

ANALYSIS OF DIFFICULTIES OF STUDENTS IN LEARNING CALCULUS**Christian S. Domondon^{1*}, Corazon G. Pardo², Elmarie T. Rin³**^{1,2,3} University of Northern Philippines, Vigan City, Philippines*For Correspondence; Tel. +639067248285, Email: dransiruno@gmail.com

ABSTRACT: *Calculus has been crucial to developing many scientific advancements, particularly in physics and engineering. This quantitative-qualitative study analyzed the difficulties in Basic Calculus of the Grade 12 Science, Technology, Engineering, and Mathematics students in Senior High School. The instrument used consisted of Multiple Choice and problem-solving questions. The study used frequency, percentages, and mean to analyze the data. Results revealed that the SHS students have satisfactory performance in Basic Calculus. Students have difficulty solving Basic Calculus problems, specifically in the general integral of functions. Some of the difficulties experienced by the students were on lack of knowledge of the concepts, poor application, complicated formulas and processes, and confusion in understanding mathematical problems. The study recommends that qualitative analysis of the errors committed by students in Basic Calculus should be conducted.*

Keywords: performance, mathematics, problem-solving, learning, education

1. INTRODUCTION

Educators have repeatedly emphasized the importance of mathematical learning. Mathematics equips learners with powerful tools to understand and change the world. These tools include logical reasoning, problem-solving skills, and the ability to think in abstract ways. The Department of Education (DepEd) did many revisions and innovations to improve learners' performance in Mathematics, but it is still one of the most difficult subjects in Basic Education.

The DepEd implemented the K-12 curriculum because of the poor quality of the Philippine Basic Education with the hope that it will help learners face the real world [1]. It is evidenced by the low achievement scores of Filipino learners in the National Achievement Test (NAT) and the international test known as the Third International Mathematics and Science Study (TIMSS) in 2003. The Philippines placed 34th out of 38 countries in High School mathematics and 43rd out of 46 in High School Science. The quality of mathematics and science subjects was slightly better in higher education. The country ranked 67th of 140 countries in 2015-2016 and 79th of 138 in 2016-2017 in the World Economic Forum's Global Competitiveness Report. Weaknesses were also reported in these areas in the National Achievement Test (NAT) mean percentage score (MPS) for high school in 2012-2013 was 51.41% or 23.59% away from the target. The MPS in science was 41.35%, and 46.83% in mathematics [2].

Mathematics is one of the subjects that is taken very seriously in the school system, irrespective of country or level of education [3]. Several related studies in learning Mathematics were conducted. Findings revealed that learners' performance developed from the individuals' perspective [4] and was influenced by teacher factors [5]. They met problems and difficulties for several reasons, such as the complex nature of Math [6] and the complexity of using symbols and computations [7]. Some of these were the lack of abilities in basic skills and algebraic manipulation, recalling knowledge facts, coordinating multiple procedures, and answering non-routine questions [8] are problems met. Their previous experiences [9], attitude [10, 11], personality [12], disposition [13], learning behavior and learning styles [8], and self-efficacy and anxiety [14, 15], also contributed to their difficulties. Learners face more difficulty with the

mathematics problems that were underrepresented and the least difficult that was overrepresented [16]. Moreover, calculus is a difficult subject among mathematics students, who often misunderstand the notion of function [17].

Despite the wide field applications in other disciplines and their constitution as a part of the curriculum, most secondary students face serious problems in understanding Basic Calculus concepts [18]. Recent efforts to reform the understanding of Basic Calculus using graphics calculator was conducted [19, 20]. The study developed descriptions of calculus learners' understandings of functions and limits to explore the interrelationships among their knowledge of these conceptual areas and to determine instances where there is proof that the student's understanding has been influenced by technological availability.

Basic Calculus prepares students who will be future scientists, engineers, and physicians. This subject requires students to determine the limit of a function, differentiate and integrate algebraic, exponential, logarithmic, and trigonometric functions in one variable, and formulate and solve problems involving continuity, extreme values, related rates, population models, and areas of plane regions. Calculus has generally been crucial to developing many scientific advancements, particularly in physics and engineering [21].

The inclusion of the Basic Calculus for Science, Technology, Engineering, and Mathematics (STEM) strand of the academic track of the Senior High School in the K-12 Basic Education Program prompted another challenge in teaching and learning. As a new subject, teachers are expected to adjust and be well-prepared for the content and standards of Basic Calculus, so their students understand its concepts and processes.

While several studies have been conducted about students' problems or difficulties in mathematics, very few of these are related to Basic Calculus. This study analyzed the difficulties of Senior High School students in learning Basic Calculus. It helps Mathematics teachers understand their students' situation and help them cope with problems and difficulties. Also, this serves as an eye opener for the curriculum planners and Mathematics educators to be aware of and consider the needs of the students in teaching Basic Calculus.

2. METHODS

This study used the quantitative-qualitative research design. The Grade 12 STEM students of the University of Northern Philippines were chosen as participants. The students who joined in the focus group discussions were purposively selected from the low scorers, comprising eight students who were willing to participate in the study. The instrument consisted of multiple choice and problem-solving tests. It was subjected for validation and had a reliability index of 0.87 and an average difficulty index of 0.40. The study adequately observed the research ethics protocol. The study used frequency, percentages, and mean to analyze the data.

3. RESULTS AND DISCUSSION

The researchers analyzed the test results given to the respondents based on the topics in Basic Calculus. The results of the analysis of the test results in Basic Calculus given to the respondents are presented below.

Table 1. Performance of students in Basic Calculus

Type of Test	Topics	Mean Percentage Score	DR
Multiple Choice	Concepts on Basic Calculus	45.61	S
	Limits	69.65	VS
Problem Solving	Derivatives	49.30	S
	Integral	1.17	P
	Overall Problem Solving	44.45	S
Overall		45.03	S

Overall, the students have "Satisfactory" performance in Basic Calculus (MPS=45.03). They performed highest along limits (MPS=69.65) and poorest along integral (MPS=1.17). This implies that the students have a difficult understanding of the integration of functions due to more complicated and confusing formulas and processes used on integrals.

This is true with the findings that students have satisfactory performance in Mathematics [22]. It is contradictory, however, to the TIMSS 2019 that the Philippines was ranked lowest out of 55 countries [2]. Also, another study found that Grade 11 STEM students have poor to very poor overall concept image in functions [23].

Results of the FGD supported the low performance of many students in their claim that Basic Calculus is difficult due to a lack of knowledge and poor foundation in Basic Mathematics, confusing and complicated, which made them fear it and not confident in solving the problems.

Item 1: With the function $y = x^2 + 4x + 1$, find the limit of the function as x approaches 3.

Figure 1: Failed to Find the Limit of Algebraic Function

The student knew how to solve the limits of the given algebraic function but failed to get the square of a number. Instead of 9, he answered 6 to 3^2 , confusion evidence about the square of a number which led to a wrong answer.

Students also claimed that some basic concepts are confusing, especially in their application. It is also a wrong notion for them to multiply 2 when it comes to the square of a number.

Students did not master the prerequisites and struggled with basic mathematical operations [24], and lacked conceptual understanding of the given problem, leading to poor skills in manipulating expressions, calculation mistakes, and technical errors [25].

Item 4: What is the limit of the function $y = \frac{1}{\sin \theta}$ as x approaches 0?

Figure 2: Failed to Do the Limit of Trigonometric Function

Even though the student followed the correct procedure, the final answer was erroneous, because one divided by zero should be undefined, leading to an infinite answer.

A student mentioned, "I had a poor mathematical foundation." This is the reason for the incorrect answer, aside from a lack of knowledge and confusion about the problems. Some errors and misconceptions were due to a lack of conceptual and procedural understanding [26].

Item 5: Find the derivative of the function $y = x^3 + 3x + 1$ with respect to x .

Figure 3: Failed to Derive Algebraic Function

A student correctly subtracted 1 in the exponent but failed to multiply the exponent by the numerical coefficient, which is a violation of the Power Rule. Students emphasized that lack of knowledge contributed most to their mistakes. Other students mentioned that they have a lack of interest in the subject because of their thinking that math is complicated.

The power rule is the most commonly used in derivatives [27]. Focusing on procedural rather than conceptual understanding contributes to learners' difficulties in calculus problems [28].

Item 8: Compute for the derivative of $y = 4^{2x}$

Figure 4: Failed to Derive Exponential Function

The student failed to use the appropriate procedure to solve the exponential function's derivative. This is due to a lack of understanding of the concept of the exponential function. The student used the Power Rule instead of another process appropriate to the given problem. Hence, the answer was wrong.

Students said that mathematics has many processes to follow, which confused them in applying those processes.

The derivation is one of the fundamental concepts in mathematics [29], and students have difficulties learning this concept because of a lack of conceptual understanding [30, 31].

Item 11: Evaluate $\int (5x^4 + 3x^2 + 6)dx$

Figure 5: Failed to Integrate Algebraic Function

Few students knew some of the processes in getting the integral of the given function but failed in other aspects. They could not divide each term by the exponent, which is necessary to solve an algebraic function correctly. "Integration is more complicated than derivative, and it is very confusing." This was mentioned by a student who believed that integration is more difficult to understand.

Learners' difficulties in integration are attributed to the students' acquired mathematical knowledge and skills from basic mathematics [32].

Item 13: Find $\int 4^{3x} dx$

Figure 6: Failed to Integrate Exponential Function

The student failed to do the proper process of getting the integral of the exponential function, resulting in an incorrect solution and answer.

Students said that numbers with exponents were challenging, and confessed that they have little knowledge of the

exponential function and hate solving exponents and fractions.

Exponential functions are pivotal mathematical concepts that give students serious difficulty [33].

4. CONCLUSIONS AND RECOMMENDATIONS

The students perform satisfactorily in Basic Calculus. They find Basic Calculus difficult, especially along Integrals. Moreover, some of the difficulties experienced by the students were on lack of knowledge of the concepts, poor application, complicated formulas and processes, and confusion in understanding the problem. A qualitative analysis of the errors committed by students in Basic Calculus should be undertaken.

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