IDENTIFY DIFFICULT COURSE OUTCOME IN LINEAR ALGEBRA USING RASCH MODEL

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ABSTRACT*—Linear Algebra, the common Engineering Mathematics subjects contains abstract concepts that students find it difficult to understand. This study aimed to identify the specific difficult course outcomes or difficult topics for the Linear Algebra subject. The difficult course outcomes are identified from a pre-test which was given to hundred Electrical Engineering students at the end of a fourteen-week course in a public university in Malaysia. The pre-test questions cover the entire course outcomes of the Linear Algebra subject. The pre-test questions consist of the level of Bloom Taxonomy (Knowledge, Comprehension, Application, Analysis, Synthesis, and Evaluation). Rasch model is used to analyze the results of the pre-test. The output achieved from the Rasch model is summary statistics for the item, fit statistics, and item dimensionality test. Summary statistics for items indicated a good item difficulty of spread. Fit statistics identified two questions as a misfit. These questions need to be rephrased. The item dimensionality test indicates the pre-test questions are within the scope of measuring students' problem-solving ability. Rasch model analysis illustrated that two-course outcome identified as the difficult course outcomes in Linear Algebra subject. They are the concepts of vector space, diagonalization, quadratic forms, and power series. On the other hand, questions related to comprehension and application level of Bloom Taxonomy are difficult for engineering students. This study provides an insight view of the Linear Algebra subject by under-pinning the difficult area of the subject. Efforts should be taken by lecturers to illustrate these topics in a much simpler way.*

KEYWORDS—Course Outcome; Difficul , Engineering Mathematics; Linear Algebra; Rasch model

I. INTRODUCTION

Engineering Mathematics is one important foundation subject to all departments in engineering. The essence of Engineering Mathematics will be applied in many other engineering subjects throughout the engineering program.

Teaching for understanding means students will be able to see the connections between mathematical ideas in various situations. Mathematics is taught base on the theories and then followed by examples. The examples include direct application of the formula and also based on case studies. This situation includes either mathematics problem-solving scenarios or real-life scenarios [1].

The assessment and documentation on how students are learning need to be included in a tertiary institution [2]. This will help to improve the level of education.

A faculty or department must formulate the Programme Outcomes and this should be followed by the Course Outcomes [3]. Course Outcomes are designed according to the syllabus of the course offered. Upon deriving the Programme Outcomes and Course Outcomes, students' achievement can be obtained at the end of the semester. This achievement includes marks from coursework and marks from the final examination. Coursework marks include assignment, project, quiz, test, report, and presentation.

In [4], the author supports that the evaluation of the Course Outcomes and Programme Outcomes depends on students' performance in quizzes, final examinations, capstone projects, and submissions of assignments that contribute to their learning achievements.

Since there is no specific method to measure Course Outcomes, it is quite difficult to measure the performance of each Course Outcome. A modern measurement method, which is the Rasch model method, was introduced to measure the Course Outcomes performance of each student [5].

The Rasch model analysis provides a reliable and reputable measurement rather than establishing the 'best-fit line'. Rasch model results provide the lecturers with more precise data on students' learning ability achievement. This in fact due to the fact that the Rasch model focuses on constructing the measurement accurately [6].

The Rasch model output will be used as a guideline for lecturers to monitor students' performance in each Course Outcome as to gauge the degree of effectiveness of the teaching and learning plans for any course based on the Course Outcomes [3].

This study concentrates on outlining the difficult Course Outcomes in Linear Algebra subjects for engineering students. Rasch model was used to analyze the output. This study will help lecturers to identify which areas that the engineering students are weak in Linear Algebra subject. This will help the former to take remedial actions to help the latter to widen their knowledge in the Linear Algebra subject.

II. METHODOLOGY

In a particular public university in Malaysia, Linear Algebra which is known as Engineering Mathematics II is a 4 credit hour subject taken by the Electrical, Mechanical, Civil, and Chemical Engineering students in the second semester of 8 semesters. Prior to the Linear Algebra subject, students take Vector Calculus subject in their first semester. In the third semester, students take Ordinary Differential subject. All three Engineering Mathematics subjects are common to Electrical, Mechanical, Civil, and Chemical Engineering students.

A pre-test session that covers all the Course Outcomes for Linear Algebra was conducted in semester II 2015/2016. This two-hour test comprises five questions and the questions are subjective.

One hundred students from the Electrical Engineering department had sat for this pre-test. The pre-test was validated by two lecturers who teach the subject. The Course Outcome and Programme Outcome are used as the basis to construct the pre-test questions. In addition, in each question, one level of Bloom Taxonomy (Knowledge, Comprehension, Application, Analysis, Synthesis, and Evaluation) was examined.

Table 1 lists the Course Outcome for the Linear Algebra subject.

TABLE 1 Course Outcome for Linear Algebra subject

Course Outcome	Description
1	Understand the fundamental concepts of the matrix and its basic operations and applications.
2	Able to use the concepts of vector space, linear independence in the space dimension, and matrix transformation.
3	Able to apply the eigenvector and eigenvalue in engineering problems.
4	Able to use the diagonalization and quadratic forms in the matrix solution for engineering problems.
5	Able to understand the concepts of Power Series.

Table 2 shows the Programme Outcome for Linear Algebra subject.

TABLE 2 Programme Outcome for Linear Algebra subject

programme outcomes	Description
1	Engineering knowledge
2	Problem analysis
3	Design/development of solutions
4	Investigation
5	Modern tool usage
6	The engineer and society
7	Environment and sustainability
8	Ethics
9	Communication
10	Individual and teamwork
11	Lifelong learning
12	Project management and finance

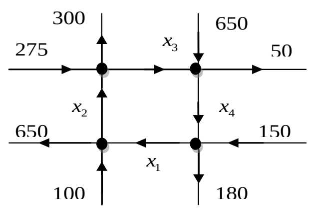
The pre-test question numbers together with the Course Outcome, Bloom Taxonomy description are labeled in Table 3.

TABLE 3 Distribution of pre-test questions

Question	Course Outcome	Bloom Taxonomy Description
1 (i)	1	Knowledge
1(ii)	1	Comprehension
1(iii)	1	Application
2	2	Comprehension
3 (i)	3	Application
3 (ii)	3	Comprehension
4 (i)	4	Application
4 (ii)	4	Comprehension
5	5	Comprehension

Table 4 illustrates the distribution of pre-final questions together with the marks.

TABLE 4 Pre-final questions

Q	Description	Marks
1	The accompanying figure shows a network of one-way streets with traffic flowing in the directions indicated. The flow rates along the streets are measured as the average number of vehicles per hour.  (i) Setup a linear system which solution provides the unknown flow rates.	4
1	(ii) Determine whether or not the system has a solution. Justify your answer	6
1	(iii) Explain how you might use the least square methods to estimate the flow rates on each street. Show all your work.	10
2	Determine whether or not the set of vectors under addition and scalar multiplication defined by $< a_1, 0, a_3 > + < b_1, 0, b_3 > = < a_1 + b_1, 0, a_3 + b_3 >$ $k < a_1, 0, a_3 > = < ka_1, 0, a_3 >$ is a vector space. Use all 10 axioms.	15
3	Suppose that the temperature at a point (x, y) on a metal plate is $T(x, y) = 4x^2 - 4xy + y^2$. A ladybug walks on the plate along a circle of radius 5 centered at the origin. (i) Use the Quadratic Forms to determine the highest and the lowest temperature encountered by the ladybug and state the point where it attains the highest and the lowest temperature.	13
3	(ii) Give the location and classification of the critical $T(x, y)$ points.	7
4	Given the following matrix: $A = \begin{bmatrix} -13 & -60 & -60 \\ 10 & 42 & 40 \\ -5 & -20 & -18 \end{bmatrix}$ (i) Show that matrix A is diagonalizable.	5
4	(ii) Find a matrix P that diagonalizes A.	5
5	Use the geometric series $\frac{1}{1-x} = 1 + x + x^2 + x^3 + ...$ to find the series representation of $\ln\lvert 1 + x \rvert$.	5

III. RESULTS AND DISCUSSION

Grades that are compiled in the Excel **prn* format were transferred using Bond & Fox Steps which is a customized WINSTEPS [7]. The WINSTEPS program provides detailed statistics on the Summary Statistics, Fit Statistics, and Item Dimensionality Test.

Figure 1 shows the measurement of items involved in this study. 'Item' represents the questions tested on the pre-test of the Linear Algebra subject. Item separation is the distance in logits between items of different levels of difficulty [8]. The higher values of separation represent the spread of items along the continuum and lower values indicate item redundancy [9].

The item reliability is 0.97 which indicates good item difficulty spread. It can be noted that the item mean is 0 and it also gives a good statistics summary for item with the item separation noted at 5.37 logit. A logit is a unit derived from transforming ordinal data into an interval scale [10]. From Figure 1, the maximum item or the highest location of the item on the logit ruler is +0.82 logit and the minimum or the lowest item on the ruler is at -0.73 logit.

	TOTAL SCORE	COUNT	MEASURE	MODEL ERROR	INFIT MNSQ	INFIT ZSTD	OUTFIT MNSQ	OUTFIT ZSTD
MEAN	225.4	79.0	.00	.09	1.05	.0	1.11	.1
S.D.	92.3	.0	.54	.02	.23	1.0	.44	1.0
MAX.	351.0	79.0	.82	.13	1.40	1.4	2.02	1.6
MIN.	106.0	79.0	-.73	.07	.78	-1.2	.60	-1.3

REAL RMSE .10 TRUE SD .53 SEPARATION 5.37 Item RELIABILITY .97
MODEL RMSE .09 TRUE SD .53 SEPARATION 5.85 Item RELIABILITY .97
S.E. OF Item MEAN = .22

Figure 1 Summary statistics for the item

According to Bond and Fox, the fit statistics result will determine whether or not the data fit a construct [7]. Rasch model experts examine the item fit by both infit and outfit [11]. The acceptable region for the Point-Measure Correlation, MNSQ, and z-Standard are $0.4 < x < 0.8$, $0.5 < MNSQ < 1.5$ and $-2 < z < 2$.

From Figure 2, items 2, 5, 3(i), 1(ii), 1(iii) and 1 (i) are out of range of Point-Measure Correlation (PT-MEASURE CORR.). Item 2 and 5 are out of range of outfit *MNSQ*. Item 2 and 5 are also out of range of outfit z-Standard.

Since items 2 and 5 are out of range for all three Point-Measure Correlation, *MNSQ* and z-Standard, therefore items 2 and item 5 are considered as a misfit. The Course Outcomes related to these questions are considered as difficult Course Outcomes in Linear Algebra subject.

ENTRY NUMBER	TOTAL SCORE	TOTAL COUNT	MEASURE	MODEL S.E.	INFIT MNSQ	INFIT ZSTD	OUTFIT MNSQ	OUTFIT ZSTD	PT-MEASURE CORR.	PT-MEASURE EXP.	EXACT MATCH OBS%	EXACT MATCH EXP%	Item
4	79	79	3.42	1.78	MAXIMUM MEASURE				.00	.00	100.0	100.0	2
9	79	79	3.42	1.78	MAXIMUM MEASURE				.00	.00	100.0	100.0	5
5	106	79	.82	.13	1.40	1.2	2.02	1.6	.21	.40	78.5	78.7	3(i)
8	149	79	.39	.08	.85	-.8	.69	-1.2	.65	.49	43.0	40.4	4(ii)
7	167	79	.27	.08	1.05	.4	.95	-.1	.55	.49	19.0	21.9	4(i)
2	173	79	.23	.08	1.05	.4	1.18	.9	.32	.49	17.7	21.8	1(ii)
3	292	79	-.35	.07	.87	-1.2	1.01	.1	.32	.37	20.3	17.9	1(iii)
6	340	79	-.64	.09	.78	-1.1	.60	-1.3	.44	.28	44.3	37.4	3(ii)
1	351	79	-.73	.10	1.36	1.4	1.29	.8	.22	.25	31.6	37.8	1(i)
MEAN	192.9	79.0	.76	.46	1.05	.0	1.11	.1			36.3	36.6	
S.D.	101.6	.0	1.50	.70	.23	1.0	.44	1.0			20.0	19.2	

Figure 2 Fit statistics

The dimensionality of the instruments is determined by the standard residual variance of the eigenvalues which indicate the 'direction' of the measurement. Figure 3 shows that the raw variance unexplained by measures is 50.1% which is higher than the 40% minimum. Therefore, the instrument is within the acceptable dimension and it showed enough reliability in measuring the students' problem-solving skills in the pre-test questions. The unexplained variance of the 1^{st} contrast is 13.4%. This value is within the 5% to 15% of the acceptable range. This item dimensionality is unidimensionality. This means the pre-test questions measure the students' problem-solving skills only.

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Table of STANDARDIZED RESIDUAL variance (in Eigenvalue units)
                                                 -- Empirical --    Modeled
Total raw variance in observations     =         14.0 100.0%         100.0%
  Raw variance explained by measures   =          7.0  50.1%          50.4%
    Raw variance explained by persons  =          1.4   9.9%          10.0%
    Raw Variance explained by items    =          5.6  40.2%          40.4%
  Raw unexplained variance (total)     =          7.0  49.9% 100.0%   49.6%
    Unexplned variance in 1st contrast =          1.9  13.4%  26.7%
    Unexplned variance in 2nd contrast =          1.5  10.5%  21.0%
    Unexplned variance in 3rd contrast =          1.1   8.2%  16.4%
    Unexplned variance in 4th contrast =          1.0   7.3%  14.6%
    Unexplned variance in 5th contrast =           .8   5.6%  11.3%
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Figure 3 Item Dimensionality Test

From the Fit Statistics from Figure 2, Course Outcomes 2 and Course Outcome 5 are identified as difficult Course Outcomes for Linear Algebra subject. Table 5 summarizes the difficult Course Outcomes for Linear Algebra subjects. In the future, the lecturer who teaches this batch should rephrase the questions for these difficult Course Outcomes. Perhaps instead of comprehension questions, the lecturer can propose an application level of questions for both the Course Outcomes. This is because students may find it easier to solve application problems rather than trying to recall the theories of Linear Algebra.

TABLE 5 Difficult Course Outcome

Course Outcome	Description	Level of Question	Bloom Taxonomy Description
2	Able to use the concepts of vector space, linear independence in the space dimension, and matrix transformation.	Difficult	Comprehension
5	Able to understand the concepts of Power Series.	Difficult	Comprehension

IV. CONCLUSION

The pre-test questions for Linear Algebra (KKKQ 1223) subject were valid except question 2 and question 5 as both questions identified as a misfit. The eigenvalue of the raw variable explained by the measure is 50.1% which is more than 40%. This indicates that the sample and questions are reliable in measuring the students' ability to answer questions.

The Rasch model identified that the students are weak in Course Outcome 2 and Course Outcome 5. More tutorials should emphasize these areas. In conclusion, the Linear Algebra pre-test questions cover well all the Course Outcomes, Programme Outcomes, and the level of Bloom Taxonomy. The Rasch model identifies the misfit questions and tests the validity and reliability of questions.

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