ON NON-COMMUTING GRAPHS OF DICYCLIC GROUP

A. Mahboob[1], Taswer Hussian[2], ,M. Javaid[3], Sajid Mahboob[4]

[1,2]Department of Mathematics, University of ducation, Lahore, Pakistan. aabidmahboob@gmail.com
shergarhcity@gmail.com
[3]Department of Mathematics,School of Science, University of Management and Technology (UMT), Lahore, Pakistan.
muhammad.javaidmath@umt.edu.pkjavaidmath@gmail.com
[4]Departmentof Mathematics,MinhajUl-Quran University,Lahore.
sajid.mehboob@gmail.com

ABSTRACT: *For a given group G and a subset $A\subseteq G - Z(G)$, the non-commuting graph $\Delta = [G, A]$ of the group G is a graph with vertex-set $V(\Delta) = A$ such that two distinct vertices $x, y \in V(\Delta)$ are connected by an edge i. e $xy \in E(\Delta)$ if and only if $xy = yx$ in G. In this paper, we study the certain properties of non-commuting graph on Dicyclic group $DiC_n = < a, x : a^{2n} = 1, x^2 = a^n, a^x = a^{-1} >$ of order 4n and obtained certain parameters of graph theory as chromatic number, clique number and perfect matching.*

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INTRODUCTION:

As we know, the algebraic structures pay a very important rule in the studies of different branches of Mathematics. The involvement of these structures in Graph Theory has become one of attractive research activities for Mathematicians of modern era. Therefore, in the literature, there are many graphs like zero-divisor graphs, total graphs, commutative graphs and non-commutative graphs. These graphs have been constructed from commutative rings and finite groups under certain algebraic properties of idealization, modules, commutatively and non-commutatively, respectively. For a detail study, reader can see [2,3, 5–7]. Now, we focus on the graphs, constructed from finite groups. Segevand Seitz [13, 14] investigated the commuting graphs from non abelian simple groups. Iranmanesh and Jafarzadeh [9] associated the commuting graphs with symmetric and alternating groups. In 2011, Chelvametal. [8] and Mashkourietal. [10] found commutating graphs from dihedral groups. Abdullah oetal. [1] and Moghaddam faretal. [11] constructed non-commuting graphs on different types of groups. In particularly, Asghar Talebi [4] constructed then on-commuting graphs of the Dihedral group.

In this paper, we construct the non-commuting graphs from non-abelian and finite groups called dicyclic groups DiC_n with respect to its some specified sub sets. The of paper is organized as follows ; the section 2 contains some notations and basic definitions, while section 3 includes main results of commuting graphs.

2 Preliminaries:

Consider simple, finite and undirected graphs. For a graph Δ, we denote the vertex-set and edge-set by $V(\Delta)$ and $E(\Delta)$, respectively. Moreover, $m = |E(\Delta)|$and $n = |V(\Delta)|$ are called size and order of the graphΔ. The degree of a vertex $v \in V(\Delta)$, denoted by $\deg\Delta(v)$ is number of incident edges on v. A graph is regular if all the vertices are of same degree. A graph of order n is called a complete graph if each pair of vertices is adjacent. It is denoted by K_n and is $n-1$ regular. For n=2, it is also called a path of order 2 such that both vertices are of degree 1. A friendship graph F_m consists of m triangles with exactly one common vertex, this common vertex is called the center of F_m.

A subset X of $V(\Delta)$ is said to be clique if the sub graph induced by X is a complete graph. The maximum size of a clique in a graph Δ is called clique number of Δ. It is denoted by $\omega(\Delta)$. For k>0 (an integer), k-vertex coloring of the graph Δ is an assignment of k-colors to the vertices of Δ such that no two adjacent vertices have same color. The chromatic number of Δ (denoted by $\chi(\Delta)$) is the minimum k for which Δ has k-vertex coloring. A graph Δ is k-colorable (edge), if its edges can be colored with k-colors such that no two adjacent edges have same color. It is denoted by $\chi^0(\Delta)$ and called chromatic index of Δ. Length of minimal cycle in a graph Δ is called girth of Δ. If u and v are the vertices in Δ, the d(u, v) denotes the distance between u and v (length of shortest path between u and v). The largest distance between all the pairs of vertices of Δ is called the diameter of Δ and is denoted by $dia(\Delta)$. A matching on a graph Δ is a sub set of Δ such that no two edges share a common vertex in it and order of this subset is called matching number of Δ denoted by $\gamma(G)$. The largest possible matching on a graph with n nodes consists of $n/2$ edges and such a matching is called a perfect matching. The center of a group G is denoted by Z(G) and defined as $Z(G) = \{x \in G: xy = yx \text{ for all } y \in G\}$. Let a be any non-identity element to f G, then centralizer of a in G is the set of elements of G which commutes with a and it is denoted by $C_a(G)$. On the other hand, $\overline{C_a(G)}$ presents the set of elements which do not commute with a in G.

Moreover, for a $\in A\subseteq \underline{G}$, $C_a(G,A)$ is set of elements of A which commutes with a and $\overline{C_a(G,A)}$ is set of elements of A which do not commute with a . Thus, if A=G, then $C_a(G,A)=C_a(G)$ and $\overline{C_a(G,A)} = \overline{C_a(G)}$.The Dicylic group is represented by

$DiC_n = \langle a, x: a^{2n}=1, x^2 = a^n, a^x=a^{-1} \rangle$ under the following multiplication rules; (i) $a^s a^t=a^{s+t}$, (ii) $a^s a^t x=a^{s+t}x$,
(iii) $a^s x a^t=a^{s-t}x$,
(iv) $a^s x a^t x=a^{s-t+n}$.
It is a member of a class of non-abelian groups of order 4n, where n>1. Every element of Dicyclic group can be written in the form $a^i x^j$, where $i\in\{0,1,2,...,2n-1\}$ and j is 0 or 1. Suppose that
$I_1= \{1,2,...,n-1,n+1,...,2n-1\}$,
$I_2= \{1,2,...,n\}$, $I_3=\{n+1,n+2,...,2n\}$
and $I_4=I_2\cup I_3$ then
$A_1 = \{a^i: i\in I_1\}$, $B_i=\{a^i x, a^{n+I} x: i\in I_2\}$,
$B=\bigcup_{i\in I} B_i C_1= \{a^i x: i\in I_2\}$,
$C_2 = \{a^i x: i\in I_3\}$,
$D_i = \{a^i, a^i x: i\in I_1\}$,
are suitable subsets of D_iC_n.

3 MAIN RESULTS

In this section, we present the main results of the non-commuting graphs $\Delta = [DiC_n, A]$, Where $A \subseteq DiC_n$.

Lemma3.1. For n≥2 and $A = DiC_n$, if $\Delta = [DiC_n, A]$ is a non-commuting graph. Then,
2n, for $v \in A_1$

$$\deg\Delta(v)=\begin{cases} 2n, \text{for } v \in A_1 \\ 4(n-1), \ v \in B. \end{cases}$$

Proof: Let $v = a^i \in A_1$, where $i \in I_1$.Then, $C_{a^i}(DiC_n) = \{a^j x: j= 1,2,...,2n\}$.
Therefore, for $i\in I_1$, we have $\deg\Delta(a^i) = 2n$. Consequently, $\deg\Delta(v) = 2n$ for each $v \in A_1$.
For $v = a^i x \in B$, where $i \in \{1, 2, 3,..., 2n\}$, we have $C_{a^i x}(DiC_n) = \{a^j x: j = 1, 2, ..., 2n, j\neq I$ and $j \neq n +i\}\cup\{a^j : j\in I_1\}$. Thus, $\deg\Delta(a^i x) = 4(n-1) \Rightarrow \deg\Delta(v) = 4 (n-1)$ for $v\in B$ $\deg\Delta (a^i x) = 4(n-1)\Rightarrow\deg\Delta(v) = 4 (n-1)$ for $v \in B$

Lemma3.2. For n≥2 and $A\subseteq DiC_n$, if $\Delta =[DiC_n,A]$ is a non-commuting graph. Then, the following are true;
(i) If A is an abelian subgroup of DiC_n, then $dia(\Delta) = \infty$.
(ii) If $A = DiC_n - Z(DiC_n)$, then $dia(\Delta) = 2$.
Proof; (i) Since A is an abelian subgroup of DiC_n, then the non-commuting graph.
$\Delta = [DiC_n, A]$ is an empty graph and hence $dia(\Delta) = \infty$.
(ii) If $A = DiC_n - Z(DiC_n)$, then non-commuting graph is multipartite graph. So, $dia(\Delta) = 2$.

Theorem3.3. For $n \geq 2$ and $A \subseteq DiC_n$, if $\Delta = [DiC_n, A]$ is a non-commuting graph. Then, (i) Δ is an empty graph of order 2 $\Leftrightarrow A= B_i$ for some $i\in I_2$ or $A = X$, where $X\subset A_1$ such that $|X| = 2$.
(i) Δ is an empty graph of order $2n-2\Leftrightarrow A = A_1$.
(ii) $\Delta = K_{n+1} \Leftrightarrow A= C_1\cup\{u\}$ or $C_2\cup\{u\}$ for some $u\in A_1$.
Proof: (i)(case-i)
Assume that $\Delta = [DiC_n,A]$ is an on-commuting graph such that $A = B_i$ for some $i\in I_2$. As $\overline{C_{a^i}(DiC_n, B_i)} = \Phi = \overline{C_{a^{i+n}x}(DiC_n, B_i)}$ for each $i\in I_2 \Rightarrow \deg\Delta(a^i) = 0 = \deg\Delta(a^i x)$.

Moreover, $|B_i| = 2$ for each $i\in I_2$. Thus, Δ is an empty graph of order 2. (case-ii)Assume that $\Delta=[DiC_n, A]$ is a non-commuting graph such that A=X, where $X \subset A_1$ with $|X| = 2$. As each element of A_1 commutes with others in A_1, there for both element of X, say a and b commutes with each other. Thus, $\overline{C_a(DiC_n, X)}=\Phi=\overline{C_b(DiC_n, X)} \Rightarrow \deg\Delta(a) =0 = \deg\Delta(b)$. Thus, Δ is an empty graph of order
2. Conversely, assume that $\Delta = [DiC_n, A]$ is a non-commuting graph such that Δ is an empty graphoforder2. Let $v \in A\Rightarrow v \in V(\Delta)\Rightarrow \deg \Delta(v) = 0 \Rightarrow$. Thus, $v \in B_i$ for some $i\in I_2$ or $v \in X$. If $v \in B_i$ for some $I \in I_2 \Rightarrow A \subseteq B_i$ and also $B_i\subseteq A$. Consequently, $A=B_i$ for some i. If $v \in X \Rightarrow A \subseteq X$ and also $X \subseteq A$. Consequently, A=X.
(ii) Assume that $\Delta =[DiC_n,A]$ is a non-commuting graph such that $A=A_1$.As
$\overline{C_a(DiC_n, A_1)}=\Phi$ for each $a \in A_1 \Rightarrow \deg \Delta(a) = 0$ for each $a\in A_1$.Thus, Δ is an empty graph. Also $|A| = 2n-2 \Rightarrow \Delta$ is anemptygraphoforder2n−2.Conversely, assume that $\Delta = [DiC_n, A]$ is a non-commuting graph such that
Δ is an empty graph of order $2n-2$. Let $v\in A \Rightarrow v \in V(\Delta) \Rightarrow \deg \Delta(v) = 0\Rightarrow$ v commutes with each element of A. Thus, $v \in A_1 \Rightarrow A\subseteq A_1$ and also $A_1 \subseteq A$. Consequently, $A = A_1$.

(iii) Assume that $\Delta = [Di\ C_n,A]$ is a non-commuting graph such that $A= C_1\cup\{u\}$, where $u \in A_1$. Then, $|\overline{C_v(DiC_n, A)}|= n$ for each $v \in C_1\cup\{u\}\Rightarrow\deg\Delta(v) = n$ for each $v \in C_1\cup\{u\}$. Thus, Δ is a complete graph of order $n+1\Rightarrow\Delta = K_{n+1}$. Conversely, assume that $\Delta = [DiC_n,A]$ is a non-commuting graph such that $\Delta = K_{n+1}$. Let $v \in A \Rightarrow v \in V(\Delta)\Rightarrow v \in K_{n+1}\Rightarrow \deg\Delta(v) = n\Rightarrow v$ does not commute with n elements of A. Thus, $v\in C_1\cup\{u\}$ for some $u \in A_1 \Rightarrow A\subseteq C_1\cup\{u\}$ and also $C_1\cup\{u\}\subseteq A$. Consequently, $A= C_1\cup\{u\}$. Similarly, we can prove for $A = C_2\cup\{u\}$.

Corollary 3.4. For $n \geq 2$ and $A \subseteq DiC_n$, if $\Delta = [DiC_n,A]$ is a non-commuting graph. Then $\Delta = K_n \Leftrightarrow A = C_1$ or C_2.

Theorem3.5. For $n \geq 2$ and $A \subseteq DiC_n$, if $\Delta = [DiC_n, A]$ is a non-commuting graph. Then, (i)Δ is a star graph $S(2n-2)$ $\Leftrightarrow A = A_1 \cup \{u\}$ for some $u \in B$.

(ii) $\Delta = K_{2n-2,2,2,2,\ldots,}$ $\Leftrightarrow A = A_1 \cup B$

Proof: (i) Assume that $\Delta = [DiC_n, A]$ is a non-commuting graph such that $A = A_1 \cup \{u\}$ for Some $u \in B$. As $|\overline{C_u(DiC_n, A)}| = |A_1| = 2n-2$ for each $u \in B$ and $|\overline{C_v(DiC_n, A)}| = 1$ for each $v \in A_1$. Thus, Δ is a graph such that one vertex is of degree $2n-2$ and remaining vertices are pendent$\Rightarrow \Delta = S(2n-2)$.

Conversely, assume that $\Delta = [DiC_n, A]$ is a non-commuting graph such that $\Delta = S(2n-2)$. Let $v \in A \Rightarrow v \in V(\Delta) \Rightarrow v \in S(2n-2) \Rightarrow$ either $\deg\Delta(v) = 2n-2$ or $\deg\Delta(v) = 1$. If $\deg \Delta(v) = 2n-2 \Rightarrow v$ does not commute with $2n-2$ elements of $A \Rightarrow v \in A_1 \cup \{u\}$ for some $u \in B$. If $\deg\Delta(v) = 1$, where $v \in A$ and $|A| = 2n-1 \Rightarrow v$ does not commute with exactly one elements of $A \Rightarrow v \in A_1 \cup \{u\}$ for some $u \in B$ $\Rightarrow A \subseteq A_1 \cup \{u\}$ and also $A_1 \cup \{u\} \subseteq A$ for some $u \in B$. Consequently, $A = A_1 \cup \{u\}$.

(ii)Assume that $\Delta = [DiC_n, A]$ is a non-commuting graph such that $A = A_1 \cup B$. Then, for $i \in \{0,1,2,...,2n-1\}$, $C_{a^i}(X)$ $(DiC_n, A) = \{a^j x$: $j = 0,1,2,..., 2n-1$, $j \neq i$ and $j \neq n+i\} \cup \{a^j$: $j = 1,2,3,...,n-1,n+1,...,2n-1\} \Rightarrow \deg\Delta(a^i x) = 2(n-1) + (2n-2) = 4n-4$ for each $a^i x \in A$. Similarly, for $i \in \{1,2,..., n-1,n+1,..., 2n-1\}$, $C_{a^i}(DiC_n, A) = \Phi \cup \{a^j x$: $j = 0,1,2,3,...,2n-1\} \Rightarrow \deg\Delta(a^i) = 2n$ for each $a^i \in A$. Moreover, as $a^i x$ commutes with $a^{i+n} x$ for each i in A. Therefore, there are n+1 partitions in A such that (a) one of order $2n-2$ and others n are of same order 2.

(b) all elements of the same partition are isolated.

(c)each element of a partition is adjacent to all the elements of the remaining partitions. Thus, $K_{2n-2,2,2,...,2}$.

Conversely, assume that $\Delta = [DiC_n, A]$ is a non-commuting graph such that $\Delta = K_{2n-2,2,2,...,2}$. Let $v \in A \Rightarrow v \in V(\Delta)$ $\Rightarrow \deg\Delta(v) = 2n$ or $\deg\Delta(v) = 4n-4$. If $\deg\Delta(v) = 2n \Rightarrow v = a^i$ for some $i \in \{1, 2, 3,..., n-1, n+1,..., 2n-1\} \Rightarrow v \in A_1 \Rightarrow v \in A_1 \cup B \Rightarrow A \subseteq A_1 \cup B$. If $\deg\Delta(v) = 4n-4 \Rightarrow v = a^i x$ For some $i \in \{0,1,2,3,...,2n-1\} \Rightarrow v \in B \Rightarrow v \in A_1 \cup B \Rightarrow A \subseteq A_1 \cup B$. Similarly, $A_1 \cup B \subseteq A$. Consequently, $A = A_1 \cup B$.

Corollary 3.6. For $n \geq 2$ and $A \subseteq DiC_n$, if $\Delta = [DiC_n, A]$ is a non-commuting graph. Then,

(i)$\Delta = K_{2n-2,2} \Leftrightarrow A = A_1 \cup B_i$ for some $i \in I_2$ (ii)$\Delta = K_{2,2}$ if $A = B_i \cup B_j$, for $i \neq j$ and $i, j \in I_2$. (iii)$\Delta = K_{2,2,...,2} \Leftrightarrow A = B$.

Theorem3.7. For $n \geq 2$ and $A \subseteq DiC_n$, if $\Delta = [DiC_n, A]$ is a non-commuting graph. Then,

(i) $\Delta = P_2 \Leftrightarrow A = \{u, v\}$ for some $u \in A_1$ and $v \in B$ or $A = \{u, v\}$ for $u \in B_i$, $v \in B_j$, $i = j$ and $i, j \in I_2$.

(ii) $\Delta = P_3 \Leftrightarrow A = B_i \cup \{u\}$ for some $u \in A_1$ and $i \in I_2$ or $A = \{u, v\} \cup \{x\}$, where $u, v \in A_1$ and $x \in B$ or $A = B_i \cup \{x\}$, where $x \in B_j$, $i = j$ and $i, j \in I_2$.

Proof. (i) (Case-i) Assume that $\Delta = [DiC_n, A]$ is a non-commuting graph such that $A = \{u, v\}$ for some $u \in A_1$ and $v \in B$. Since each element of A_1 does not commute with any element of B, therefore $u \in A_1$ does not commute with $v \in B$ .As $|A| = 2$, therefore $|C_u(DiC_n, A)| = 1 = |C_v(DiC_n, A)| \Rightarrow \deg\Delta(u) = 1 = \deg\Delta(v) \Rightarrow \Delta$ is a path graph of order 2, i.e $\Delta = P_2$.

(Case-ii) Assume that $\Delta = [DiC_n, A]$ is a non-commuting graph such that $A = \{u, v\}$ for some $u \in B_i$,$v \in B_j$, $i \neq j$ and $i, j \in I_2$. Since each element of B_i does not commute with any element of B_j for $i \neq j$ and $i, j \in I_2$, therefore $u \in B_i$ does not com- mute with $v \in B_j$ for $i \neq j$ and $i, j \in I_2$. As $|A| = 2$, therefore $\overline{C_u(DiC_n, A)} = 1 = \overline{C_v(DiC_n, A)} \Rightarrow \deg\Delta(u) = \deg\Delta(v) \Rightarrow \Delta$ is a path graph of order 2, i.e $\Delta = P_2$. Conversely, assume that $\Delta = [DiC_n, A]$ is a non-commuting graph such that $\Delta = P_2$. Let $x \in A \Rightarrow x \in V(\Delta) \Rightarrow x \in P_2$. As P_2 consists of exactly two vertices such that each vertex is of degree1, therefore $x \in \{u, v\}$ such that $\deg\Delta(u) = 1 = \deg\Delta(v)$. Since each element of A_1 does not commute with any element of B and each element of B_i does not commute with any element of B_j for $i \neq j$ and $i, j \in I_2$, therefore $x \in \{u, v\}$, where $u \in A_1$ and $v \in B$ or $u \in B_i$ and $v \in B_j$ for $i \neq j$ and $i, j \in I_2$. Consequently, $A = \{u, v\}$ for some $u \in A_1$ and $v \in B$ or $A = \{u, v\}$ for $u \in B_i$, $v \in B_j$, $i \neq j$ and $i, j \in I_2$.

(ii) (Case-i) Assume that $\Delta = [DiC_n, A]$ is a non-commuting graph such that $A = B_i \cup \{u\}$ for some $u \in A_1$ and $i \in I_2$. As, $|B_i| = 2$ and $a^i x$ commutes with $a^{i+n} x$ for each $i \in I_2$.

Moreover, $C_u(DiC_n, A) = B_i$ for each $u \in A_1$ and $i \in I_2$. Thus, two elements in A commute with each other and third does not commute with them$\Rightarrow$ degree of two vertices is1 and third vertex is of degree 2 in then on-commuting graph $\Delta = [DiC_n, A] \Rightarrow \Delta = P_3$.

Case-ii Assume that $\Delta = [DiC_n, A]$ is a non- commuting graph such that $A = \{u, v\} \cup \{x\}$, where $u, v \in A_1$ and $x \in B$. Consider, $C_u(DiC_n, A) = \{x\} = C_v(DiC_n, A)$ and $C_x(DiC_n, A) = \{u, v\} \Rightarrow \deg\Delta(u) = 1 = \deg\Delta(v)$ and $\deg\Delta(x) = 2$in $\Delta = [DiC_n, A] \Rightarrow \Delta = P_3$.

(Case-iii) Assume that $\Delta = [DiC_n, A]$ is a non-commuting graph such that $A = B_i \cup \{x\}$, for $x \in B_j$, $i = j$ and $i, j \in I_2$. Since both the elements of B_i commute with each other for $i \in I_2$ and do not commute with $x \in B_j$, where $i = j$ and $i, j \in I_2 \Rightarrow \deg\Delta(u) = 1 = \deg\Delta(v)$ and $\deg\Delta(x) = 2$ in $\Delta = [DiC_n, A]$, where u, v are in B_i for some $i \in I_2 \Rightarrow \Delta = P_3$.

Conversely, assume that $\Delta = [DiC_n, A]$ is a non-commuting graph such that $\Delta = P_3$. Let $u \in A \Rightarrow u \in V(\Delta) \Rightarrow u \in P_3$. As two vertices are of degree 1 and one vertex is of degree 2 in $P_3 \Rightarrow$ either $\deg\Delta(u)$ is 1 or 2. If $\deg\Delta(u) = 1$, then for other two vertices of P_3 say v and x are such that $\deg\Delta(v) = 1$and $\deg\Delta(x) = 2$. Consequently, $A = \{u, v\}\cup\{x\}$, where $u, v \in A_1$ and $x \in B$ or $A = B_i\cup\{x\}$, where $x \in B_j$, i=j and $i, j\in I_2$. If $\deg\Delta(u)=2$ then for other two vertices of P_3 say v and x are such that $\deg\Delta(v) = 1 = \deg\Delta(x) \Rightarrow$ v and x commute but u does not commute with them in $A\Rightarrow$ v and $x \in B_i$ for some $i \in I_2$ and $u \in A_1 \Rightarrow A= B_i\cup\{u\}$for some $u \in A_1$ and $i\in I_2$.

Corollary3.8. For $n\geq 2$ and $A\subseteq DiC_n$, if $\Delta = [DiC_n,A]$ is a non-commuting graph. Then, $\Delta = P_2$ if $A = D_i = \{a^i, a^i x\}$ for each $i\in I_1$.

Theorem3.9. For $n\geq 3$ and $A \subseteq DiC_n$, if $\Delta = [DiC_n,A]$ is a non-commuting graph. Then,

(i)$\Delta = C_3 \Leftrightarrow A = \{a^i x, a^j x\}\cup \{u\}$, where $i, j \in I_4$, $i\neq j$, $j\neq n+I$ and $u \in A_1$ or $A = \{a^i x, a^j x, a^k x\}$, where $i, j, k \in I_4$, $i\neq j\neq k, j\neq n+i, k\neq n+i$ and $k\neq n+j$.

(ii) $\Delta = C_4 \Leftrightarrow A= B_i\cup\{u, v\}$for $u, v \in A_1$ and $i\in I_2$ or $A= B_i\cup B_j$,for $i\neq j$and $i, j\in I_2$.

Proof.(i)(Case-i)Assume that $\Delta = [DiC_n, A]$ is a non-commuting graph such that $A = \{a^i x, a^j x\} \cup\{u\}$, where $i, j\in I_4$, $i\neq j$, $j\neq n + i$ and $u \in A_1$. Since, under the supposed conditions $a^i x$ and $a^j x$ do not commute, also u does not commute with $a^i x$ and $a^j x$. Therefore, $|C_v (DiC_n, A)| = 2$ for each $v \in A\Rightarrow$ each element of A has degree 2 in $\Delta = [DiC_n, A]$ .As, $|A| = 3 \Rightarrow \Delta = [DiC_n, A] = C_3$.

(Case-ii) Assume that $\Delta = [DiC_n, A]$ is a non-commuting graph such that $A = \{a^i x, a^j x, a^k x\}$, where $i, j, k \in I_4$, $i\neq j \neq k$, $j\neq n+i$, $k \neq n+I$ and $k \neq n+j$. Under the supposed conditions, we note that each element of A do not commute with remaining two elements of $A\Rightarrow|C_V(DiC_n, A)| = 2$ for each $v \in A\Rightarrow$ each element of A has degree 2 in $\Delta = [DiC_n, A]$. Consequently, $\Delta = [DiC_n, A] = C_3$. Conversely, Assume that $\Delta = [DiC_n, A]$ is a non-commuting graph such that $\Delta = [DiC_n, A] = C_3$. Let $v_1,v_2,v_3 \in A\Rightarrow v_1,v_2,v_3 \in V(\Delta)\Rightarrow v_1, v_2, v_3\in C_3 \Rightarrow v_1, v_2$ and v_3 do not commute with each other.

(Case-i) We have possibilities as (i) v_1, v_2 and $v_3 \in A_1$, (ii) any two say v_2 and $v_3\in A_1$ and (iii) only one say $v_3 \in A_1$. The possibilities (i) and (ii) are not possible as each pair of elements commute in A_1. The possibility (iii) holds, thus others elements v_1, v_2 are must in B such that if $v_1= a^i x$ then $v_2 = a^{n+I} x$ for $i\in I_4$. Thus, $v_3\in A_1$ and $v_1, v_2 \in \{a^i x, a^j x\}$, where $i,j \in I_4$, $i= j$, $j = n+i$. Clearly, $A \subseteq \{a^i x, a^j x\}\cup\{u\}$ and also $\{a^i x, a^j x\}\cup\{u\}\subseteq A$. Consequently, $A = \{a^i x, a^j x\}\cup \{u\}$, where $i, j\in I_4$, $i= j$, $j = n +i$ and $u\in A_1$.

(Case-ii) if v_1,v_2 and $v_3 \notin A_1\Rightarrow v_1,v_2$ and $v_3 \in B$ such that if $v_1 = a^i x$, then $v_2 = a^j x$ and $v_3 = a^k x$ with $i, j, k \in I_4$, $i\neq j\neq k$, $j \neq n +i$, $k\neq n +i$ and $k \neq n + j$. Consequently, $A=\{a^i x, a^j x, a^k x\}$, where $i , j, k \in I_4$, $i\neq j \neq k$,$j \neq n +i$, $k\neq n+ I$ and $k\neq n+ j$. (ii) Proof follows by Corollary (3.6) ((i) and (ii)).

Theorem3.10. For $n\geq 2$ and $A \subseteq DiC_n$, if $\Delta = [DiC_n, A]$ is a non-commuting graph. Then,

(i) $|E(\Delta)| = 0\Leftrightarrow A \subseteq A_1$ or $A = B_i$ for some $i\in I_2$.

(ii) $|E(\Delta)| = 2n (n-1)\Leftrightarrow A= B$.

(iii) $|E(\Delta)| = n(n-1)\Leftrightarrow A = C_1$ or $A = C_2$.(iV) $|E(\Delta)| = 6n(n-1)\Leftrightarrow A = DiC_n-Z(DiC_n)$.

Proof: (i) Let $A \subseteq A_1$ or $A= B_i$ for some $i \in I_2$. Since, each element of A_1 commute with all other elements of A_1 and similarly both the elements of B_i also commute with each other, where $i\in I_2 \Leftrightarrow\Delta = [DiC_n, A_1]$ and $\Delta = [DiC_n, B_i]$ for some $i\in I_2$ are empty graphs $\Leftrightarrow|E(\Delta=[DiC_n,A_1])| = 0 = |E(\Delta = [DiC_n, B_i]) |\Leftrightarrow |E(\Delta = [DiC_n,A])| = 0\Leftrightarrow |E(\Delta)| = 0$.

(ii) By Corollary (3.6 (iii)), $\Delta = K_{2,2,...,2}$ $\Leftrightarrow A= B$. Thus, $|E(\Delta)| =\frac{1}{2}n (n-1) (4) = 2n (n-1)\Leftrightarrow A = B$.

(iii) By Corollary (3.4(iii)), $\Delta = K_n\Leftrightarrow A = C_1$ or C_2. Thus $|E(\Delta)| =\frac{1}{2}n (n-1) \Leftrightarrow A = C_1$ or C_2. (iv) By Theorem3.5 (ii) ,$\Delta = K_{2n-2,2,2,...2} \Leftrightarrow$ $A = A_1\cup B$ Thus, $|E(\Delta)| = 2n(n-1) +2n(2n-2) = 6n (n-1)\Leftrightarrow A = DiC_n-Z(DiC_n)$.

Theorem3.11. For $n \geq 2$ and $A = DiC_n- Z(DiC_n)$, if $\Delta = [DiC_n, A]$ is a non-commuting graph. Then,

(i) $\omega(\Delta) = \chi(\Delta) = n + 1$

(ii) The grith of $\Delta = [DiC_n, A]$ is 3.

(iii) The maximal matching of $\Delta = [DiC_n, A]$ is $D = \{a^i a^i x :i\in I_1\}$ with $\gamma(\Delta) = 2n-2$, which is not a perfect matching.

Proof: (i) By Theorem 3.3 (iii), $\Delta = K_{n+1} \Leftrightarrow A = C_1 \cup\{u\}$ or $C_2\cup\{u\}$ for some $u\in A_1 \Rightarrow \Delta = [DiC_n, C_1\cup\{u\}]$ or $\Delta = [DiC_n, C_2\cup\{u\}]$ is a maximal complete subgraph with or der n+1 of the non-commuting graph $\Delta = [DiC_n, A] \Rightarrow \omega(\Delta) = n + 1$. By Theorem 3.5 (ii) $\Delta = K_{2n-2,2,2,...2} \Leftrightarrow A = A_1\cup B \Rightarrow \Delta$ is a complete (n+1) – partitegraph $\Leftrightarrow A = DiC_n-Z(DiC_n)$. Thus, n + 1 colors are required to colour the complete (n + 1) -partite graph. Thus, $\chi(\Delta) = n + 1$. Consequently, $\omega(\Delta) = \chi(\Delta) = n +$

1. The non-commuting subgraph of $\Delta = [DiC_n, A]$ induced by C_1 is a complete graph of order n (Corollary3.4). So completeness of this implies that there exists at least one cycle of minimum length 3. So, $\Delta = [DiC_n, A]$ has grith 3.

(iii) By Corollary 3.8, $\Delta = P_2$ if $A = D_i = \{a^i, a^i x\}$ for each $i \in I_1$ Thus, for each $i \in I_1$, $a^i a^i x$ is P_2 with different end points and $|I_2| = 2n - 2$, where $a^i \in A_1$ and $a^i x \in B$ .As $|A_1| = 2n-2$, $|B| = 2$ n and n, $2n \notin I_2 \Rightarrow$ we leave only two vertices x and $a^n x$ which are not end of any P_2 in $D = \{a^i a^i x: i \in I_1\}$, otherwise we get edges with common vertex. Consequently the maximal matching of $\Delta = [DiC_n, A]$ is $D = \{a^i a^i x: i \in I_1\}$ with $\gamma(\Delta) = 2n - 2$. As $\gamma(\Delta)$ is not equal to the half of $|DiC_n - Z(DiC_n)|$, therefore matching is not perfect.

REFERENCES:

1. Abdullaho, A. , S. Akbari and H. R. Maimani, Non-commuting graph of a group, Journal of Algebra 298 (2006), 268- 492.
2. Anderson, D. F., and P. Livingstong, The zero-divisor graph of a commutating Ring, Journal of Algebra 112(2006), 517527.
3. Anderson, D. F. , and AymanBadawi, The total Graph of a commuting ring, Journal of Algebra 320(2008), 2706 - 2719.
4. Asghar Talebi, A. , On the Non Commuting Graphs of Group D_{2n}. International Journal of Algebra, 220 (2008) 957 - 961.
5. Axtell, M., and J. Stickles, Zero-divisor graphs of idealizations, Journal of Pure and Applied Algebra 204, 2(2006), 235 - 243.
6. Behboodi, M. , Zero divisors graphs for modules over commutative rings, J. Commut. Algebra 4, 2 (2012), 175 - 197.
7. Bundy, D., The Connectivity of Commuting Graphs, J. Comb. Theory, Ser. A113, 6 (2006), 995 - 1007.
8. Chelvama, T. T. , K. Selvakumar and S. Raja, Commuting Graphs on Dihedral Group, The Journal of Mathematics and Computer Science, 2 (2011), 402 - 406.
9. Iranmensh, A. , A. Jafarzadeh, On the commuting graphs associated with the symmetric and alternating groups. Journal of Algebra and Its Applica- tion,7,1 (2008) 129-146.
10. Mashkouri, M. and Taeri, B., On a graph Associated to Groups, Bull. Malays. Math. Sci. Soc. 2 (34) 3 (2011), 553-560.
11. Moghaddamfar, A. R. , W. J. Shi , W. Zhou and A. R. Zokayi, On the non-commuting graph associated with finite group. Siberian Mathematical Journal, 246 (2005) 325-332.
13. Segev, Y., On finite Homomorphic image of the commutative group of a division algebra, Annals of Mathematics, 149 (1999), 219-251.
14. Segev, Y. , and G. Seitz Anisotropic group of type An and commuting graph of finite simple groups Pacific Journal of Mathematics, 202(2002),125-225.