EXACT CLOSED FORM EXPRESSIONS AND PERFORMANCE EVALUATION OF COGNITIVE AMPLIFY-AND-FORWARD RELAYING IN (I.N.I.D) NAKAGAMI-*m* FADING ENVIRONMENT

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ABSTRACT: Dynamic Spectrum Acquisition is playing the key role in making the licensed band more effective. Cognitive relay network is the solution for the efficient utilization of the licensed band. This paper therefore discusses the performance evaluation of cognitive amplify-and-forward (AF) relay networks model over (i.n.i.d.) Nakagami-*m* fading channel environment. By considering a radio resource sharing environment having a secondary user or cognitive user source *S*, *K* number of secondary relays R_k, secondary destination *D* and a primary receiver *P* under best relay selection (*BRS*) strategy, several important performance parameters are investigated. For secondary user network exact closed form expressions for the average symbol error probability (*SEP*) and outage probability (*OP*) are derived. These derived analytical results are very important and useful in understanding the performance of the system against quite lengthy time consuming simulations. Asymptotic plots are also considered and sketched on the same curves in order to validate the accuracy of the derived theoretical results. This further allows the evaluation of the system performance considering flexibility in fading scenarios.

Key Words: Best relay selection, Cognitive radio, Cooperative diversity, Nakagami-m fading, Performance analysis

1. INTRODUCTION

In this modern era, communication has played a key role. The world is moving more towards fast and reliable communication sources. Wireless communication has his importance and place in the globe today. Due to its large range of geographical coverage it has got more priority than any other source of communication. As wireless communication is associated with the radio waves, with the rapid increase in the demand of radio spectrum, bandwidth has become the most valuable and expensive thing in Telecom: industry.

As radio spectrum is of limited range it is becoming quite difficult to sustain this enormous demand. Furthermore, with the crop up of new applications in wireless communication such as mobile communication, Wi-Fi etc., the demand for radio spectrum will increase in an exponential way. Typically, radio spectrum is isolated into licensed channels to telecommunication companies, internet providers and different sectors that involve the communication act as primary users (PU). Experimental exploration has indicated that more often the authorized channels utilized by the clients either don't transmit or impart. Due to increase in demand researchers are focusing on the efficient use of spectrum or bandwidth [1,3].

Cognitive radio is the reply to this demanding question. When cognitive network users try to utilize or share a licensed band they detect free spaces in spectrum, pick the right frequency bands, put together along with other users pertaining to gain access the particular free frequency band each time a primary user shows up. Cooperative Diversity is also an efficient approach in terms of improving the reliability of wireless communication. It uses an Intermediate node named as 'relay' for transmitting data between the source and the destination. This Intermediate node will not only improve network performance but also network throughput and adaptability. Combining the Cognitive Radio network and Cooperative Diversity will enable the network for enhanced usage of spectrum and its reliable communication.

Author in [6] describes a cooperative overlay cognitive network, where the secondary user exploits the primary retransmissions. It is shown that by using cooperation, the secondary rate can be considerably increased when compared to other schemes, without causing major impact on the primary performance. The outage probability of cognitive relay networks with cooperation between secondary users based on the underlay approach is evaluated in [7], while adhering to the interference constraint on the primary user in Rayleigh fading channel. However, [8] investigates the outage probability of cognitive relay networks based on the underlay approach, while considering the mutual interference between cognitive system and primary system in Nakagami-*m* fading channels. The performance of the best relay selection scheme with fixed gain relays operating in non-identical Nakagami-*m* fading channels is analyzed in [9]. Research article at [10] investigates the performance for the primary and secondary transmissions in cognitive radio networks where the amplify-and-forward (AF) secondary relay helps to transmit the signals for both the primary and secondary transmitters over independent Nakagami-*m* fading. In [11], the outage probability (*OP*) of dual-hop cognitive amplify-and-forward (AF) relay networks subject to independent non-identically distributed (*i.n.i.d.*) Nakagami-*m* fading is examined. We assume a spectrum-sharing environment, where two different strategies are proposed to determine the transmit powers of the secondary network. In [12], the outage performance of dual hop cooperative spectrum sharing system with a direct link is investigated. A selection combining receiver is employed at the destination in order to combine the signals received from the decode-and-forward (DF) relay assuming independent non-identically distributed Nakagami-*m* fading channels. In [13], performance analysis of cognitive amplify-and-forward relaying with best relay selection strategy is considered when non-identical Rayleigh fading channel is assumed. An asymptotic analysis of the symbol error rates of a selection AF network is presented and a comparison of it with the conventional all participate schemes is carried out in [14].

In a radio resource sharing environment for cognitive radio network BRS (best relay selection) strategy is normally considered with decode-and-forward (DF) relays or amplify-and-forward (AF) relays. The secondary users use the radio spectrum concurrently with the main user's communication probably employing frequency spreading or perhaps power allocation strategies. Therefore, the particular SU needs to communicate having minimal power to run down below the particular noise level of the PU, making sure sort of tolerable interference towards main user's. Underlay method has the flexibility of transmission without any interruption and doesn't need to be synchronized while using licensed user's band.

The paper is organized as follows: Section 2 describes the System Model and the channel characteristics while Section 3 derives the closed form analysis of outage probability and symbol error probability. Section 4 discusses the results obtained and finally some conclusions are drawn in Section 5.

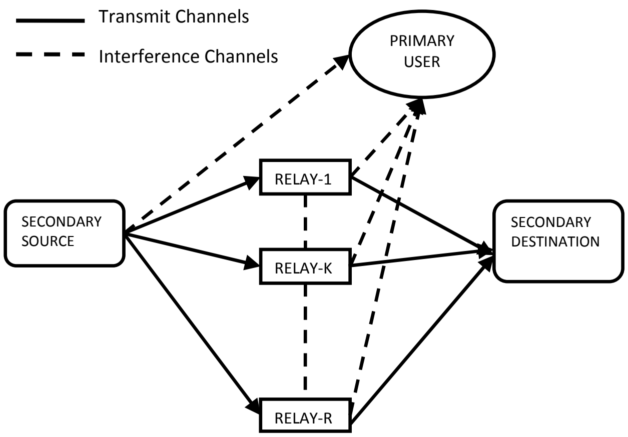

Figure 1: System Model

2. SYSTEM AND CHANNEL MODEL

The system model shown in Figure 1 assumes a dual-hop cooperative radio resource sharing environment and it further consists of one Secondary User (SU) source *S*, *K* Amplify and Forward (*AF*) *SU* relays *RL*, one *SU* destination D, and one primary User (PU) receiver P. All terminals are single-antenna devices and work in a half-duplex manner. It is supposed that there is no direct link between the source and the destination. The underlay or interference tolerant approach is applied in this Cognitive relay network.

Transmit power at *SU* source is $\mathbf{P}_s = I_P/|h_{s,p}|^2$, where $h_{s,p}$ represent the channel coefficient between *S* & *P* and I_P is threshold interference at *PU* receiver *P*. While transmit power at *K* Relays is $P_R = I_P/|h_{k,p}|^2$, where $h_{k,p}$ represent the channel coefficient between R_K & *P*. R_K amplifies the signal with gain G_K. The gain in Amplify and Forward based relay is dependent on the channel behavior:

$$1/{G_K}^2 = |h_{K,P}|^2\left(\frac{|h_{S,K}|^2}{|h_{S,P}|^2} + \frac{N_0}{I_P}\right) \tag{1}$$

The relay thus forwards the amplified signal to the destination and the signal received at destination (*D*) is given by:

$$y_D = \sqrt{P_s}\, G_K h_{K,D} h_{S,K} + G_K h_{K,D} \eta_{R_K} + \eta_{D_K} \tag{2}$$

Where η_{R_K} & η_{D_K} is *AWGN* at R_K and *D*. Thus End-to-End signal to noise ratio (*SNR*) is

$$\gamma_K = \frac{\frac{\bar{\gamma}|h_{S,K}|^2 \bar{\gamma}|h_{K,D}|^2}{|h_{S,P}|^2\ |h_{K,P}|^2}}{\frac{\bar{\gamma}|h_{S,K}|^2}{|h_{S,P}|^2} + \frac{\bar{\gamma}|h_{K,D}|^2}{|h_{K,P}|^2} + 1} \tag{3}$$

Now, for the Best Relay Selection (*BRS*) strategy the relay having the highest value of γ_K is selected. End-to-End *SNR* at *D* is given by:

$$\gamma_D = \max_{K=1\ldots k} \gamma_K \tag{4}$$

For upper bound analysis the Best Relay that maximizes the minimum *SNR* related to link S→ R_K & R_K → *D* is

$$\gamma_{up} = \max_{K=1\ldots k}\left(\min\left(\gamma_{1K}, \gamma_{2K}\right)\right) \tag{5}$$

Where γ_{1K} = S→ R_K link and γ_{2K} = R_K → *D* link

$$\gamma_{up} = \max_{K=1\ldots k}\left(\min\left(\frac{\bar{\gamma}|h_{S,K}|^2}{|h_{S,P}|^2}, \frac{\bar{\gamma}|h_{K,D}|^2}{|h_{K,P}|^2}\right)\right) \tag{6}$$

Here in (6), we are assuming that all channel coefficients suffer from (*i.n.i.d*) Nakagami-*m* fading. As a result channel gains $|h_{S,P}|^2$, $|h_{S,K}|^2$, $|h_{K,D}|^2$ and $|h_{K,P}|^2$ follow gamma distribution. $m_{S,P}$, $m_{S,K}$, $m_{K,D}$ and $m_{K,P}$ are the fading severity parameters having mean channel powers equal to $\Omega_{S,P}$, $\Omega_{S,K}$, $\Omega_{K,D}$ and $\Omega_{K,P}$ respectively.

3. CLOSED FORM ANALYSIS

Performance Analysis of the system is done through deriving the closed form expressions for Outage Probability and Symbol Error Probability.

3.1 Outage Probability

Outage Probability is described as the probability that the output *SNR* at destination γ_D is less or equivalent to a threshold *SNR* γ_{th} level, i.e.

$$P_{out} = P_r\left(\gamma_D \le \gamma_{th}\right) \tag{7}$$

The *CDF* of γ_{up} is written as

$$F_{\gamma_{up}}(\gamma) = \prod_{K=1}^{k} F_{\gamma_K^{up}}(\gamma) \tag{8}$$

Applying the conditional statistics on the channel from *S*→*P*. *CDF* of γ_{up} conditioned on $h_{S,P}$ is

$$F_{\gamma_{up}}\left(\gamma|h_{S,P}\right) = \prod_{K=1}^{k} F_{\gamma_K^{up}}\left(\gamma|h_{S,P}\right)$$

$$= \prod_{K=1}^{k}\left[1 - \left(1 - F_{\gamma_{1K}}\left(\gamma|h_{S,P}\right)\right)\left(1 - F_{\gamma_{2K}}\left(\gamma|h_{S,P}\right)\right)\right] \tag{9}$$

where,

$$F_{\gamma_{1K}}(\gamma|h_{S,P}) = 1 - \frac{\Gamma(m_{S,K}, \alpha_{1K}|h_{S,P}|^2\gamma/\bar{\gamma})}{\Gamma(m_{S,K})} \tag{10}$$

and

$$F_{\gamma_{2K}}(\gamma|h_{S,P}) = \int_0^\infty F_{|h_{K,D}|^2}\left(\frac{\gamma}{\bar{\gamma}}x\right) f_{|h_{K,P}|^2}(x)\,dx$$
$$= 1 - \sum_{q=0}^{m_{K,D}-1} \frac{\alpha_{4K}^{m_{K,P}}\Gamma(m_{K,P}+q)}{q!\Gamma(m_{K,P})(\alpha_{4K}+\alpha_{2K}\gamma/\bar{\gamma})^{m_{K,P}+q}}\left(\frac{\alpha_{2K}\gamma}{\bar{\gamma}}\right)^q \tag{11}$$

$$F_{\gamma_{up}}(\gamma|h_{S,P}) = \prod_{K=1}^{R}\left[1 - \left(\frac{\Gamma\left(m_{S,K}, \frac{\alpha_{1K}|h_{S,P}|^2\gamma}{\bar{\gamma}}\right)}{\Gamma(m_{S,K})}\right)\left(\sum_{q=0}^{m_{K,D}} \frac{\alpha_{4K}^{m_{K,P}}\Gamma(m_{K,P}+q)}{q!\Gamma(m_{K,P})(\alpha_{4K}+\alpha_{2K}\gamma/\bar{\gamma})^{m_{K,P}+q}}\left(\frac{\alpha_{2K}\gamma}{\bar{\gamma}}\right)^q\right)\right] \tag{12}$$

For more simplification, we need to apply this identity

$$\prod_{K=1}^{k}(1 - x_K) = \sum_{K=0}^{k}\frac{(-1)^K}{K!}\sum_{n_1,\dots,n_K}^{R}\prod_{t=1}^{K} x_{n_t} \tag{13}$$

The equation in (12) becomes and gets the form of

$$F_{\gamma_{up}}(\gamma|h_{S,P}) = \sum_{K=0}^{R}\frac{(-1)^K}{K!}\sum_{n_1,\dots,n_K}^{R}\prod_{t=1}^{K}\left[\left(\frac{\Gamma\left(m_{S,K}, \frac{\alpha_{1n_t}|h_{S,P}|^2\gamma}{\bar{\gamma}}\right)}{\Gamma(m_{S,K})}\right)\bar{\Theta}(\gamma)\right] \tag{14}$$

where,

$$\bar{\Theta}(\gamma) = \left(\sum_{q=0}^{m_{K,D}-1}\frac{\alpha_{4n_t}^{m_{K,P}}\Gamma(m_{K,P}+q)}{q!\Gamma(m_{K,P})(\alpha_{4n_t}+\alpha_{2n_t}\gamma/\bar{\gamma})^{m_{K,P}+q}}\left(\frac{\alpha_{2n_t}\gamma}{\bar{\gamma}}\right)^q\right) \tag{15}$$

Finally the ***OP*** can be expressed as

$$P_{out} \simeq \int_0^\infty F_{\gamma_{up}}(\gamma_{th}|h_{SP}) f_{|h_{S,P}|^2}(x)\,dx \tag{16}$$

Replacing 'γ' with 'γ_{th}' and put this in the eq. (5)

$$P_{out} \simeq \sum_{K=0}^{R}\frac{(-1)^K}{K!}\sum_{n_1,\dots,n_K=1}^{R}\prod_{t=1}^{K}(A1) \times B1 \tag{17}$$

where,

$$A1 = \sum_{q=0}^{m_{K,D}-1}\frac{\alpha_{4n_t}^{m_{K,P}}\Gamma(m_{K,P}+q)}{q!\Gamma(m_{K,P})\left(\alpha_{4n_t}+\frac{\alpha_{2n_t}\gamma_{th}}{\bar{\gamma}}\right)^{m_{K,P}+q}}\left(\frac{\alpha_{2n_t}\gamma_{th}}{\bar{\gamma}}\right)^q \tag{18}$$

and

$$B1 = \frac{(m_{S,K}-1)!}{\Gamma(m_{S,K})\Gamma(m_{S,P})}\sum_{p=0}^{m_{S,K}-1}\frac{1}{p!}\Gamma(m_{S,P}+p)\frac{\alpha_{3K}^{m_{S,P}}\sum_{t=1}^{K}\alpha_{1n_t}^{p}\left(\frac{\gamma_{th}}{\bar{\gamma}}\right)^p}{\left(\sum_{t=1}^{K}\alpha_{1n_t}\frac{\gamma_{th}}{\bar{\gamma}}+\alpha_{3K}\right)^{m_{S,P}+p}} \tag{19}$$

3.2 Symbol Error Probability

Utilizing the method applied in [1], expression for *SEP* is written as

$$P_e = \frac{a\sqrt{b}}{2\sqrt{\pi}}\int_0^\infty \frac{e^{-b\gamma}}{\sqrt{\gamma}} F_{\gamma_{up}}(\gamma)\,d\gamma \tag{20}$$

Here "*a*" and "*b*" are dependent on the type of modulation scheme selected, *e.g.* $a = 2$ and $b = \sin\left(\frac{\pi}{M}\right)^2$ for *M*-PSK.

Replacing γ_{th} with γ in equation (17) for obtaining *CDF* expression for γ_{up}:

$$F_{\gamma_{up}}(\gamma) \simeq 1 + \sum_{K=1}^{R}\frac{(-1)^K}{K!}\sum_{n_1,\dots,n_K}^{R}\prod_{t=1}^{K}(A2) \times \frac{\alpha_{3K}^{m_{S,P}}}{\Gamma(m_{S,K})\Gamma(m_{S,P})}(m_{S,K}-1)!\,(B2) \tag{21}$$

where,

$$A2 = \sum_{q=0}^{m_{K,D}-1}\frac{\alpha_{4n_t}^{m_{K,P}}\Gamma(m_{K,P}+q)}{q!\Gamma(m_{K,P})\left(\alpha_{4n_t}+\frac{\alpha_{2n_t}\gamma}{\bar{\gamma}}\right)^{m_{K,P}+q}}\left(\frac{\alpha_{2n_t}\gamma}{\bar{\gamma}}\right)^q \tag{22}$$

$$B2 = \sum_{p=0}^{m_{S,K}-1}\frac{1}{p!}\Gamma(m_{S,P}+p)\frac{\sum_{t=1}^{K}\alpha_{1n_t}^{p}\left(\frac{\gamma}{\bar{\gamma}}\right)^p}{\left(\sum_{t=1}^{K}\alpha_{1n_t}\frac{\gamma}{\bar{\gamma}}+\alpha_{3K}\right)^{m_{S,P}+p}} \tag{23}$$

Put this in eq. (20)

$$P_e = \frac{a}{2} + \frac{a\sqrt{b}}{2\sqrt{\pi}}\sum_{K=1}^{R}\frac{(-1)^K}{K!}\sum_{n_1,\dots,n_K}^{R}\prod_{t=1}^{K}A3 \times B3\int_0^\infty\left(\frac{e^{-b\gamma}(\gamma)^{p+q-\frac{1}{2}}}{\left(\frac{\alpha_{4n_t}}{\alpha_{2n_t}c}+\gamma\right)^{m_{K,P}+q}\left(\frac{\alpha_{3K}}{\sum_{t=1}^{k}\alpha_{1n_t}c}+\gamma\right)^{m_{S,P}+p}}\right)d\gamma \tag{24}$$

where,

$$A3 = \sum_{q=0}^{m_{K,D}-1}\frac{\alpha_{4n_t}^{m_{K,P}}\Gamma(m_{K,P}+q)}{q!\Gamma(m_{K,P})}\frac{1}{(\alpha_{2n_t}c)^{m_{K,P}}}\frac{\alpha_{3K}^{m_{S,P}}}{\Gamma(m_{S,K})\Gamma(m_{S,P})}(m_{S,K}-1)! \tag{25}$$

$$B3 = \sum_{p=0}^{m_{S,K}-1}\frac{1}{p!}\Gamma(m_{S,P}+p)\frac{1}{\left(\sum_{t=1}^{k}\alpha_{1n_t}c\right)^{m_{S,P}}} \tag{26}$$

Only taking the integral part:

$$\int_0^\infty\left(\frac{e^{-b\gamma}(\gamma)^{p+q-\frac{1}{2}}}{\left(\frac{\alpha_{4n_t}}{\alpha_{2n_t}c}+\gamma\right)^{m_{K,P}+q}\left(\frac{\alpha_{3K}}{\sum_{t=1}^{k}\alpha_{1n_t}c}+\gamma\right)^{m_{S,P}+p}}\right)d\gamma \tag{27}$$

By using the equation from [2], since the integral is not represented in the common form we need to do partial fraction of the equation, we apply the [[4], Eq. (2.102)], and then use [[3], Eq. (8.4.2.5)]. Using [4], Eq. (8.13.1)], the equation becomes:

$$= \left(\frac{1}{b}\right)^{p+q+\frac{1}{2}}\left[\sum_{i=1}^{m_{S,P}+p}\Delta\left(m_{S,P}+p-i, m_{K,P}+q, \frac{\alpha_{4n_t}}{\alpha_{2n_t}c}, \frac{\alpha_{3K}}{\sum_{t=1}^{k}\alpha_{1n_t}c}\right)\left(\frac{\sum_{t=1}^{k}\alpha_{1n_t}c}{\alpha_{3K}}\right)^i\frac{1}{\Gamma(i)}G_{2,1}^{1,2}\left(\left(\frac{\sum_{t=1}^{k}\alpha_{1n_t}c}{\alpha_{3K}b}\right)\middle|\begin{matrix}\frac{1}{2}-p-q, 1-i\\ 0\end{matrix}\right) + \sum_{j=1}^{m_{K,P}+q}\Delta\left(m_{K,P}+q-j, m_{S,P}+p, \frac{\alpha_{3K}}{\sum_{t=1}^{k}\alpha_{1n_t}c}, \frac{\alpha_{4n_t}}{\alpha_{2n_t}c}\right)\left(\frac{\alpha_{2n_t}c}{\alpha_{4n_t}}\right)^j\frac{1}{\Gamma(j)}G_{2,1}^{1,2}\left(\left(\frac{\alpha_{2n_t}c}{\alpha_{4n_t}b}\right)\middle|\begin{matrix}\frac{1}{2}-p-q, 1-j\\ 0\end{matrix}\right)\right] \tag{28}$$

Here $\Delta(n, m, \varsigma, \tau) = \frac{1}{n!}\frac{\partial^n(\gamma+\varsigma)^{-m}}{\partial\gamma^n}\Big|\gamma = -\tau$

Doing some more simplification

$$P_e \simeq \frac{a}{2} + \frac{a\sqrt{R}}{2\sqrt{\pi}} \sum_{K=1}^{R} \frac{(-1)^K}{K!} \sum_{n_1 \ldots n_K}^{R} \sum_{q=0}^{m_{K,D}-1} \sum_{p=0}^{m_{S,K}-1} (m_{S,K} - 1)! \frac{\Gamma(m_{K,P}+q)\Gamma(m_{S,P}+p)}{p!q!\Gamma(m_{K,P})\Gamma(m_{S,K})\Gamma(m_{S,P})} \left(\frac{1}{b}\right)^{p+q+\frac{1}{2}} \times [A4 + B4] \quad (29)$$

This is the closed form for the symbol error probability.
Where **A4** and **B4** are,

$$\mathbf{A4} = \sum_{i=1}^{m_{S,P}+p} \left(\begin{array}{c} \left(\prod_{t=1}^{K} \Delta\left(m_{S,P}+p-i, m_{K,P}+q, \frac{\alpha_{4n_t}}{\alpha_{2n_t}c}, \frac{\alpha_{3K}}{\sum_{t=1}^{k}\alpha_{1n_t}c}\right)\right) \\ \left(\frac{\alpha_{4n_t}}{\alpha_{2n_t}c}\right)^{m_{K,P}} \left(\frac{\sum_{t=1}^{k}\alpha_{1n_t}c}{\alpha_{3K}}\right)^{(i-m_{S,P})K} \\ \frac{1}{\Gamma(i)} G_{2,1}^{1,2}\left(\left(\frac{\sum_{t=1}^{k}\alpha_{1n_t}c}{\alpha_{3K}b}\right) \middle| \begin{array}{c} \frac{1}{2}-p-q, 1-i \\ 0 \end{array} \right) \end{array} \right) \quad (30)$$

$$\mathbf{B4} = \sum_{j=1}^{m_{K,P}+q} \left(\sum_{t=1}^{K} \left(\begin{array}{c} \Delta\left(m_{K,P}+q-j, m_{S,P}+p, \frac{\alpha_{3K}}{\sum_{t=1}^{k}\alpha_{1n_t}c}, \frac{\alpha_{4n_k}}{\alpha_{2n_k}c}\right) \\ \left(\frac{\alpha_{3K}}{\sum_{t=1}^{k}\alpha_{1n_t}c}\right)^{m_{S,P}} \left(\frac{\alpha_{2n_t}c}{\alpha_{4n_t}}\right)^{(j-m_{K,P})K} \\ \frac{1}{\Gamma(j)} G_{2,1}^{1,2}\left(\left(\frac{\alpha_{2n_t}c}{\alpha_{4n_t}b}\right) \middle| \begin{array}{c} \frac{1}{2}-p-q, 1-j \\ 0 \end{array} \right) \end{array} \right) \right) \quad (31)$$

4.

UMERICAL RESULTS AND DISCUSSION

In this section numerical results are presented to validate the accuracy of our closed form analysis on (*i.n.i.d*) Nakagami-*m* fading. In Figs. 2,3,4 & 5, outage probability (***OP***) and symbol error probability (***SEP***) against signal to noise ration (***SNR***) are ploted. The following parameters are assumed for this purpose: K = {1, 2, 3, 4}, $\Omega_{S,P}$ = {4}, $\Omega_{S,K}{}_{k=1}^{4}$ = {1.2, 2.3, 3.1,4.0}, $\Omega_{K,D}{}_{k=1}^{4}$ = {0.5, 0.9, 0.7,0.6}, $\Omega_{K,P}{}_{k=1}^{4}$ = {1.1, 3.2, 2.1,2.5} and γ_{th}= 3dB. In Fig. 2 analytical and asymptotic results for outage probability (***OP***) employing best relay selection strategy is plotted. The fading parameters are set to Rayleigh fading channel by assigning $m_{S,P} = 1, m_{S,K} = 1, m_{K,D} = 1$ and $m_{K,P} = 1$. From this result it can be analyzed that as the number of relays increases, the system's performance also becomes better. Asymptotic results are used for analyzing the diversity and coding gains of the system.

In Fig 3, outage probability is again implemented for different fading scenarios between the channels. Here *R = 2* is constant for this case and only the fading parameter between the channels is varied. As we take the case of (*i.n.i.d*) Nakagami-*m* fading, it means that we can have different types of fading in the four channels. The $m_1 = m_{S,P} = 1$, $m_2 = m_{S,K} = 2$ and the rest of the fading parameters $m_3 = m_{K,D}$ and $m_4 = m_{K,P}$ are being varied. It can be observed that as the fading parameter '*m*' exceeds between the channels, the system is giving better and improved performance due to the presence of low fading between the channels.

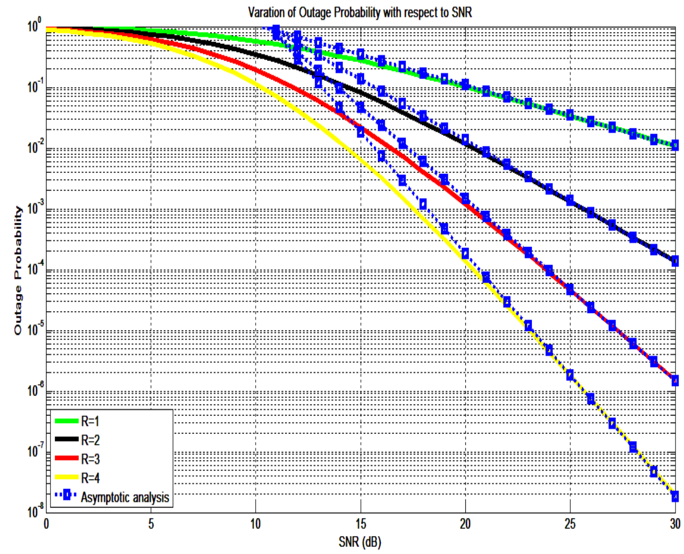

Fig. 2: Outage Probability of cognitive *AF* relaying in (*i.n.i.d*) Nakagami-*m* fading channel by varying number of R_k

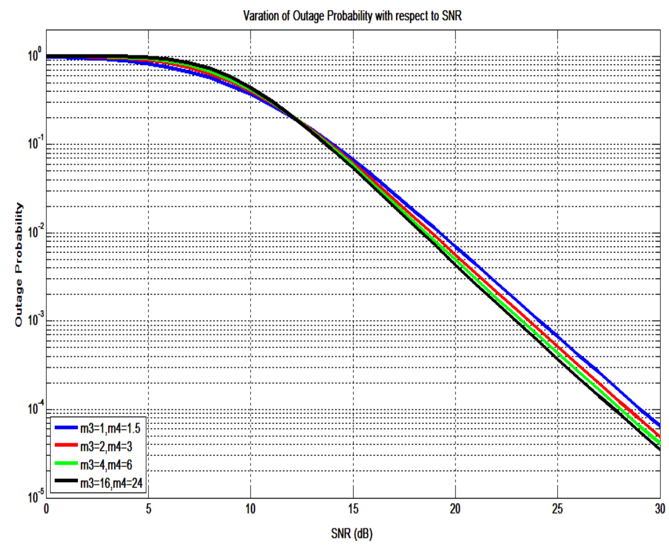

Fig 3: Outage Probability of cognitive *AF* relaying in (*i.n.i.d*) Nakagami-*m* fading channel by varying fading parameter '*m*'

In Fig 4 analytical and asymptotic results for symbol error probability (*SEP*) employing best relay selection strategy is plotted for *Q-PSK*. The performance of the system gets better and better as the value of *R* is increased from 1 to 4 respectively. We have also shown asymptotic plot on the same graph to identify the accuracy of the derived analytical results.

Now in Fig 5, the modulation scheme is changed to 4-*PAM*. Here, variation of relays is applied from R = 1 to 4. We can observe that as the number of relay increases, the system performance becomes better. This shows that diversity gain is also improved. In all the figures shown it is observed that at low and medium *SNR* values, the portion of asymptotic curves do not overlap or converge to the exact ones but at higher *SNR* values (preferably ≥ *20 dBs*), it falls accurately and coincides with their exact ones. These results deduce that asymptotic curves converge accurately towards their exact values and thus validate the accuracy of our mathematical analysis.

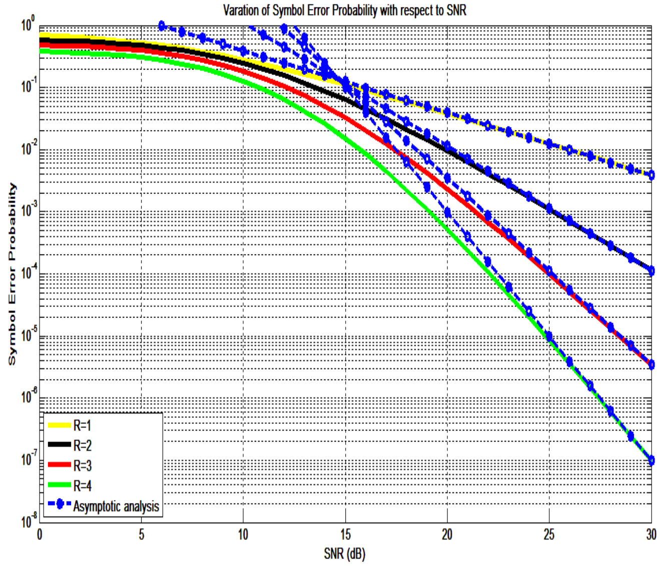

Fig 4: Symbol Error Probability of cognitive *AF* relaying in (*i.n.i.d*) Nakagami-*m* fading channel for *QPSK* by varying number of R_k

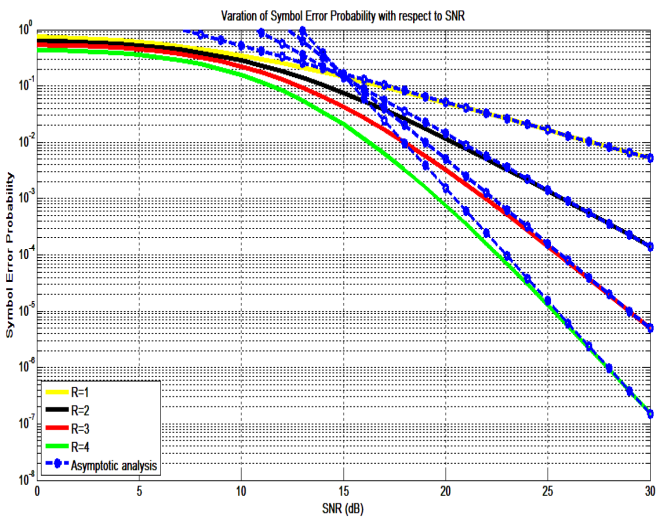

Fig 5: Symbol Error Probability of cognitive *AF* relaying in (*i.n.i.d*) Nakagami-*m* fading channel for 4-*PAM* by varying number of R_k

5. CONCLUSIONS

This work studied the performance of several cooperative relay networks in a cognitive radio environment. For deriving the closed form expressions for the probability density function (*PDF*) and cumulative distribution function (*CDF*), tight lower bound is assumed for the *SNR* between source and destination. Closed form expressions were derived for the outage Probability and symbol error probability for (*i.n.i.d*) Nakagami fading channels. Asymptotic expressions were also considered for the outage probability and symbol error probability to observe the system performance affected by the key parameters. From these derived asymptotic expressions, it was shown that that cognitive *AF* system had the same diversity as the conventional *AF* system.

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