to this model. A three-dimensional numerical method is carried out for different densities that change as a function of the liquid depth, and the numerical method is validated by comparing simulation results with existing numerical data.

This paper investigates the extent to which heterogeneity affects the sloshing behavior in a 3-D rectangular tank subjected to horizontal excitation. In section 2, we set the theoretical model (Static study). Section 3 conducts a numerical simulation (Dynamic study) of the sloshing to ensure the method's validity and discuss heterogeneity's effect on the sloshing response. Meantime, section 4 examines in detail the effects of heterogeneity on free surface elevation. Lastly, the conclusions are presented in section 5.

2. STATIC STUDY

Assume that a heterogeneous liquid partially fills an immovable tank, and in equilibrium, it occupies a domain Ω that is surrounded by the solid boundary S and the free surface Γ. Γ is orthogonal to the acceleration $\vec{g}$ of the gravitation field. We choose the system of coordinates $(Ox_1x_2x_3)$ such that $\vec{g}=-g\vec{x}_3$ and its center O is located on the equilibrium surface Γ. (See Fig. 1).

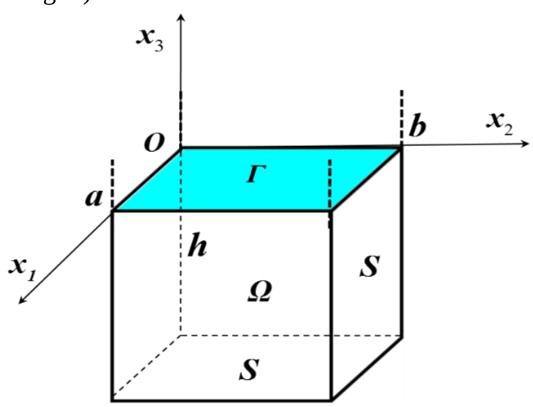

Figure 1. The geometry of the system

2.1. Notions

At equilibrium, the surface of the liquid is carried by the horizontal plane (O, x_1, x_2) of the equation $x_3 = 0$.

The container is parallelepiped in shape.

The parallelepiped has sides a, b, and h.

We denote by Ω the volume occupied by the fluid (liquid).

S is the area of each face.

2.2. Equations of the motion

The governing equations of the liquid in the tank are given:

With:

$$\rho\ddot{\vec{u}} = -\overrightarrow{grad}\, p - \rho\beta g u_3 \vec{x}_3 \quad (1)$$

$$div\vec{u} = 0 \quad \text{in } \Omega \quad (2)$$

$$u_n\big|_S = 0 \quad (3)$$

$$\int_\Gamma u_n\big|_\Gamma d\Gamma = 0 \quad (4)$$

$$p\big|_\Gamma = \rho g u_n\big|_\Gamma \quad (5)$$

By projection on the axes(Ox_1), (Ox_2), (Ox_3), we get the following system of equations:

$$\rho\ddot{u}_1 = -\frac{\partial p}{\partial x_1} \quad (6)$$

$$\rho \ddot{u}_2 = -\frac{\partial p}{\partial x_2} \quad (7)$$

$$\rho \ddot{u}_3 = -\frac{\partial p}{\partial x_3} - \rho \beta g u_3 \quad (8)$$

We seek the solutions of the Eqs. (6), (7), and (8) that depend on the law $e^{i\omega t}$ (ω is real):

$$\vec{u} = \vec{U}(x_1, x_2, x_3) e^{i\omega t} \quad (9)$$

$$p = P(x_1, x_2, x_3) e^{i\omega t} \quad (10)$$

With ω is a real and t is time.
We have:

$$\rho \omega^2 U_1 = \frac{\partial P}{\partial x_1} \quad (11)$$

$$\rho \omega^2 U_2 = \frac{\partial P}{\partial x_2} \quad (12)$$

$$\rho \left(\omega^2 - \beta g\right) U_3 = \frac{\partial P}{\partial x_3} \quad (13)$$

We have $div\vec{u} = 0$, therefore:

$$\frac{1}{\omega^2}\left(\frac{\partial^2 P}{\partial x_1^2} + \frac{\partial^2 P}{\partial x_2^2}\right) + \frac{1}{\omega^2 - \beta g}\frac{\partial^2 P}{\partial x_3^2} = 0$$

(14)

With the kinematic boundary conditions:

$u_1 = 0$ for $x_1 = 0$, $x_1 = a$, therefore $\frac{\partial P}{\partial x_1} = 0$ for $x_1 = 0$, $x_1 = a$.

$u_2 = 0$ for $x_2 = 0$, $x_2 = b$, therefore $\frac{\partial P}{\partial x_2} = 0$ pour $x_2 = 0$, $x_2 = b$.

$u_3 = 0$ for $x_3 = -h$, therefore $\frac{\partial P}{\partial x_3} = 0$, for $x_3 = -h$.

This problem was solved by the method of separating variables.

$$P(x_1, x_2, x_3) = X_1(x_1).X_2(x_2).X_3(x_3) \quad (15)$$

Setting:

$$\alpha = \sqrt{\frac{\beta g}{\omega^2} - 1} \quad (16)$$

Where β is the heterogeneity parameter, and g is the gravitational constant.
With:

$$\begin{cases} \alpha \text{ is real if } \omega^2 \langle \beta g \ (stable\ zone) \\ \alpha \text{ is imaginary if } \omega^2 \rangle \beta g \ (unstable\ zone) \end{cases} \quad (17)$$

Then the expression of the pressure $P(x_1, x_2, x_3)$ is provided by the following formula:

$$P(x_1, x_2, x_3) = \sum_{n=1}^{\infty}\sum_{m=1}^{\infty} A_{mn} \cos\frac{n\pi}{a}x_1 \cos\frac{m\pi}{b}x_2 \times \left[e^{i\alpha\pi\sqrt{\frac{n^2}{a^2}+\frac{m^2}{b^2}}(x_3+h)} + e^{-i\alpha\pi\sqrt{\frac{n^2}{a^2}+\frac{m^2}{b^2}}(x_3+h)}\right] \quad (18)$$

3. NUMERICAL STUDY

The computational fluid dynamics (CFD) technique can better describe complex sloshing behaviors. In this work, the Comsol-Multiphysics CFD platform was adopted to investigate sloshing. This software is based on the Finite Element Method (FEM). The numerical method and model problems are introduced in this section. This 3D model aims to simulate the dynamics of free surface flow using the moving mesh module (moving mesh) . The model handles fluid motion with the incompressible Navier-Stokes equations. The fluid is initially at rest in a rectangular tank.

A three-dimensional partially filled rectangular tank is considered, with length L, width W, and still liquid depth h.

The tank is a Cartesian coordinate system $0, x, y, z$ with the origin located as illustrated in figure 2. Irrotational and incompressible homogeneous-heterogeneous fluid is considered and excited by horizontal excitation: $X = X_0 sin(\omega t)$.

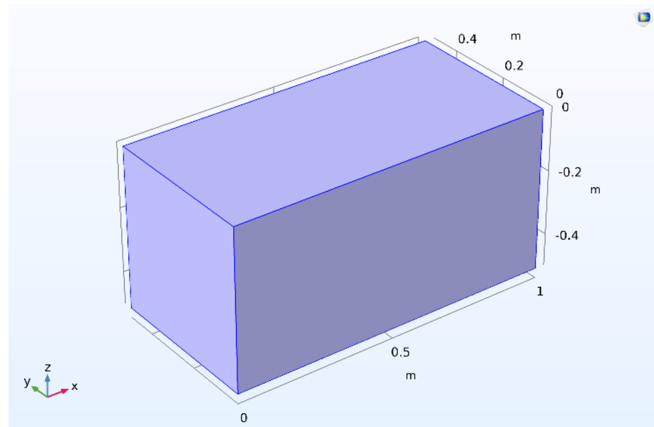

Figure 2. The geometry of the numerical model

A mesh of 23374 triangular elements considered in COMSOL Multiphysics as a coarse mesh is sufficient to discretize the geometry considered. Refinement to a normal mesh of 33116 triangular elements shows the same results.

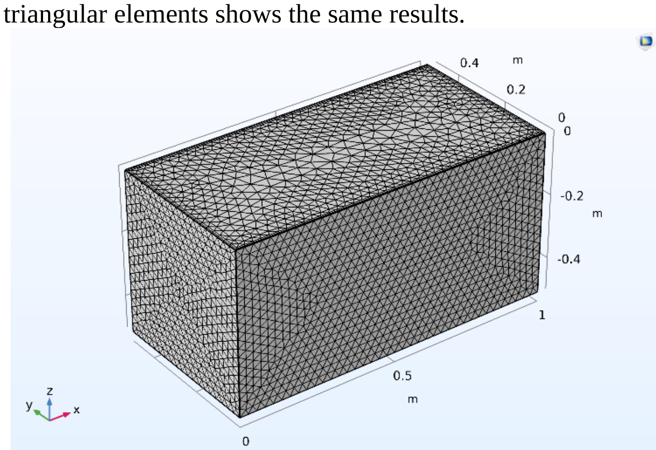

Figure 3. Mesh of the model

2.3. Investigation and Verification Schemes

The numerical model's efficiency is validated by comparing simulation results with 3-D numerical existing data.

The tank is determined with a length of $L = 0.92\ m$, a width of $W = 0.46m$, a height of $H = 0.62\ m$, and the liquid's depth is $h = 0.465\ m$. While in the second case, the tank's geometry is $L = 1\ m$, $W = 1\ m$, and $H = 1\ m$, and liquid depth is $h = 0.5\ m$. The tank was horizontally excited in the sinusoidal form: $x = 0.002\, sin(5.29t)$.

It was supposed to be water with a density of $\rho_0 = 1000 kg / m^3$ of the liquid in the tank.

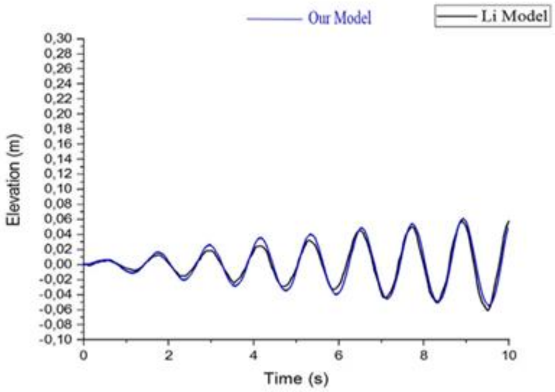

Figure 4. Comparisons of free surface elevation at the left wall of the container for $X_0 = 0.0004\, m$ and $\omega = 5.0502\, rad/s$, between the present numerical model (black line) and the numerical results by Lu et al. (red line).

Figure 4 represents a comparative analysis of the free surface elevation of the liquid between the findings of the numerical data by Li et al. [10] and the present numerical model. The results show the same tendency, with almost no errors being observed, which confirms the accuracy of this study.

4. RESULTS AND DISCUSSIONS

In this section, the sloshing wave profiles are simulated for a 3-D rectangular tank excited horizontally in the following form $X_0\, sin(\omega t)$, in which $X_0 = 0.05$ m and ω are the amplitude and the pulsation amplitude of the excitation, respectively. The tank's dimensions are supposed to be the length is $L = 1\, m$, and the width is $W = 0.5\, m$. In this section, we suggest studying the behavior of a heterogeneous fluid in a partially filled 3-D rectangular tank. A numerical study using COMSOL-MULTIPHYSICS software analyzes the influence of varying liquid density on sloshing. The numerical analysis was conducted using homogeneous liquid for the first time and heterogeneous liquid for the second time. An almost homogeneous liquid density form is considered. In all these cases, the container has the same length and width. The varying parameters are the pulsation amplitude of excitation and the heterogeneity parameter. We present the results obtained for a homogeneous liquid compared to a heterogeneous liquid.

General parameters and properties used for the model are given in table 1:

Table 1. Parameters of the numerical model

Parameters	Values
Gravity constant	9.81 N/kg
Water density	$1000 kg / m^3$
Heterogeneous liquid density	Variable

3.1. Effect of Liquid Heterogeneity on the Free Surface Behaviour of the Liquid

In this part, we present a comparative study to show the effect of liquid heterogeneity on the free surface behavior of the liquid. Sloshing behavior is compared in three cases of different liquid densities. The liquid is assumed to be irrotational and incompressible. The density of our liquid is supposed to be variable and depends on the physical properties of the liquid. We varied the heterogeneity parameter (β) to see its effect on the elevation of the liquid's free surface. The results are presented for the almost homogeneous liquid.

From Eq. (17), we distinguish the following two cases:

i) $\omega^2 \langle \beta g$ (stable zone)

ii) $\omega^2 \rangle \beta g$ (unstable zone): This case has been excluded from the discussion because it corresponds to a resonance phenomenon.

This paragraph represents the results of a detailed numerical study aimed at investigating the influence of liquid density on the characteristics of the free surface profiles in the form of an almost homogeneous liquid. The fluid density varies according to the z-axis of the tank.

We assume that the variable density in equilibrium takes the following form:

$$\rho(z) = \rho_0 (1 - \beta z) \quad (25)$$

where ρ and β is are positive constants, representing the density of the liquid and the heterogeneity parameter, respectively.

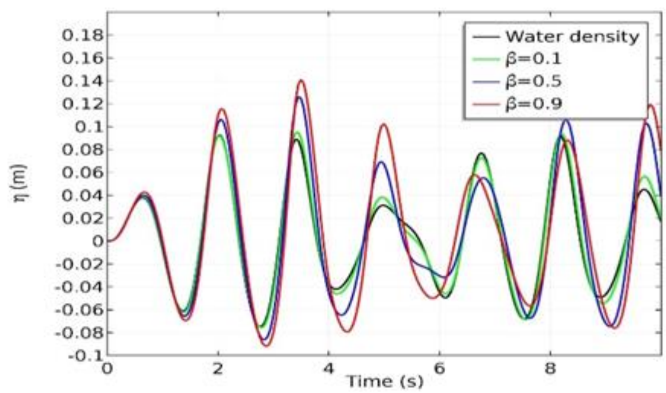

Figure 5. Time histories of free surface profiles for various beta-value (Almost-homogeneous liquid), at position (x = L, y = W / 2) with ω=4 rad/s

It can be observed from Fig. 5 that the elevation of the liquid's free surface changes as a function of the heterogeneity augmentation. We can easily see that the free surface profiles increased in the heterogeneous liquid case. Considering Fig. 5, we can notice that the heterogeneous liquid ($\beta = 0.9$) reaches a height elevation of $0.14\, m$ at $t = 3.44$ s, while the maximum wave height is $0.083\, m$ in water sloshing, showing an apparent difference between the sloshing of low and high heterogeneity fluids.

3.2. Effect of the Frequency on Sloshing

The effect of the frequency amplitude on the free surface profiles of the almost homogeneous liquid is studied by varying the external excitations through the parameter ω, where ω is the frequency of the periodic horizontal excitation.

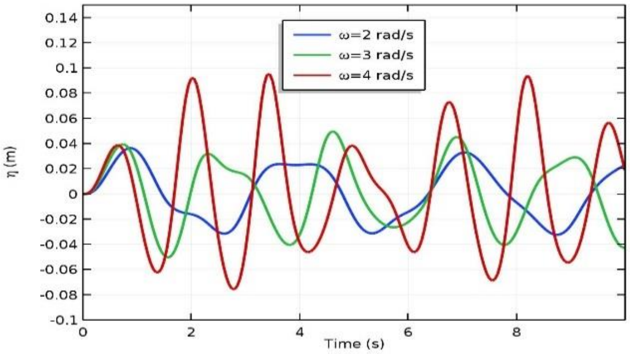

Figure 6. Wave profiles comparison for various values of the amplitude of horizontal excitation, with heterogeneity parameter β=0.3.

In Fig. 6, the wave profiles are compared for various horizontal excitation amplitudes, and it can be seen that the wavelength profiles increase as the amplitude of excitation increases. The liquid's behavior is relative to the amplitude of the excitation, and the fluid tends to move in a lateral periodic motion.

3.3. Dynamic Pressure

The following figures illustrate simulations of pressure acting on the container at the point $(x = L, y = W/2,\ z = -0.25\ m)$ for various beta parameter values. The tank excited horizontally $X_0\, sin(\omega t)$, with $X_0 = 0.05$ m and $\omega = 3rad/s$.

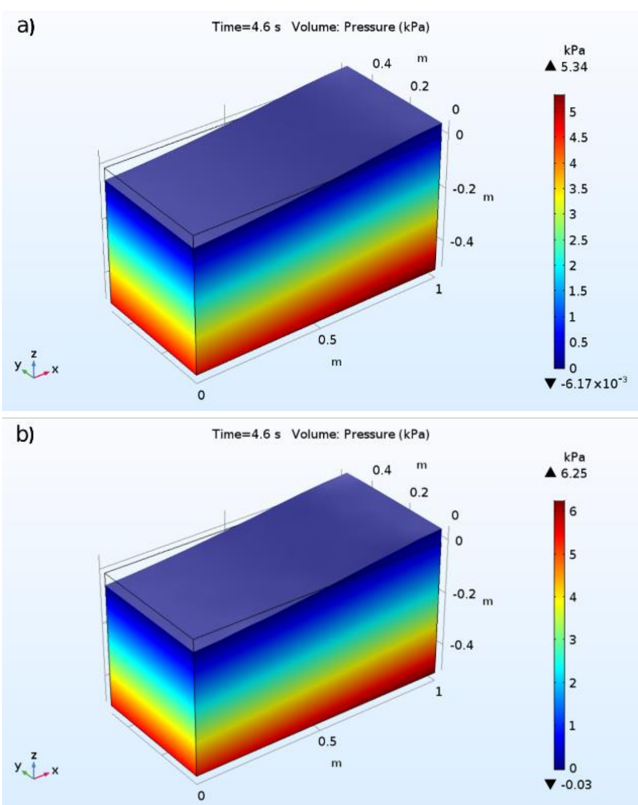

Figure 7. Pressure simulations for various β parameters (β=0 for cases a, b and β=0.5 for cases c, d), with ω=3 rad/s.

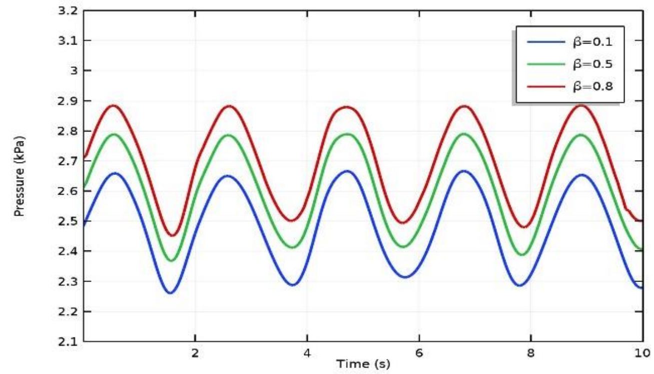

Figure 8. Pressure comparison for various β parameters at position (x=L, y=W/2, Z=-0.25 m).

Figure 7 show the pressure results for the different heterogeneity parameter value for almost heterogeneous liquid density. A significant pressure decrease is observed in the state of liquids with high heterogeneity. The maximum pressure value for the almost heterogeneous liquid at $\beta = 0.5$ is nearly ($Pmax\ = 6{,}25\ kPa$), but in the case of a homogeneous liquid, this value is decreased to ($Pmax = 5{,}34\ kPa$).
The time series of the dynamic pressure under horizontal excitation are plotted together for different heterogeneity cases of liquid sloshing in Fig. 8.

5. CONCLUSION

The proposed 3-D numerical method is used to investigate the effect of heterogeneity on the sloshing waves inside a partially filled 3-D rectangular tank, excited horizontally. The following conclusions can be drawn from the findings of the study:
We show that the heterogeneity of the liquid represented by the small parameter β is the case of new physical effects that are not characteristic of a homogeneous liquid. Notably, the free surface of the liquid increases remarkably for low heterogeneity coefficients and tends towards a large limit (instability) when the excitation frequency becomes closer and closer to $[0,\ \beta g]$ (resonance interval).

The heterogeneity of the liquid significantly affects the free surface profile elevation. When the beta parameter reaches a high value, the free surface profiles take the highest level compared to water's free surface profiles.
Pressure increases significantly in liquids with high heterogeneity.

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