LOOP QUANTUM GRAVITY AND BLACK HOLE COMPLEMENTARITY

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ABSTRACT: *The one-way evolution of the black hole entropy towards a maximum can lead to truncation of the relevant operators to a subspace in a quantum theory of gravity. It is argued that the boundary Hilbert space in loop quantum gravity pertinent to the maximal entropy state of the horizon should lead to non-commutativity of the otherwise compatible area and volume operators. This may provide a clue towards understanding the black hole complementarity and the dimensional reduction inherent in the holographic principle.*

Keywords: Loop quantum gravity, black hole complementarity, quantum Hall effect, Immirzi parameter, truncation of Hilbert space, black hole entropy

INTRODUCTION

Loop quantum gravity (LQG) seems to be the sole theory that has produced results from first principles that geometrical quantities such as area and volume are quantized [1-3]. It uses spin networks as a basis for its Hilbert space. Spin networks are graphs with edges carrying spin labels $\{j \in 0, 1/2, 1, ...\}$, i.e., the representations of *SU*(2) group which serves as the gauge group of the theory. When a set of edges labelled with spins j_i puncture a surface $\mathbf{S}$, it acquires an area

$$A(j_i) = 8\pi G\hbar\gamma \sum_{j_i} \sqrt{j_i(j_i+1)}\,, \qquad (1)$$

which is an eigenvalue of the area operator $\mathbf{A}$. Since such areas lie at the boundaries of volumes, the eigenstates corresponding to the area operator also diagonalize the volume operator [1-3]. The theory, however, carries the burden of an unknown free parameter γ in the predicted geometric spectra, called the Immirzi parameter [4]. No satisfactory physical explanation of this parameter is available till date, but its value is mainly fixed [5] by the requirement that the LQG computation produces the Bekenstein–Hawking entropy,

$$S = \frac{A}{4G\hbar}, \qquad (2)$$

for a black hole with the horizon area A [6].

There has been competing approaches in the literature to fix γ, including some controversy over the true gauge of the theory and the counting of the true degrees of freedom living on the boundary, see for instance [7-11]. In this note, however, we wish to take the discussion to another angle that has not been considered yet. We argue that choosing the appropriate Hilbert space corresponding to the maximal entropy state of an isolated horizon will give rise to non-commutativity of the of the otherwise commuting area and volume operators. This non-commutativity can be regarded as a result close to the black hole complementarity [12], which says that an observer can only detect the information at the horizon, or inside, but never both simultaneously. We recall that the black hole complementarity was proposed in response to the famous information loss problem, first realized by Hawking [13]. Our proposal is augmented by the strikingly similar example of the physics of the lowest landau levels (LLL) in the quantum Hall effect.

Since our argument is linked with how the value of the Immirzi parameter is fixed, we first briefly review some of the relevant approaches to fixing the value of this parameter in section 2. In section 3, we present our argument. Section 4 concludes this paper.

BLACK HOLE ENTROPY AND THE IMMIRZI PARAMETER

In LQG the entropy of a spherically symmetric horizon is determined via counting the states of the boundary Hilbert space. Considering a horizon as an isolated sphere $\mathbf{S}^2$, the problem is reduced to counting the number of different ways the boundary can be punctured yielding the horizon area close to a given value [14-16]. The entropy of the horizon is measured as the logarithm of the dimension of the boundary Hilbert space $\mathscr{H}_{\text{boundary}}$, i.e.,

$$S = \ln\left(\prod_i^N \dim\mathscr{H}_{j_i}\right), \qquad (3)$$

where $\dim\mathscr{H}_j$ is $(2j+1)$ for a puncture with spin j and N is the number of edges puncturing the horizon. For large horizon area A the leading contribution to the entropy comes from punctures with minimal spins $j_{\min} = 1/2$ only. The maximum entropy can thus be approximated as

$$S \approx N_{1/2} \ln 2\,. \qquad (4)$$

Here $N_{1/2}$ is the total number of spin-half edges puncturing the boundary. Comparison of this result with the Bekenstein-Hawking formula fixes the Immirzi parameter at the value $\ln 2/\pi\sqrt{3}$.

Dreyer [7] arrived at a different value, $\gamma = \ln 3/2\pi\sqrt{2}$, by exploiting the quasi-normal mode (QNM) spectrum of a Schwarzschild black hole [17]. But at the same time his approach also suggested that the dominant contribution should come from edges with $j_{\min} = 1$ and that the true gauge group of the theory should therefore be considered as $SO(3)$ rather than $SU(2)$. However, it was argued that one should be restricted to $SU(2)$ if Fermions were to be accommodated in the theory [9]. Based on a combinatoric formulation of the black hole entropy, it was claimed that contribution from all positive half-integer values of j should be taken into account, thereby, resulting yet in a different value of γ [8]. However, this proposal also received significant criticism. For instance, it was argued that admitting arbitrary distribution of spins over the punctures may result in an arbitrarily complicated geometry, and that spherical symmetry of the horizon in the continuum limit would be hard to achieve [10]. It was also put forth that since a black hole, even if initiated by an excited state, will always evolve to a state of equilibrium with the maximal entropy, one should therefore be restricted to only $j_{\min}$ edges from the very beginning. Punctures by higher j edges could be thought as representing locally excited states of the black hole without spherical symmetry. See also reference [11], for arguments in favor of $j_{\min}$ edges puncturing the horizon.

BLACK HOLE OMPLEMENTARITY IN LQG

In light of the above discussion, we are now able to present the main argument of this paper. Ordinarily, the LQG area and volume operators commute, but the fate of these operators in the environment of a black hole remains unexplored. It is of utmost importance to embark on such a question because we have a convincing concept of black hole complementarity on the table from other tentative theories of quantum gravitation [12], according to which the boundary and the bulk degrees of freedom of a black hole are incompatible. If the area and volume operators of LQG were to commute for a black hole too, it would make possible simultaneous measurements of the boundary and the bulk degrees of freedom---a clear violation of the black hole complementarity. We argue that the one-way evolution of the boundary Hilbert space to the space of minimal spins would cause the area and volume operators to no longer commute. This is an inevitable outcome that has been bypassed in the earlier literature. The guiding principle in favor of the argument is that two commuting matrices in a full Hilbert space need not commute upon their truncations to a subspace. Thus, restriction of the horizon Hilbert space to a subspace would cause the area and volume operators to become non-commuting whence, in theory, they are diagonalized by a common set of eigenstates. A concrete example of such an effect is provided by the physics of the infinitely degenerate lowest Landau levels (LLL) in the quantum Hall effect. Electrons confined to a plane at extremely low temperature and high transvers magnetic field occupy only the degenerate ground level. In this process the problem is essentially reduced to a one-dimensional problem, where the X and Y coordinates, which are commuting by postulate, become a canonically conjugate pair. Such an effect of a black hole could also provide a mechanism behind the dimensional reduction intrinsic to the holographic principle.

It remains to work out the exact commutator of the area and volume operators in the black hole environment. A constant commutator would turn these operators into a canonical pair that could be conjectured as the LQG version of the black hole complementarity [12], according to which information could be detected only at the horizon or in the bulk inside the horizon, but never both simultaneously.

CONCLUSION

We have pointed out that truncations of the area and volume operators to a subspace at the horizon would cause these operators to become non-commuting. This non-commutativity is not imposed from the outset, but rather emerges as a result of the one-way evolution of a black hole to the state of maximal entropy, which may be regarded as the mechanism behind the black hole complementarity and the dimensional reduction inherent in the holographic principle [18].

Complementarity of the area and volume degrees of freedom would cause them to no longer communicate. This will suggest revisiting the Gauss's constraint because punctures on the horizon would cease to join edges in the bulk.

Finally, choosing the minimal spins seems to be the only way to achieve non-commutativity of the area and volume operators. Therefore, the many-spin hypothesis advocated in [8] can be ruled out.

ACKNOWLEDGEMENT

The author acknowledges the support from the University of Tabuk, Saudi Arabia.

REFERENCE

1. Rovelli, C. and Smolin, L., "Knot theory and quantum gravity," *Phys. Rev. Lett.*, **61**: 1155(1995)
2. Rovelli, C. and Smolin, L., "Loop space representation of quantum general relativity," *Nucl. Phys. B*, **331**: 80-152(1990)
3. Thiemann, T., "Loop quantum gravity: an inside view, in: Approaches to fundamental physics," Springer, 185-263(2007)
4. Immirzi, G., "Quantum gravity and Regge calculus," *Nucl. Phys. B- Proceedings Supplements*, **57**: 65-72(1997).
5. Rovelli, C., "Black hole entropy from loop quantum gravity," *Phys. Rev. Lett.*, **77**: 3288(1996); Ashtekar, A., Baez, J., Corichi, A., Krasnov, K., "Quantum geometry and black hole entropy," *Phys. Rev. Lett.*, **80**: 904(1998); Ashtekar, A., Baez, J., Krasnov, K., "Quantum geometry of isolated horizons and black hole entropy," *arXiv preprint gr-qc/0005126*, 2000
6. Bekenstein, J. D., "Black holes and entropy," *Phys. Rev. D*, **7**: 2333(1973); Hawking S. W., "Particle creation by black holes," *Comm. Math. Phys.*, **43**: 199-220(1975)
7. Dreyer, O., "Quasinormal modes, the area spectrum, and black hole entropy," *Phys. Rev. Lett.*, **90**: 081301(2003)
8. Domagala, M., Lewandowski, J., "Black-hole entropy from quantum geometry," *Class. Quant. Grav.*, **21**: 5233(2004); Meissner, K. A., "Black-hole entropy in loop quantum gravity," *Class. Quant. Grav.*, **21**: 5245(2004)
9. Swain, J., "The Pauli exclusion principle and SU(2) versus SO(3) in loop quantum gravity," *Int. J. Mod. Phys. D*, **12**: 1729-1736(2003); Corichi, A., "Quasinormal modes, black hole entropy, and quantum geometry," *Phys. Rev. D*, **67**: 087502(2003); Ling, L.,

Zhang, H., “Quasinormal modes prefer supersymmetry?” *Phys. Rev. D,* **68**: 101501(2003); Sadiq, M., “A correction to the Immirzi parameter of SU (2) spin networks,” *Phys. Lett. B*, **741**: 280-283(2015); Abreu, E. M., Neto, J. A., Barboza, E. M. Jr, Soares, B. B., “Black holes quasinormal modes, Loop Quantum Gravity Immirzi parameter and nonextensive statistics,” *Phys. Lett. B*, **798**: 135011(2019)

10. Alexandrov, S., “On the counting of black hole states in loop quantum gravity,” *arXiv preprint gr-qc/0408033*, 2004; Alexandrov, S., Roche, P., “Critical overview of loops and foams,” *Phys. Rep.*, **506**: 41-86(2011).
11. Dreyer, O., Markopoulou, F., Smolin, L., “Symmetry and entropy of black hole horizons,” *Nucl. Phys. B,* **744**: 1-13(2006)
12. Susskind L., Thorlacius, L. and Uglum, J., “The stretched horizon and black hole complementarity,” *Phys. Rev. D*, **48**(8): 3743(1993); Hooft, ‘t. G., “On the quantum structure of a black hole,” *Nucl. Phys. B,* **256**: 727-745(1985)
13. Hawking, S. W., “Breakdown of predictability in gravitational collapse.” *Phys. Rev. D,* **14**(10): 2460(1976)
14. Rovelli, C., “Black hole entropy from loop quantum gravity,” *Phys. Rev. Lett.,* **77**: 3288(1996)
15. Ashtekar, A., Baez, J., Corichi, A., Krasnov, K., “Quantum geometry and black hole entropy,” *Phys. Rev. Lett.,* **80**: 904(1998)
16. Ashtekar, A., Baez, J., Krasnov, K., “Quantum geometry of isolated horizons and black hole entropy,” *arXiv preprint gr-qc/0005126*, 2000
17. Andersson, N., “On the asymptotic distribution of quasinormal mode frequencies for Schwarzschild black holes,” *Class. Quant. Grav.*, **10**: L61(1993); Hod, S., “Bohr's correspondence principle and the area spectrum of quantum black holes,” *Phys. Rev. Lett.*, **81**: 4293(1998); Motl, L. “An analytical computation of asymptotic Schwarzschild quasinormal frequencies,” *arXiv preprint gr-qc/0212096*, 2002
18. Hooft ‘t. G., “The holographic principle,” in: *Basics and Highlights in Fundamental Physics*, *World Scientific*, 72-100(2001)