COMPARATIVE STUDY OF TEST SIZES OF DIFFERENT ESTIMATORS OF BOX'S CORRECTION IN THE UNI-VARIATE REPEATED MEASURES DESIGNS

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ABSTRACT: *The main objective of repeated measures design is to control and reduce the maximum variation. In this paper, we have focused to control type-I error under the violation of the sphericity assumption. In this regard, the test sizes i.e. 0.01, 0.05, and 0.10 of different estimators of Box's correction in the univariate repeated measures designs are to compare through the simulation process. A simulation study will be conducted to compare the four estimators: Geisser-Greenhouse, Huynh-Feldt, lower bound* $\frac{1}{(k-1)}$ *and Rauf Khan (*ε_{GG}*,* ε_{HF}*,* ε_{LB}*, and* ε_{RK}*) and two newly proposed estimators: which are* $\bar{\varepsilon}_{EK1}$ *and* $\bar{\varepsilon}_{EK2}$ *are discussed. Where* $\bar{\varepsilon}_{EK1}$ *is the average of* ε_{HF}*, and* ε_{RK}*. While* $\bar{\varepsilon}_{EK1}$ *is also the average of* ε_{HF}*, and* ε_{LB} *in Box's correction for univariate repeated measures designs under non-sphericity. Several combinations of basic design parameters, number of treatments, Sample size, a measure of violation of sphericity assumption, and Skewness will be used as input parameters to generate data through STATISTICA, statistical software. The generated numerical information will follow all the assumptions of the univariate repeated measures model except the assumption of sphericity. The estimators will be compared on several grounds but their performances in terms of controlling type I error will be the main focus. Our newly proposed estimators perform better under moderate and high violation of sphericity assumption where the skewness does not affect the performance of these estimators. This implies that the behavior of parameters is the same at each level of skewness.*

Keywords: Box's correction, Monte Carlo Methods, Sphericity, Type-I error

1. INTRODUCTION

The repeated measures design is used when each subject or individual is repeatedly measured for each treatment or factor level. Since here individual works like a block of (a set of) treatments, so blocking is at its extreme. In repeated measures designs, the same participants are used in all conditions. This is like an extreme matching. This allows for the reduction of error variance due to subject factors. Fewer participants can be used in a repeated measures design. Repeated measures designs make it easier to see an effect of the independent variable on the dependent variable (if there is such an effect). In the univariate analysis, the within factor is treated as an independent variable and *F*-test can be conducted to determine if the effect of time varies by group.

The advantage of repeated measures designs is that the measurements obtained under the different treatment conditions will in many experiments be highly correlated since they are made on the same subjects. The presence of these correlations will reduce the experimental error and reside in the number of the subject it may be more economical in terms of time and effort to test the same subject under each treatment.

In this paper, we deal with repeated measure experiments and similar situations. The *F*-test depends on the special covariance structure of the data under consideration. The required co-variance structure is very well defined by the assumption of circularity condition. The assumption is violated not only in repeated measures experiments but also in other similar experiments, for example, a series of experiments, the experiments in which there are some repeated factors and some non-repeated factors, etc. However, the study focuses on the experiments in which the assumption of sphericity is violated.

The univariate analysis requires an additional kind of homogeneity assumption that's called the Mauchly [1] sphericity assumption. Sphericity refers to a special relationship between scores at the levels of the within-subjects variables that is revealed in the pattern of the variance-covariance matrix. To be spherical, the data must meet the criteria of either compound symmetry or circularity. For compound symmetry, is a property of variance-covariance matrix of order ($k\times k$) where k is the number of treatments (repeated measures). It is the population variance-covariance matrix for a vector $[x_{1j}, x_{2j}, \dots, x_{kj}]$ and this matrix, under the stated assumptions, can be presented in a special form as follows:

$$\Sigma = \begin{pmatrix} 1 & \rho & . & . & . & \rho \\ \rho & 1 & . & . & . & \rho \\ . & . & & . & & . \\ . & . & . & . & . & . \\ . & . & & . & & . \\ \rho & \rho & . & . & . & 1 \end{pmatrix}$$

The variances on the main diagonal and the co-variances on the off-diagonal are equal. In literature, this type of matrix is known as having Compound Symmetry. Hence the assumption that all treatments (repeated measures) have a common variance and each pair of populations ($i \neq j$) have the same covariance is known as compound symmetry assumption. Under the null hypothesis of equality of all treatment means, compound symmetry of the population variance-covariance matrix, $\sum$, is a sufficient condition for the *F* ratio, $F = \frac{MS_{Treatments}}{MS_{Residual}}$ to follow an *F*-distribution with (*k-1*) and (*r-1*)(*k-1*) degrees of freedom.

The compound symmetry assumption is not a necessary condition for the *F* ratio to follow an *F*-distribution. Huynh and Feldt [2] in 1970 concluded that the *F* ratio will also have an *F*-distribution if the following condition holds:

$$var(x_{ij} - x_{lj}) = var(x_{ij}) + var(x_{lj}) - 2covar(x_{ij}, x_{lj})$$

Where $var(x_{ij} - x_{lj})$ is constant for every *i≠j*. This is known as a necessary and sufficient condition for the validity of the *F* test. Originally this assumption was stated as that all variance is equal and all covariance are equal in the matrix Σ_x. This assumption, known as compound symmetry is very strong and rarely met. This assumption is called the assumption of circularity and the matrix Σ_x, satisfying this assumption, is called circular. Any circular matrix can be transformed into a spherical matrix by the transformation, $C\Sigma_x C' = \Sigma_y = \lambda I$, where, $C_{(k-1)\times k}$ is a matrix of (*k-l*) orthogonal contrasts of k treatments and $I_{(k-1)\times(k-1)}$ is an identity matrix. The matrix Σ_y is called the spherical matrix satisfying the assumption of sphericity. Under the sphericity assumption, the usual ANOVA procedure was used. The *F*-test in this case, called unadjusted *F*, is valid and follows the *F* distribution.

ε is a measure of the violation of sphericity It is computed from the following formula:

$$\varepsilon = \frac{k^2(\bar{\sigma}_{ij} - \bar{\sigma}_{..})^2}{(k-1)\Sigma\Sigma(\sigma'_{ij} - \bar{\sigma}_{i.} - \bar{\sigma}_{.j} + \bar{\sigma}_{..})^2}$$

It can also be computed from the transformed matrix, Σ_y as:

$$\varepsilon = \frac{(\Sigma\lambda_i)^2}{(k-1)(\Sigma{\lambda^2}_i)}$$

where, λ_i is denoted as the Eigenvalues of the sphericity matrix.

$$\varepsilon_{GG} = \frac{k^2(\bar{s}_{ij} - \bar{s}_{..})^2}{(k-1)\Sigma\Sigma(s'_{ij} - 2k\Sigma s^2_{i.} - k^2\overline{\bar{s}'}_{..})^2}$$

The Geisser and Greenhouse (ε_{GG})[4] in 1958 *extended* Box's [11] ε to repeated measures designs involving one or more between-group variables. In 1959, they also suggested the use of the lower bound estimate of Box's [11] ε. This use of a lower bound estimate for the Greenhouse-Geisser [4] conservatively distributes the *F* test ratio: *F* (1, *k*-1). While one of the strengths of the Greenhouse-Geisser [4] ε is its computational ease, the conservativeness of the Greenhouse-Geisser [4] ε may be too "severe," perhaps increasing Type II error "McCall & Appelbaum [7] in 1973". In responding to the conservativeness, Greenhouse, and Geisser [4] in 1959 offer two suggestions for conceptualizing this issue:

(1) If *F* is found to be significant using the Greenhouse-Geisser [3] ε adjustment, then it would also be significant using Box's ε adjustment or no adjustment at all.

(2) If *F* is found to be non-significant using no adjustment at all, then no evidence exists to suggest a significant effect.

Also, an extension of Huynh-Feldt's [2] ε adjustment is used to adjust for less severe violations of sphericity. Using unbiased estimators of the numerator and denominator of Box's [11] ε, Huynh and Feldt [2] in 1976 proposed an alternative estimator: Huynh-Feldt's [2] ε_{HF}, which can be outside the range of 0 to 1 and is less conservative than the lower bound.

$$\varepsilon_{HF} = \frac{n(k-1)\hat{\varepsilon} - 2}{(k-1)\{(n-1) - (k-1)\hat{\varepsilon}\}}$$

The problem with these two estimators is that the Geisser-Greenhouse [3] estimator underestimates the true value while the Huynh-Feldt [2] estimator overestimates it. Even the Huynh-Feldt [2] estimate sometimes exceeds 1. The third estimate is Lower bound i.e. $\frac{1}{(k-1)}$. This is not an estimate this is an adjustment. But Lower bound is highly conservative.

This procedure helps the investigator to decide what he can do

1) Assuming no violation of sphericity assumption, use unadjusted *F*-Test.
2) If unadjusted *F* is rejected, then multiplied the numerator and denominator degree with ε.

Box's [11] correction applied but there is some problem in the estimates that are purposed by Box. Geisser-Greenhouse [3] is negatively biased and Huynh-Feldt [2] is positively biased, the third one is Lower bound which is highly conservative.

So it is expected that the average estimate of Geisser-Greenhouse [3] and Huynh-Feldt [2] provide a balanced estimate in terms of controlling the type-I error rate. They are of the view that this estimator will balance the bias that this present in Geisser-Greenhouse [3] estimator and Huynh-Feldt [2] estimator. Through the simulation study, they observed that in the presence of a moderate violation of sphericity these newly proposed estimators perform better than Geisser-Greenhouse [3] estimator and Huynh-Feldt [2] estimator.

In this research paper, we have proposed two new estimators of Box's [11] correction in the univariate repeated measures designs. The first estimator is the average of Geisser-Greenhouse [3] estimator and Huynh-Feldt [2] estimator that is $\bar{\varepsilon}_{EK1}$ and the second estimator is the average of Huynh-Feldt [2] estimator and Lower Bound of $\bar{\varepsilon}_{EK2}$. There are some other relevant studies in the repeated measures design reveal the effects of non-sphericity, associated other parameters in the design, for example, power (Muller and Barton [8] in 1989), sample size (Vonesh and Schork [10] in 1986), efficiency and optimality (Kushner [6] in 1997).

2. Simulation Strategy

The simulation techniques also called Monte Carlo Methods which have been applied to many problems in the various sciences and are useful in situations where direct experimentation is not possible, the cost of experimenting is very high or the experiment takes too much time

The simulation is performed keeping in view the situations in real-life problems, i.e., different sample sizes with different treatment sizes. Moreover in each type of data set three subsets are generated to display varying levels of average correlation among subjects' item score. The three levels are independence, moderate, and high correlation. Then within each subset, the data sets are generated to reflect variability according to skewness i.e., low, moderate, and high skewness.

3. Model and Design Parameters

To develop a research plan and simulation study is conducted. Generation of data for the univariate repeated measures model is:

$$x_{ij} = \mu + \alpha_i + S_j + \eta_{ij}$$

Where *i=1, 2….k* and *j=1, 2…..n*.

As α_i is a treatment effect, S_j is a random subject effect (assume subjects are randomly selected from a large population) and η_{ij} is a random error (within-subject error).

Assume $S_j \sim NID(0,\sigma_j^2)$ and $\eta_{ij} \sim NID(0,\sigma_\eta^2)$ under a random effect model.

In regards to the simulation framework, fifty-four thousand (54,000) data sets are generated for every four basic parameters (*k*= number of treatments, *n*= Sample size, *ε*= measure of violation of sphericity assumption, and s_k= Skewness). For each *ε*-value, a variance-covariance matrix Σ (see Annex) of order *k*×*k* is generated and used as input to generate data.

Hence, the subsequent Table 1 indicates the *λ*, values as measures of departure from sphericity assumption. The λ values were used to generate each matrix until the desired level of violation of the sphericity assumption was achieved.

Table 1: Table of λ-Values along with Intensity of Violation

K	λ-Value	ε-Value	Intensity of Violation
3	4	1.000	No
	3, 7, 11	0.847	Moderate
	3, 5, 11	0.634	Severe
6	2	1.000	No
	1, 2	0.743	Moderate
	1, 2, 3	0.456	Severe
9	3	1.000	No
	1,3,5, 7, 9	0.629	Moderate
	2, 4, 9, 13, 19	0.343	Severe

To analyzing the Univariate analysis of variance and several variables are recorded. These variables are classified as *P-value* for unadjusted F, adjusted with GG, LB, and RK. Two newly proposed estimators are the weighted average of ε_{GG} and ε_{HF}, given by:

$$\bar{\varepsilon}_{EK1} = \frac{\varepsilon_{GG} + 3\varepsilon_{HF}}{4} = \frac{\varepsilon_{RK} + \varepsilon_{HF}}{2}$$

Now the $\bar{\varepsilon}_{EK2}$ is the average of ε_{HF}andε_{LB}, given below:

$$\bar{\varepsilon}_{EK2} = \frac{\varepsilon_{LB} + \varepsilon_{HF}}{2}$$

These averages are also then used to find the *P-value*s of the *F*-test when its degrees of freedom are corrected by these averages. The proposed estimators are then compared with the other estimators ε_{GG}, ε_{HF}, ε_{LB}, and ε_{RK} on several bases specifically concerning the control of type I error rate. For each quadruple of parameters (*k, n, ε,* s_k) a table is constructed to depict the comparisons of type I error controls by all six estimators.

4. DISCUSSION AND CONCLUSIONS

The *P-value*s of Box's [11] estimators ε_{GG} and ε_{HF} and one old ε_{RK} and two newly purposed estimators $\bar{\varepsilon}_{EK1}$ and $\bar{\varepsilon}_{EK2}$ are observed and results about these estimators are given below.

1. When the sphericity assumption holds, the closest competitor of unadjusted *F* is the Hyun-Feldt [2] estimator, $\tilde{\varepsilon}$. We may not be sure about whether the assumption of sphericity holds or not, then in such cases, the Hyun-Feldt [2] estimator can be used to adjust the degrees of freedom of F. We know that the Hyun-Feldt [2] estimator usually overestimates the true value of ε, and under sphericity (*ε = 1.0*), this estimator exceeds 1.0. In this case, we set the estimated value to be 1.0, therefore, artificially we reduce the bias it has incurred in the estimation of ε. This reduction of bias makes it the best estimator of ε and hence the *P-value*s are tightly controlled.
2. Geisser-Greenhouse [3] estimator, excellently perform when there is a severe violation of sphericity because it estimates the true value of ε and it is also negatively biased.
3. The Probability value of the Geisser-Greenhouse (GG) [3] estimator is lower than the Probability value of the Hyun-Feldt (HF) [2] estimator.
4. *k=3* are generally lower than the nominal values irrespective of sample size. But for *k = 6* and *k = 9*, the *P-value*s are larger than the nominal values.
5. Reduction in the degrees of freedom, un-adjusted *F* ratio increases, and hence the *P-value* underestimates the nominal value.
6. Under violation of sphericity, the unadjusted F gives misleading results. Due to this type-I error inflates and seriously damages the analysis. Even under moderate violation of sphericity assumption, its use will provide too many significant results than occur.

- Lower Bound (LB) provides too less significant results than occur. While giving comments on the performance of LB and its effects on the *P-value*s we must be clear about what picture does the use of this LB provides in the analysis of repeated measures designs. Multiplying the degrees of freedom of *F*-test, for the present study, (*k - 1*) and (*n-1*) (*k-1*), $\frac{1}{(k-1)}$ with LB, simply reduces them to (*1, n-1*). It makes us pretend that the repeated measures factor (for our case, k) has only two levels (where it is not the case) and proceed as usual for the analysis of the design. Due to these limitations associated with the LB, the statisticians limit the use of the conservative test to
- Ensure that α is below a certain level.
- When an approximate test is not available,

- When the covariance matrices (in multifactor repeated measures designs) are different from group to group, (Muller and Barton [8] in 1989).

1. There is no violation of sphericity when there are three treatments, unadjusted F
2. Performs better than the other competing estimators and gives the *P-value*s closest to the nominal rates. Under sphericity, the *F*-test needs no adjustment, and if an adjustment is made using any estimator, the degrees of freedom of the *F*-test get markedly reduced. As the reduction in the degrees of freedom of F increases the critical value of F and hence the *P-value* underestimates the nominal value.
3. Our studies show that at each nominal level of significance behavior the parameter different at each level of intensity of violation.
4. For small size for treatments are 6 and 9 the *P-value*s lies between the Geisser-Greenhouse [3] estimator and Hyun-Feldt [2] estimator of the estimator proposed by Rauf-Khan [9]. Because it is an average of the Geisser-Greenhouse [3] estimator and Hyun-Feldt [2] estimator.
5. The first newly purposed estimator $\bar{\varepsilon}_{EK1}$ is a weighted average of Geisser-Greenhouse [3] and Hyun-Feldt [2] estimator with weights 1 and 3. The performance of this new estimator, $\bar{\varepsilon}_{EK1}$ is very close to the Rauf-Khan [9] proposed estimator, ε_{RK} because Rauf-Khan's [9] estimator is a simple average of Geisser-Greenhouse [3] and Hyun-Feldt [2] estimator. These two estimators $\bar{\varepsilon}_{EK1}$ and ε_{RK} behave like for the given set of parameters. Our results show that two estimators perform best under moderate violation of sphericity assumption.
6. The second new estimator$\bar{\varepsilon}_{EK2}$, a simple average of Hyun-Feldt [2] and LB [5]. This new estimator $\bar{\varepsilon}_{EK2}$ is generally close to the *P-value*s of the GG estimator. Then under high violation of sphericity, the performance of $\bar{\varepsilon}_{EK2}$ is very good.
7. *P-value*s show that skewness does not affect the performance of the estimators. It is implied behavior of parameters is the same at each level of skewness under study.

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Annex-I
Variance Covariance Matrices for simulation

No Violation of Sphericity $\varepsilon = 1.000$

$$\begin{bmatrix} 4.0 & 2.5 & 6.0 \\ 2.5 & 9.0 & 8.5 \\ 6.0 & 8.5 & 16.0 \end{bmatrix}$$

Moderate Violation of Sphericity $\varepsilon = 0.847$

$$\begin{bmatrix} 4.0 & 3.5 & 3.0 \\ 3.5 & 9.0 & 1.5 \\ 3.0 & 1.5 & 16.0 \end{bmatrix}$$

Severe Violation of Sphericity $\varepsilon = 0.634$

$$\begin{bmatrix} 4.0 & 3.5 & 5.0 \\ 3.5 & 9.0 & 1.5 \\ 5.0 & 1.5 & 16.0 \end{bmatrix}$$

No Violation of Sphericity $\varepsilon = 1.000$

$$\begin{bmatrix} 4.0 & 4.5 & 8.0 & 12.5 & 18.0 & 24.5 \\ 4.5 & 9.0 & 10.5 & 15.0 & 20.5 & 27.0 \\ 8.0 & 10.5 & 16.0 & 18.5 & 24.0 & 30.5 \\ 12.5 & 15.0 & 18.5 & 25.0 & 28.5 & 35.0 \\ 18.0 & 20.5 & 24.0 & 28.5 & 36.0 & 40.5 \\ 24.5 & 27.0 & 30.5 & 35.0 & 40.5 & 49.0 \end{bmatrix}$$

Moderate Violation of Sphericity $\varepsilon = 0.743$

$$\begin{bmatrix} 4.0 & 0.5 & 4.0 & 8.5 & 8.0 & 2.5 \\ 0.5 & 9.0 & 0.5 & 5.0 & 10.5 & 17.0 \\ 4.0 & 0.5 & 16.0 & 8.5 & 1.5 & 8.5 \\ 8.5 & 5.0 & 8.5 & 25.0 & 6.5 & 13.0 \\ 8.0 & 10.5 & 1.5 & 6.5 & 36.0 & 18.5 \\ 2.5 & 17.0 & 8.5 & 13.0 & 18.5 & 49.0 \end{bmatrix}$$

Severe Violation of Sphericity $\varepsilon = 0.456$

$$\begin{bmatrix} 4.0 & 0.5 & 4.0 & 8.5 & -2.0 & 2.5 \\ 0.5 & 9.0 & 6.5 & 11.0 & -1.5 & 5.0 \\ 4.0 & 6.5 & 16.0 & 14.5 & 2.0 & 8.5 \\ 8.5 & 11.0 & 14.5 & 25.0 & 6.5 & 13.0 \\ -2.0 & -1.5 & 2.0 & 6.5 & 36.0 & 18.5 \\ 2.5 & 5.0 & 8.5 & 13.0 & 18.5 & 49.0 \end{bmatrix}$$

No Violation of Sphericity $\varepsilon = 1.000$

$$\begin{bmatrix} 4.0 & 5.0 & 9.0 & 13.5 & 19.0 & 25.5 & 33.0 & 41.5 & 51.0 \\ 5.0 & 9.0 & 8.0 & 11.5 & 16.0 & 21.5 & 28.0 & 35.5 & 44.0 \\ 9.0 & 8.0 & 16.0 & 19.5 & 25.0 & 31.5 & 39.0 & 47.5 & 57.0 \\ 13.5 & 11.5 & 19.5 & 25.0 & 29.5 & 36.0 & 43.5 & 52.0 & 61.5 \\ 19.0 & 16.0 & 25.0 & 29.5 & 36.0 & 41.5 & 49.5 & 57.5 & 67.0 \\ 25.5 & 21.5 & 31.5 & 36.0 & 41.5 & 49.0 & 55.5 & 64.0 & 73.5 \\ 33.0 & 28.0 & 39.0 & 43.5 & 49.5 & 55.5 & 64.0 & 71.5 & 81.0 \\ 41.5 & 35.5 & 47.5 & 52.0 & 57.5 & 64.0 & 71.5 & 81.0 & 89.5 \\ 51.0 & 44.0 & 57.0 & 61.5 & 67.0 & 73.5 & 81.0 & 89.5 & 100.0 \end{bmatrix}$$

Moderate Violation of Sphericity $\varepsilon = 0.629$

$$\begin{bmatrix} 4.0 & -3.5 & 0.0 & 4.5 & 10.0 & -3.5 & 4.0 & 2.5 & 12.0 \\ -3.5 & 9.0 & 2.5 & 7.0 & 12.5 & -1.0 & 6.5 & 5.0 & 14.5 \\ 0.0 & 2.5 & 16.0 & 10.5 & 16.0 & 2.5 & 0.0 & 8.5 & 18.0 \\ 4.5 & 7.0 & 10.5 & 25.0 & 0.5 & 7.0 & 4.5 & 3.0 & 12.5 \\ 10.0 & 12.5 & 16.0 & 0.5 & 36.0 & 12.5 & 10.0 & 8.5 & 18.5 \\ -3.5 & -1.0 & 2.5 & 7.0 & 12.5 & 49.0 & 6.5 & 15.0 & 24.5 \\ 4.0 & 6.5 & 0.0 & 4.5 & 10.0 & 6.5 & 64.0 & 22.5 & 32.0 \\ 2.5 & 5.0 & 8.5 & 3.0 & 8.5 & 15.0 & 22.5 & 81.0 & 40.5 \\ 12.0 & 14.5 & 18.0 & 12.5 & 18.5 & 24.5 & 32.0 & 40.5 & 100.0 \end{bmatrix}$$

Severe Violation of Sphericity $\varepsilon = 0.343$

$$\begin{bmatrix} 4.0 & -3.5 & 0.0 & 4.5 & 10.0 & 11.5 & -11.0 & -7.5 & 2.0 \\ -3.5 & 9.0 & 2.5 & 7.0 & 12.5 & 14.0 & -8.5 & -5.0 & 14.5 \\ 0.0 & 2.5 & 16.0 & 10.5 & 16.0 & 17.5 & -5.0 & -1.5 & 8.0 \\ 4.5 & 7.0 & 10.5 & 25.0 & 20.5 & 22.0 & -0.5 & 3.0 & 12.5 \\ 10.0 & 12.5 & 16.0 & 20.5 & 36.0 & 27.5 & 0.0 & 8.5 & 18.5 \\ 11.5 & 14.0 & 17.5 & 22.0 & 27.5 & 49.0 & 6.5 & 15.0 & 24.5 \\ -11.0 & -8.5 & -5.0 & -0.5 & 0.0 & 6.5 & 64.0 & 22.5 & 32.0 \\ -7.5 & -5.0 & -1.5 & 3.0 & 8.5 & 15.0 & 22.5 & 81.0 & 40.5 \\ 2.0 & 14.5 & 8.0 & 12.5 & 18.5 & 24.5 & 32.0 & 40.5 & 100.0 \end{bmatrix}$$