3D NUMERICAL STUDY OF ACOUSTIC WAVES USING D3Q19-LATTICE BOLTZMANN MODEL

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ABSTRACT: *This paper proposes a three-dimensional (3D) lattice Boltzmann method (LBM) to analyze the propagation of acoustic waves. Indeed, this numerical method uses a stable and accurate scheme, the D3Q19 multiple relaxation model. The work carried out focuses on studying the vibration of a square sound source located at the center of the left surface of a 3D enclosure filled with water. The main objective of this numerical study is to investigate the waves using a statistical technique and to visualize how acoustic waves emitted by a square source propagate instantaneously in three dimensions. The numerical code used is verified by studying the usual problem of flows generated by a lid-driven cavity. The two-dimensional LBM approach is introduced in a second step to compare the results obtained in two and three dimensions.*

Keywords: Lattice Boltzmann method, acoustic waves, propagation, 3D simulation.

1. INTRODUCTION

The applications of sound waves in many fields such as industry, medicine, and even in everyday life constitute well-known research topics. They have been treated in the literature for many years. Generally, the production and propagation of sound waves are associated with the existence of a vibratory motion. In the present work, the vibration is applied to a square source placed at the left facet of a three-dimensional cavity. Thus, the particles of the medium (water) enter in vibration one after the other around their equilibrium position. This idea allowed us to use a sinusoidal function of the density to model the waves by the LBM approach. The basic principle is to make the density vibrate around its equilibrium position by referring to the acoustic point source method [1,2]. This technique is effortless to implement and makes LB a powerful numerical way to simulate acoustic waves.

The lattice Boltzmann technique is a numerical method derived from lattice gas Automata (LGA) and the kinetic theory of gas [3]. It is presented as an alternative numerical approach to model fluid dynamics. It is based on the probability of the presence of a particle in a lattice. This probability is defined by the particle distribution density function. Indeed, in the LB method, the fluid is not considered a continuous matter but rather as a discrete set of fictive particles that interact with each other.

In recent years, the lattice Boltzmann method has made significant progress in the numerical solution of various physical problems. For example, in the acoustic domain, the LBM has been used in the literature to simulate different types of waves (shock waves, sound waves, aeroacoustic waves...). In this work, this technique is applied in three dimensions to study acoustic phenomena. In the first step, the propagation of waves generated by a square source has been reviewed, and in the second step, the interaction of waves with an obstacle has been treated.

The present work is arranged into six sections. In the first, a general introduction to sound waves and the LB method is given. The second part represents a description of the LBM in two and three dimensions. The third section describes the boundary conditions employed. The validation of our computer code is given in the fourth section. After this validation, the results found are discussed. Finally, the conclusions are reported in the last quarter.

2. LATTICE BOLTZMANN METHOD

Due to its accuracy [1], the multiple relaxation time scheme is used in 2D and 3D instead of the single relaxation time scheme (SRT) to study acoustic waves. In 2D, the D2Q9 model (Figure 1.0) is applied, and in 3D, the D3Q19 model (Figure 2.0) is used. For these two models, the behavior of the fluid and the acoustic waves is given by the following Boltzmann equation [2,3]:

$$f_j\left(\vec{x}_j+\vec{c}_j\Delta t, t+\Delta t\right)-f_j\left(\vec{x}_j,t\right)=M^{-1}S[m_j^{eq}-m_j]$$

Equation 1

where f_j, Δt, and $\vec{c}_j$ are the distribution functions, the time step, and the vector of lattice velocities in direction j. m_j and m_j^{eq} are moments. M^{-1} and S are the inverse and collision matrices, respectively.

The vector $\vec{c}_j$ defines the direction of the particles in the LBM lattice. It is given in the references [2,4]. M^{-1} is the inverse matrix of the transformation matrix M. These two matrices relate the distribution function to the moments [3]:

$$m = Mf \text{ and } f = M^{-1}m$$ Equation 2

The 2D and 3D mathematical expressions of M^{-1}, M, and the moments m_j are given in the references [3,4]. The equilibrium moments can be delivered directly from the equilibrium distribution function (f^{eq}) as:

$$m^{eq} = Mf^{eq}$$ Equation 3

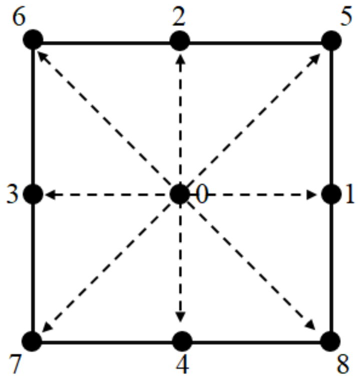

Figure 1.0: The D2Q9 model

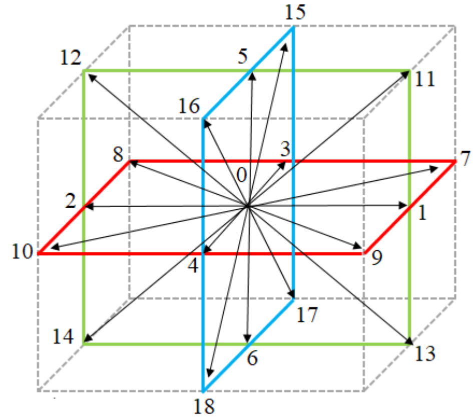

Figure 2.0: The D3Q19 model

The function f^{eq} depends on the speed of sound (c_s), the fluid density (ρ), the physical velocity vector $\vec{U} = (u, v, w)$, the LBM velocity vector $\vec{c}_j$, and discretization weights (W_j) [3]:

$$f_j^{eq} = W_j \rho \left[1 + \frac{1}{c_s^2} \vec{c}_j . \vec{U} + \frac{1}{2c_s^4} \left(\vec{c}_j . \vec{U}\right)^2 - \frac{1}{2c_s^2} \left|\vec{U}\right|^2\right]$$

Equation 4

where u, v, and w are the macroscopic velocities in $\vec{x}$, $\vec{y}$, and $\vec{z}$ directions, respectively.

The matrix S is a diagonal matrix. It is composed of fixed and variable relaxation times (s_j). The fixed ones are maintained at constant values between 0 and 2. On the other hand, the varying times s_j are generally related to the kinematic viscosity (ν) of the fluid. The mathematical expression of S can be given as:

$$S(D2Q9) = \mathrm{diag}(s_0, s_1, s_2, s_3, \dots, s_8) \quad \text{Equation 5}$$

$$S(D3Q19) = \mathrm{diag}(s_0, s_1, s_2, s_3, s_4, s_5, \dots, s_{18}) \quad \text{Equation 6}$$

In the present study, the relaxation times utilized in 2D and 3D are listed in Ref. [2] and Ref. [4], respectively.

3. BOUNDARY CONDITION

In the lattice Boltzmann method, the boundary conditions are not necessarily based on the velocity or pressure, for example, but rather on the distribution function. There are different types of boundary conditions in LBM simulations, such as periodic conditions, absorption conditions, open boundary conditions, and so on [3]. In this work, simple requirements are used to define the solid walls of the cavity. It concerns the Bounce-back boundary conditions (BBC). The principle of this condition is to reverse the direction of the unknown distribution functions at the boundaries. They are implemented as:

$$f_j(\vec{x}_B) = f_{\bar{j}}(\vec{x}_B) \quad \text{Equation 7}$$

where $f_j(\vec{x}_B)$ is the unknown function at the node $\vec{x}_B$ and $f_{\bar{j}}(\vec{x}_B)$ is the known function in the inverse direction of j $(\bar{j} = -j)$.

4. VALIDATION

In our study, we have proposed the D3Q19-LBM model to simulate acoustic waves in 3D. This model is validated by studying the lid-driven cavity problem. This type of physical problem is considered in the literature as one of the benchmark problems. It concerns a cubic cavity with a surface driven with a constant velocity. In our case, the upper wall is in uniform motion with a speed $u_0 = 0.1$ (Figure 3.0).

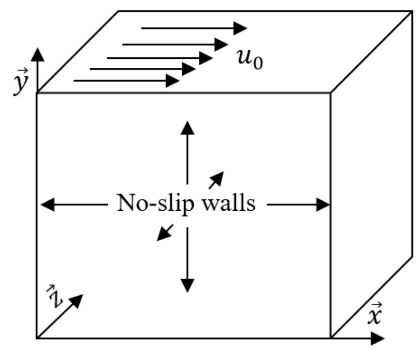

Figure 3.0: Illustration of the validation problem

The flow created by the entrained cavity can be produced numerically by using the boundary conditions of Bouzidi et al. [5]. The Reynolds number $(Re = 400)$ has been chosen to compare our results with those of Ku et al. [6], Jiang et al. [7], and Ding et al. [8]. Figure 4.0 represents the adimensional velocities along the x $(U = u/u_0)$ and y $(V = v/u_0)$ axes. This figure illustrates that the velocity $U(y)$ starts with a zero value, then, takes a negative value and arrives at the maximum value 1. However, the velocity $V(x)$ varies according to x between 0 and 1, and takes a minimum value of about -0.4 and another maximum of about 0.2. In this figure, we also note a good correspondence between our results and those of the references [6–8].

In the present work, only the 3D numerical code is validated. However, the 2D code is already verified in our previous work [2].

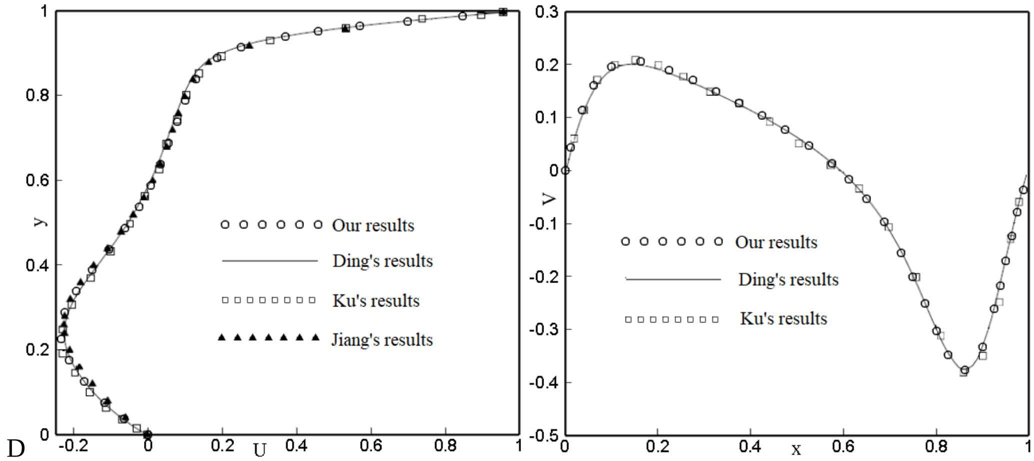

Figure 4.0: Velocity profiles $U(y)$ and $V(x)$ along the cavity centerlines for Re=400.

5. RESULTS AND DISCUSSION

As previously mentioned, the physical problem considered in this work is a square acoustic source, which vibrates with a frequency of $200KHz$ (Figure 5.0). It is placed in the center of the left wall of a three-dimensional enclosure filled with water. The walls of the cavity are considered solid. The BBC conditions are applied to these boundaries. The length, width, and height of the hole are denoted by Lx, Ly, and Lz, respectively. The node numbers which correspond to these dimensions are $\mathrm{Lx} = 300$, $\mathrm{Ly} = 240$, and $\mathrm{Lz} = 240$.

The LB method is a non-dimensional technique. However, performing physical simulations requires parameters in natural units. The space step (Δx) and the lattice time step (Δt) can be used to make the conversions [1]. In the present study, the waves propagate with a wavelength (λ_{ph}) of $7.4\ 10^{-3}$ m. To obtain an accurate simulation of the waves in the cavity, we have chosen to get at least $20\Delta x$ in $1\lambda_{ph}$. This guided us to choose $\Delta x = 3.2\ 10^{-4}$ m. Step Δt can be obtained from the physical (c_{ph}) and LBM (c_{lbm}) speeds of sound as $\Delta t = \Delta x c_{lbm}/c_{ph} = 1.248\ 10^{-7}$ s. From these space and time steps, several LBM and physics quantities can be related to each other. For example, the LBM period and wavelength can be found as $T_{lbm} = T_{ph}/\Delta t = 40.053$ and $\lambda_{lbm} = \lambda_{ph}/\Delta x = 23.125$, respectively.

The numerical technique used to generate the waves is the acoustic point source method (APSM). For this technique, the vibration of the density around its equilibrium value produces the waves as follows [1,2]:

$$\rho = \rho_0 + \rho_a \sin\left(\frac{2\pi}{T_{lbm}} t\right) \qquad \text{Equation 8}$$

Our numerical study started with verifying the APSM technique described in equation 8. This check studies the waves generated by a point source localized in the center of a cubic enclosure. Figure 6.0 shows the distribution of the density in the cavity. The waves thus caused propagate in a spherical form in 3D (Figure 6.0(A)). A vertical section at $y = Ly/2$ is given in Figure 6.0(B) to clarify the propagation of the waves in 2D. In this case, the wave propagation takes place in a circular form. The points of the LBM lattice used in this test are $150 * 150 * 150$ nodes, where ρ_0 and ρ_a are the equilibrium density and the amplitude. The value of ρ_a used in this work is employed in the references [1,2] ($\rho_a = 0.01$).

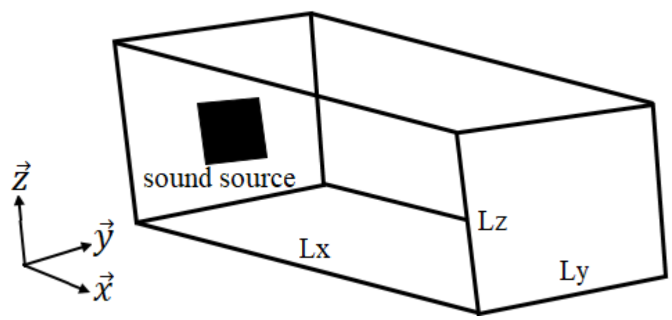

Figure 5.0: Acoustic problem studied.

Figure 7.0 shows the waves generated by the source represented in Figure 5.0 at time $t = 500$. The surface of the square source is defined by $\mathrm{Ly}/3 * \mathrm{Lz}/3$. The discretization of this source gives $80 * 80$ point sources. The waves emitted by each point interfere between them and provide an acoustic beam in the cavity. The three-dimensional density field (Figure 7.0(A)) shows that the global acoustic beam is directed towards the right wall. However, the obtained waves are not too focused on the central axis of the source, as in the case of a circular source. Remarkable reflections can be observed near the lateral and horizontal walls. The general shape of the waves can be considered as circular with a tendency to take a flat form. The vertical cross-section of the density field shown in Figure 7.0(B) illustrates that the waves propagate in two dimensions as semicircles in the near field and assume a flat form in the far-field in the proximity of the wall facing away from the source.

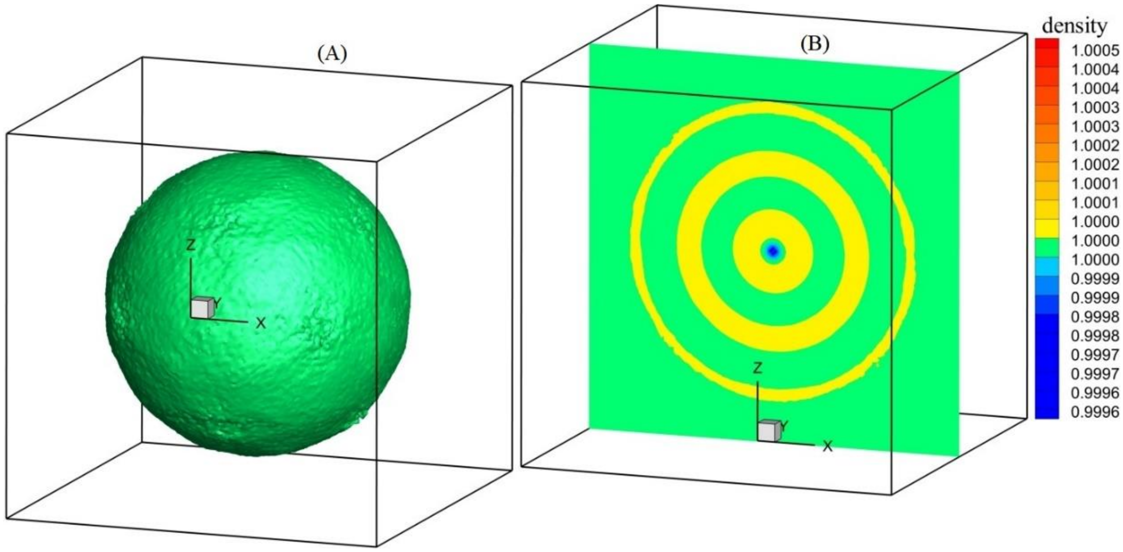

Figure 6.0: Depiction of the acoustic wave propagation produced by a point source at $t = 100$ iterations; (A) 3D illustration; (B) 2D illustration for a vertical section at $y = Ly/2$.

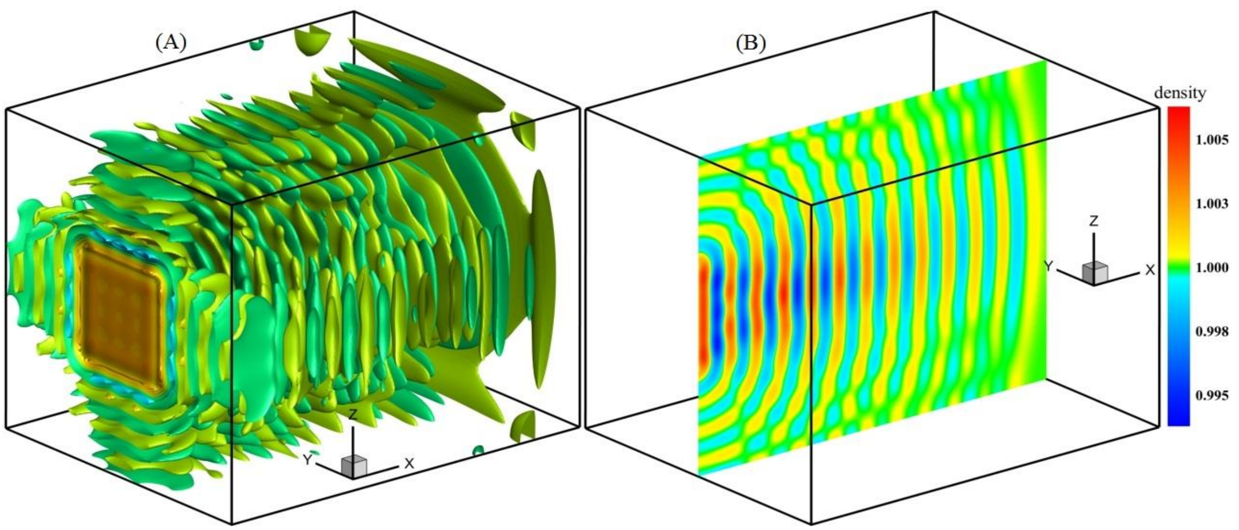

Figure 7.0: Density field obtained with a square source at $t = 500$: (A) 3D depiction and (B) 2D depiction for a vertical section at $y = Ly/2$.

As mentioned before, and to visualize the difference between 2D and 3D simulations, a 2D study is introduced in this work. The waves generated by a line of 80 point sources in 2D are compared with the 3D case. In this context, the 2D and 3D longitudinal density profiles are plotted at the center of the cavity along the x-axis (Figure 8. 0). The waves obtained from both simulations propagate in phase and are attenuated as they propagate farther from the source due to the dissipation effect and geometric spreading. However, there is a difference between the two results in terms of amplitude. The amplitude of the waves obtained in 3D is quite large than that found in 2D. This is due to the contribution of the neighboring points of the linear source in 3D.

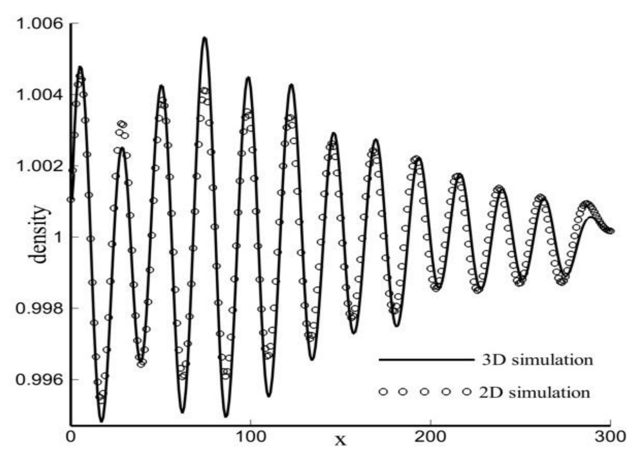

Figure 8.0: Longitudinal density profiles at the center of the cavity along the x-axis and time 500.

After comparing 2D and 3D simulations, the emphasis in this section is on the treatment of the behavior of the waves when they encounter an obstacle of size comparable to the wavelength. This phenomenon is known in the literature as diffraction. Indeed, in acoustics, the diffraction of sound waves is more pronounced with waves of large wavelengths. This means that the low frequencies around obstacles can be heard well than the higher frequencies. The large wavelength waves can pass around obstacles and reach our ears from sources in faraway places. This principle is also used in the sound insulation of a room. High-quality soundproofing requires no openings, because even a tiny space can allow sound to enter the room and, through the process of diffraction, propagate throughout the area and cause disturbance (noise). This is considered as one of the significant applications of diffraction in everyday life or, more precisely, in civil engineering [9].

Figure 9.0 shows the behavior of sound waves when they pass through a small aperture. In this figure, a plane with a central opening of 20 points (smaller than the wavelength λ_{lbm}≅23) is used to visualize the diffraction phenomenon clearly. This plane is located at the position $x = \mathrm{Lx}/3$. The waves are generated here with the vibration of the entire left wall of the cavity. The emitted waves are plane waves. When they pass through the aperture, their shape is significantly modified. The waves are no more plane but become circular.

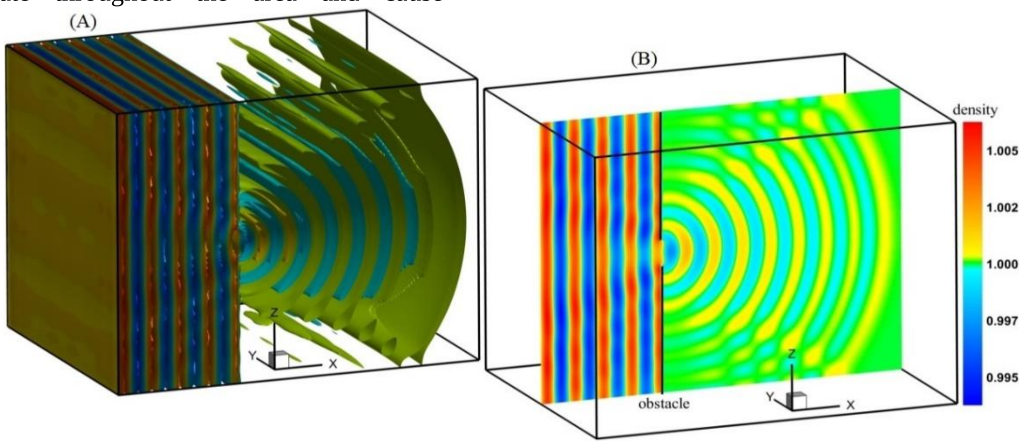

Figure 9.0: Diffraction of sound waves at time 500; (A) 3D depiction and (B) vertical section at $y = Ly/2$.

6. CONCLUSION

The lattice Boltzmann method has been applied to study the propagation of acoustic waves in three dimensions. In the first step, the waves emitted by a single point source and a square source were studied. Then, the diffraction phenomenon was investigated in the second step. These studies have shown the propagation of the waves in the cavity (spherical or circular shapes). To better visualize the wave propagation, 2D presentations have been considered. Moreover, to obtain a reliable numerical code, the three-dimensional LBM approach has been verified by simulating the lid-driven cavity problem. The results obtained show that the LBM approach can be applied confidently to different sound wave studies.

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