DYNAMICAL BEHAVIOR OF HIGHER ORDER RATIONAL DIFFERENCE EQUATION

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ABSTRACT: *In this paper we discuss the global stability of the nature of positive solutions and the periodicity of the difference equation*

$$z_{n+1}=\alpha_0 z_n+\alpha_1 z_{n-k}+\alpha_2 z_{n-s}+\frac{b_0 z_{n-t}+b_1 z_{n-l}+b_2 z_{n-m}+b_3 z_{n-p}}{b_4 z_{n-t}+b_5 z_{n-l}+b_6 z_{n-m}+b_7 z_{n-p}}$$

with non-negative initial conditions $z_{-\mu}, z_{-\mu+1},..., z_{-1}, z_0$ where $\mu=\max\{k,s,t,l,m,p\}$ and coefficients

$$\alpha_0,\alpha_1,\alpha_2,b_0,b_1,b_2,b_3,b_4,b_5,b_6,b_7\in R^+ \qquad (A)$$

Numerical examples are also given to confirm the obtained results.

Keywords: Difference Equations, Higher order difference equations, Periodic solutions.

1. INTRODUCTION:

In this paper we obtain solutions of rational difference equation

$$z_{n+1}=\alpha_0 z_n+\alpha_1 z_{n-k}+\alpha_2 z_{n-s}+\frac{b_0 z_{n-t}+b_1 z_{n-l}+b_2 z_{n-m}+b_3 z_{n-p}}{b_4 z_{n-t}+b_5 z_{n-l}+b_6 z_{n-m}+b_7 z_{n-p}} \qquad (1.1)$$

where the initial values are arbitrary real numbers.

Difference equation is a vast field which impact almost found in every branch of pure as well as applied mathematics.Recently great interest is developed in studying difference equation systems.The reason is that there is need of some techniques whose can be used in investigating problems in different fields.Recently a great effort has been made in studying the qualitative analysis of rational difference equations.Difference equations are very simple in form, but it is very difficult to understand thoroughly the behaviors of their solutions.See [1-3].

In 2006 Ramazan karatas et.al [4] studied the positive solutions and attractivity of the difference equation by considering non zero real numbers initial values.

$$x_{n+1}=\frac{x_{n-5}}{\pm 1\pm x_{n-2}x_{n-5}}, n=0,1\,...$$

In 2008 Elsayed [5] worked on the difference equation

$$x_{n+1}=\frac{x_{n-5}}{\pm 1\pm x_{n-2}x_{n-5}}, n=0,1\,...$$

He found the solution of this equation and obtained graphs of numerical examples for some values of initial conditions .In 2009 Elsayed [6] investigated the difference equation by considering real numbers initial values

$$x_{n+1}=\frac{x_{n-5}}{\pm 1\pm x_{n-2}x_{n-5}}, n=0,1\,...$$

He checked the qualitative behavior of the difference equation.

In 2010 Elsayed [7] studied the solutions of the following class of difference equation

$$x_{n+1}=\frac{x_{n-8}}{\pm 1\pm x_{n-2}x_{n-5}x_{n-8}}, n=0,1\,...$$

In 2011 Elsayed [8] investigated the rational difference equation

$$x_{n+1}=\frac{x_{n-9}}{\pm 1\pm x_{n-4}x_{n-9}}, n=0,1\,...$$

In 2011 Elsayed [9] investigated the rational difference equation

$$x_{n+1}=\frac{x_{n-3}}{\pm 1\pm x_{n-1}x_{n-3}}, n=0,1\,...$$

In 2012 Touafek and Elsayed [10] got the form of solutions of the rational difference systems

$$x_{n+1} = \frac{y_n}{x_{n-1}(\pm 1 \pm y_n)}, y_{n+1} = \frac{x_n}{y_{n-1}(\pm 1 \pm x_n)}$$

In 2011 Vu Van Khuong and Mai Nam Phong [11] investigated the difference equation

$$x_{n-3} = x_n(1 + x_{n-1}x_{n-2}), n = 0,1,\ldots$$

In 2016 Elsayedet. al [12] studied the solutions of some difference equations of the form

$$x_{n+1} = \frac{x_{n-1}x_{n-5}}{x_{n-3}(\pm 1 \pm x_{n-1}x_{n-5})}, n = 0,1 \ldots$$

Suppose that I is some interval of real numbers and F a continuous function defined on I^{k+1} (k+1 copies of I),where k is some natural number. Throughout this thesis, we consider the following difference equation

$$Z_{n+1} = f\left(Z_n, Z_{n-1}, \ldots, Z_{n-k}\right), n = 0,1,\ldots \quad (1.2)$$

For given initial values $Z_{-k}, Z_{-(k-1)}, \ldots Z_0 \in I$

1.1 Definition: (Equilibrium Point)

A point $\bar{Z} \in I$ is called an equilibrium point of difference equation (1.2) if

$$\bar{Z} = F(\bar{Z}, \ldots, \bar{Z})$$

That is,$Z_n = \bar{Z}$ for $n \geq 0$ is a solution of difference equation (1.2).

1.2 Definition: (Periodicity)

A solution$\{Z_n\}_{n=-k}^{\infty}$of Equation (1.2) is called periodic with period p if there exists an integer $p \geq 1$ is a s such that $Z_{n+p} = Z_n$for all$n \geq -k$ .If $Z_{n+p} = Z_n$ holds for smallest positive integer p then solution $\{Z_n\}_{n=-k}^{\infty}$of Equation (1.2) is called periodic period of prime.

1.3 Theorem: Consider the difference equation

$$z_{n+1} + a_k z_n + a_0 z_{n-k} = 0 , n = 0,1, \ldots$$

Where $k \in \{1,2, \ldots\}$and $a_i$real numbers for all$i$.Then $\sum_{i=0}^{k}|a_i| < 1$is a sufficient condition for the asymptotic stability of eq. (1.2).

2. Local stability

Here we discuss the local stability of Eq.(1.1).The equilibrium point of Eq.(1.1) is given by

$$\bar{z} = \alpha_0 \bar{z} + \alpha_1 \bar{z} + \alpha_2 \bar{z} + \frac{b_0 \bar{z} + b_1 \bar{z} + b_2 \bar{z} + b_3 \bar{z}}{b_4 \bar{z} + b_5 \bar{z} + b_6 \bar{z} + b_7 \bar{z}}$$

$$\bar{z}(1 - \alpha_0 - \alpha_1 - \alpha_2) = \frac{b_0 + b_1 + b_2 + b_3}{b_4 + b_5 + b_6 + b_7}$$

If $\alpha_0 + \alpha_1 + \alpha_2 < 1$, then the unique equilibrium point is

$$\bar{z} = \frac{(b_0 + b_1 + b_2 + b_3)}{(b_4 + b_5 + b_6 + b_7)(1 - \alpha_0 - \alpha_1 - \alpha_2)}$$

Let $f : (0, \infty)^7 \rightarrow (0, \infty)$ be a continuous function given by

$$f(q_0, q_1, q_2, q_3, q_4, q_5, q_6) = \alpha_0 q_0 + \alpha_1 q_1 + \alpha_2 q_2 + \frac{b_0 q_3 + b_1 q_4 + b_2 q_5 + b_3 q_6}{b_4 q_3 + b_5 q_4 + b_6 q_5 + b_7 q_6}$$

$$\frac{\partial f(q_0, q_1, q_2, q_3, q_4, q_5, q_6)}{\partial q_0} = \alpha_0$$

$$\frac{\partial f(q_0, q_1, q_2, q_3, q_4, q_5, q_6)}{\partial q_1} = \alpha_1$$

$$\frac{\partial f(q_0, q_1, q_2, q_3, q_4, q_5, q_6)}{\partial q_2} = \alpha_2$$

$$\frac{\partial f(q_0, q_1, q_2, q_3, q_4, q_5, q_6)}{\partial q_3} = \frac{(b_5 b_0 - b_1 b_4) q_4 + (b_6 b_0 - b_2 b_4) q_5 + (b_7 b_0 - b_3 b_4) q_6}{(b_4 q_3 + b_5 q_4 + b_6 q_5 + b_7 q_6)^2}$$

$$\frac{\partial f(q_0, q_1, q_2, q_3, q_4, q_5, q_6)}{\partial q_4} = \frac{(b_1 b_4 - b_0 b_5) q_3 + (b_6 b_1 - b_2 b_5) q_5 + (b_7 b_1 - b_3 b_5) q_6}{(b_4 q_3 + b_5 q_4 + b_6 q_5 + b_7 q_6)^2}$$

$$\frac{\partial f(q_0,q_1,q_2,q_3,q_4,q_5,q_6)}{\partial q_5}=\frac{(b_2b_4-b_0b_6)q_3+(b_5b_2-b_1b_6)q_4+(b_7b_2-b_3b_6)q_6}{(b_4q_3+b_5q_4+b_6q_5+b_7q_6)^2}$$

$$\frac{\partial f(q_0,q_1,q_2,q_3,q_4,q_5,q_6)}{\partial q_6}=\frac{(b_3b_4-b_0b_7)q_3+(b_5b_3-b_1b_7)q_4+(b_6b_3-b_2b_7)q_5}{(b_4q_3+b_5q_4+b_6q_5+b_7q_6)^2}$$

Thus $\frac{\partial f(\bar{z},\bar{z},\bar{z},\bar{z},\bar{z},\bar{z},\bar{z})}{\partial q_0}=\alpha_0=u_1$

$$\frac{\partial f(\bar{z},\bar{z},\bar{z},\bar{z},\bar{z},\bar{z},\bar{z})}{\partial q_1}=\alpha_1=u_2$$

$$\frac{\partial f(\bar{z},\bar{z},\bar{z},\bar{z},\bar{z},\bar{z},\bar{z})}{\partial q_2}=\alpha_2=u_3$$

$$\frac{\partial f(\bar{z},\bar{z},\bar{z},\bar{z},\bar{z},\bar{z},\bar{z})}{\partial q_3}=\frac{(b_0(b_5+b_6+b_7)-b_4(b_1+b_2+b_3))(1-\alpha_0-\alpha_1-\alpha_2)}{(b_4+b_5+b_6+b_7)(b_0+b_1+b_2+b_3)}=u_4$$

$$\frac{\partial f(\bar{z},\bar{z},\bar{z},\bar{z},\bar{z},\bar{z},\bar{z})}{\partial q_4}=\frac{(b_1(b_4+b_6+b_7)-b_5(b_0+b_2+b_3))(1-\alpha_0-\alpha_1-\alpha_2)}{(b_4+b_5+b_6+b_7)(b_0+b_1+b_2+b_3)}=u_5$$

$$\frac{\partial f(\bar{z},\bar{z},\bar{z},\bar{z},\bar{z},\bar{z},\bar{z})}{\partial q_5}=\frac{(b_2(b_4+b_5+b_7)-b_6(b_1+b_2+b_3))(1-\alpha_0-\alpha_1-\alpha_2)}{(b_4+b_5+b_6+b_7)(b_0+b_1+b_2+b_3)}=u_6$$

$$\frac{\partial f(\bar{z},\bar{z},\bar{z},\bar{z},\bar{z},\bar{z},\bar{z})}{\partial q_6}=\frac{(b_3(b_4+b_5+b_6)-b_7(b_0+b_1+b_2))(1-\alpha_0-\alpha_1-\alpha_2)}{(b_4+b_5+b_6+b_7)(b_0+b_1+b_2+b_3)}=u_7$$

The linearized equation of equation (1.1) about $\bar{z}$ is

$$z_{n+1}=u_1z_n+u_2z_{n-k}+u_3z_{n-s}+u_4z_{n-t}+u_5z_{n-l}+u_6z_{n-m}+u_7z_{n-p} \qquad (1.3)$$

2.1Global Stability:

Theorem

The $\bar{z}$ is a global attractor of Eq.(1.1) if one of following conditions holds

i) $\alpha_0+\alpha_1+\alpha_2\neq 1$, $b_0+b_1+b_2\neq b_3$ and $b_4+b_5+b_6>0$.

ii) $\alpha_0+\alpha_1+\alpha_2\neq 1$, $b_0+b_1+b_3\neq b_2$ and $b_4+b_5+b_7>0$.

Proof: We consider two cases.

Case 1:If function $f(v_0,v_1,v_2,v_3,v_4,v_5,v_6)$ is increasing in v_0,v_1,v_2,v_3,v_4,v_5 and decreasing in v_6.Suppose (m,M) is a solution of the system.

$M=g(M,M,M,M,M,M,m)$ and $m=g(m,m,m,m,m,m,,M)$

From eq.(1.1)

$$M=\alpha_0M+\alpha_1M+\alpha_2M+\frac{b_0M+b_1M+b_2M+b_3m}{b_4M+b_5M+b_6M+b_7m}$$

and

$$m=\alpha_0m+\alpha_1m+\alpha_2m+\frac{b_0m+b_1m+b_2m+b_3M}{b_4m+b_5m+b_6m+b_7M}$$

Then

$$M^2(1-\alpha_0-\alpha_1-\alpha_2)(b_4+b_5+b_6)+b_7Mm(1-\alpha_0-\alpha_1-\alpha_2)=M(b_0+b_1+b_2)+b_3m$$

and

$$m^2(1-\alpha_0-\alpha_1-\alpha_2)(b_4+b_5+b_6)+b_7Mm(1-\alpha_0-\alpha_1-\alpha_2)=m(b_0+b_1+b_2)+b_3M$$

Subtracting above two equations

$$(M-m)\{(M+m)(1-\alpha_0-\alpha_1-\alpha_2)(b_4+b_5+b_6)-(b_0+b_1+b_2-b_3)\}=0$$

By conditions $\alpha_0+\alpha_1+\alpha_2\neq 1$, $b_4+b_5+b_6>0$ and $b_0+b_1+b_2\neq b_3$.We see that $M=m$.Hence $\bar{z}$ is a global attractor of eq.(1.1).

Case 2:If function $f(v_0,v_1,v_2,v_3,v_4,v_5,v_6)$ is decreasing in v_0,v_1,v_2,v_3,v_4,v_6 and increasing v_5 .Suppose (m,M) is a solution of the system.

From eq.(1.1)

$M = g(M,M,M,M,M,m,M)$ and $m = g(m,m,m,m,m,M,m)$

Then

$$M = \alpha_0 M + \alpha_1 M + \alpha_2 M + \frac{b_0 M + b_1 M + b_2 m + b_3 M}{b_4 M + b_5 M + b_6 m + b_7 M}$$

and

$$m = \alpha_0 m + \alpha_1 m + \alpha_2 m + \frac{b_0 m + b_1 m + b_2 M + b_3 m}{b_4 m + b_5 m + b_6 M + b_7 m}$$

Then

$$M^2(1-\alpha_0-\alpha_1-\alpha_2)(b_4+b_5+b_7)+b_6 Mm(1-\alpha_0-\alpha_1-\alpha_2) = M(b_0+b_1+b_3)+b_2 m$$

and

$$m^2(1-\alpha_0-\alpha_1-\alpha_2)(b_4+b_5+b_7)+b_6 Mm(1-\alpha_0-\alpha_1-\alpha_2) = m(b_0+b_1+b_3)+b_2 M$$

Subtracting above two equations

$$(M-m)\{(M+m)(1-\alpha_0-\alpha_1-\alpha_2)(b_4+b_5+b_7)-(b_0+b_1+b_3-b_2)\}=0$$

By using conditions $\alpha_0+\alpha_1+\alpha_2 \neq 1, b_0+b_1+b_3 \neq b_2$ and $b_4+b_5+b_7 > 0$ .We see that $M = m$.Hence $\bar{z}$ is a global attractor of eq.(1.1).

2.2 Boundedness of solutions of (1.1):

Here we study the bounded nature of the solutions of Equation (1.1).

Theorem 2

Every solution of equation (1.1) is bounded if $\alpha_0+\alpha_1+\alpha_2 < 1$.

Proof: Let $\{z_n\}_{n=-\mu}^{\infty}$ be a solution of eq.(1.1).It follows from eq.(1.1)

$$z_{n+1} = \alpha_0 z_n + \alpha_1 z_{n-k} + \alpha_2 z_{n-s} + \frac{b_0 z_{n-t} + b_1 z_{n-l} + b_2 z_{n-m} + b_3 z_{n-p}}{b_4 z_{n-t} + b_5 z_{n-l} + b_6 z_{n-m} + b_7 z_{n-p}}$$

$$= \alpha_0 z_n + \alpha_1 z_{n-k} + \alpha_2 z_{n-s} + \frac{b_0 z_{n-t}}{b_4 z_{n-t} + b_5 z_{n-l} + b_6 z_{n-m} + b_7 z_{n-p}} + \frac{b_1 z_{n-l}}{b_4 z_{n-t} + b_5 z_{n-l} + b_6 z_{n-m} + b_7 z_{n-p}}$$

$$+ \frac{b_2 z_{n-m}}{b_4 z_{n-t} + b_5 z_{n-l} + b_6 z_{n-m} + b_7 z_{n-p}} + \frac{b_3 z_{n-p}}{b_4 z_{n-t} + b_5 z_{n-l} + b_6 z_{n-m} + b_7 z_{n-p}}$$

$$\le \alpha_0 z_n + \alpha_1 z_{n-k} + \alpha_2 z_{n-s} + \frac{b_0 z_{n-t}}{b_4 z_{n-t}} + \frac{b_1 z_{n-l}}{b_5 z_{n-l}} + \frac{b_2 z_{n-m}}{b_6 z_{n-m}} + \frac{b_3 z_{n-p}}{b_7 z_{n-p}}$$

$$\le \alpha_0 z_n + \alpha_1 z_{n-k} + \alpha_2 z_{n-s} + \frac{b_0}{b_4} + \frac{b_1}{b_5} + \frac{b_2}{b_6} + \frac{b_3}{b_7} \quad \forall n \ge 1$$

We have $z_{n+1} \le y_{n+1}$, where $y_{n+1} = \alpha_0 z_n + \alpha_1 z_{n-k} + \alpha_2 z_{n-s} + \frac{b_0}{b_4} + \frac{b_1}{b_5} + \frac{b_2}{b_6} + \frac{b_3}{b_7}$ is non-homogenous linear equation .It is easy to check that solution of this equation is locally asymptotically stable and converges to $\bar{z} = \frac{(b_0+b_1+b_2+b_3)}{(b_4+b_5+b_6+b_7)(1-\alpha_0-\alpha_1-\alpha_2)}$ if $\alpha_0+\alpha_1+\alpha_2 < 1$.

By comparison, we see

$\lim_{n\to\infty}\sup z_n \le \frac{b_0 b_5 b_6 b_7 + b_1 b_4 b_6 b_7 + b_2 b_4 b_5 b_7 + b_3 b_4 b_5 b_6}{b_4 b_5 b_6 b_7 (1-\alpha_0-\alpha_1-\alpha_2)} = M$ .Hence solution is bounded. Now we will prove that there also exist $m > 0$ such that $z_n \ge m \ \forall n \ge 1$.

For this use the transformation $z_n = \frac{1}{x_n}$ .So eq.(1.1) becomes

$$\frac{1}{x_{n+1}}=\frac{\alpha_0}{x_n}+\frac{\alpha_1}{x_{n-k}}+\frac{\alpha_2}{x_{n-s}}+\frac{b_0/x_{n-t}+b_1/x_{n-l}+b_2/x_{n-m}+b_3/x_{n-p}}{b_4/x_{n-t}+b_5/x_{n-l}+b_6/x_{n-m}+b_7/x_{n-p}}$$

$$\frac{1}{x_{n+1}}=\frac{\alpha_0}{x_n}+\frac{\alpha_1}{x_{n-k}}+\frac{\alpha_2}{x_{n-s}}+\frac{b_0x_{n-l}x_{n-m}x_{n-p}+b_1x_{n-t}x_{n-m}x_{n-p}+b_2x_{n-t}x_{n-l}x_{n-p}+b_3x_{n-t}x_{n-l}x_{n-m}}{b_4x_{n-l}x_{n-m}x_{n-p}+b_5x_{n-t}x_{n-m}x_{n-p}+b_6x_{n-t}x_{n-l}x_{n-p}+b_7x_{n-t}x_{n-l}x_{n-m}}$$

$$\frac{1}{x_{n+1}}=\frac{(\alpha_0x_{n-k}x_{n-s}+\alpha_1x_nx_{n-s}+\alpha_2x_nx_{n-k})\left[b_4x_{n-l}x_{n-m}x_{n-p}+b_5x_{n-t}x_{n-m}x_{n-p}+b_6x_{n-t}x_{n-l}x_{n-p}+b_7x_{n-t}x_{n-l}x_{n-m}\right]+(x_nx_{n-k}x_{n-s})(b_0x_{n-l}x_{n-m}x_{n-p}+b_1x_{n-t}x_{n-m}x_{n-p}+b_2x_{n-t}x_{n-l}x_{n-p}+b_3x_{n-t}x_{n-l}x_{n-m})}{x_nx_{n-k}x_{n-s}(b_4x_{n-l}x_{n-m}x_{n-p}+b_5x_{n-t}x_{n-m}x_{n-p}+b_6x_{n-t}x_{n-l}x_{n-p}+b_7x_{n-t}x_{n-l}x_{n-m})}$$

$$x_{n+1}=\frac{x_nx_{n-k}x_{n-s}(b_4x_{n-l}x_{n-m}x_{n-p}+b_5x_{n-t}x_{n-m}x_{n-p}+b_6x_{n-t}x_{n-l}x_{n-p}+b_7x_{n-t}x_{n-l}x_{n-m})}{(\alpha_0x_{n-k}x_{n-s}+\alpha_1x_nx_{n-s}+\alpha_2x_nx_{n-k})\left[b_4x_{n-l}x_{n-m}x_{n-p}+b_5x_{n-t}x_{n-m}x_{n-p}+b_6x_{n-t}x_{n-l}x_{n-p}+b_7x_{n-t}x_{n-l}x_{n-m}\right]+(x_nx_{n-k}x_{n-s})(b_0x_{n-l}x_{n-m}x_{n-p}+b_1x_{n-t}x_{n-m}x_{n-p}+b_2x_{n-t}x_{n-l}x_{n-p}+b_3x_{n-t}x_{n-l}x_{n-m})}$$

It follows

$$x_{n+1}\le\frac{x_nx_{n-k}x_{n-s}(b_4x_{n-l}x_{n-m}x_{n-p}+b_5x_{n-t}x_{n-m}x_{n-p}+b_6x_{n-t}x_{n-l}x_{n-p}+b_7x_{n-t}x_{n-l}x_{n-m})}{(x_nx_{n-k}x_{n-s})(b_0x_{n-l}x_{n-m}x_{n-p}+b_1x_{n-t}x_{n-m}x_{n-p}+b_2x_{n-t}x_{n-l}x_{n-p}+b_3x_{n-t}x_{n-l}x_{n-m})}$$

$$=\frac{b_4x_{n-l}x_{n-m}x_{n-p}}{(b_0x_{n-l}x_{n-m}x_{n-p}+b_1x_{n-t}x_{n-m}x_{n-p}+b_2x_{n-t}x_{n-l}x_{n-p}+b_3x_{n-t}x_{n-l}x_{n-m})}+$$

$$\frac{b_5x_{n-t}x_{n-m}x_{n-p}}{(b_0x_{n-l}x_{n-m}x_{n-p}+b_1x_{n-t}x_{n-m}x_{n-p}+b_2x_{n-t}x_{n-l}x_{n-p}+b_3x_{n-t}x_{n-l}x_{n-m})}+$$

$$\frac{b_6x_{n-t}x_{n-l}x_{n-p}}{(b_0x_{n-l}x_{n-m}x_{n-p}+b_1x_{n-t}x_{n-m}x_{n-p}+b_2x_{n-t}x_{n-l}x_{n-p}+b_3x_{n-t}x_{n-l}x_{n-m})}+$$

$$\frac{b_7x_{n-t}x_{n-l}x_{n-m}}{(b_0x_{n-l}x_{n-m}x_{n-p}+b_1x_{n-t}x_{n-m}x_{n-p}+b_2x_{n-t}x_{n-l}x_{n-p}+b_3x_{n-t}x_{n-l}x_{n-m})}$$

$$\le\frac{b_4x_{n-l}x_{n-m}x_{n-p}}{b_0x_{n-l}x_{n-m}x_{n-p}}+\frac{b_5x_{n-t}x_{n-m}x_{n-p}}{b_1x_{n-t}x_{n-m}x_{n-p}}+\frac{b_6x_{n-t}x_{n-l}x_{n-p}}{b_2x_{n-t}x_{n-l}x_{n-p}}+\frac{b_7x_{n-t}x_{n-l}x_{n-m}}{b_3x_{n-t}x_{n-l}x_{n-m}}=\frac{b_4}{b_0}+\frac{b_5}{b_1}+\frac{b_6}{b_2}+\frac{b_7}{b_3}$$

$$=\frac{b_1b_2b_3b_4+b_0b_5b_2b_3+b_0b_1b_6b_3+b_0b_1b_2b_7}{b_0b_1b_2b_3}=T\ \forall n\ge 1$$

Thus we get $z_n=\frac{1}{x_n}\ge\frac{1}{T}=\frac{b_0b_1b_2b_3}{b_1b_2b_3b_4+b_0b_5b_2b_3+b_0b_1b_6b_3+b_0b_1b_2b_7}=m\ \forall n\ge 1$

Hence every solution of (1.1) is bounded and persistent.

Theorem 3

If $\alpha_0>1$ or $\alpha_1>1$ or $\alpha_2>1$,every solution of Eq.(1.1) is unbounded.

Proof: Let $\{z_n\}_{n=-\mu}^{\infty}$ be a solution of eq.(1.1).It follows from eq.(1.1)

$$z_{n+1}=\alpha_0z_n+\alpha_1z_{n-k}+\alpha_2z_{n-s}+\frac{b_0z_{n-t}+b_1z_{n-l}+b_2z_{n-m}+b_3z_{n-p}}{b_4z_{n-t}+b_5z_{n-l}+b_6z_{n-m}+b_7z_{n-p}}>\alpha_0z_n,\ \forall n\ge 1$$

The right side can be written as

$$x_{n+1}=\alpha_0x_n\Rightarrow x_n=\alpha_0{}^n y_0$$

It is unstable as $\alpha_0>1$ and $\lim_{n\to\infty}z_n=\infty$ .Hence $\{z_n\}_{n=-\mu}^{\infty}$ is unbounded above by ratio test.The remaining cases can be prove by same technique.

2.3 Periodic solutions:

Here we discuss that periodic solutions of eq.(1.1) exists.

Theorem 4

If k,s,t,l,m are even and p is odd then eq.(1.1) has a prime period two solutions iff

$$[b_3-(b_0+b_1+b_2)]\left\{[b_3-(b_0+b_1+b_2)][(b_4+b_5+b_6)-b_7]\right\}$$
$$\left\{1+(\alpha_0+\alpha_1+\alpha_2)\right\}-4\{b_3(b_4+b_5+b_6)(\alpha_0+\alpha_1+\alpha_2)+b_7(b_0+b_1+b_2)\}>0 \qquad (1.4)$$

Proof: Suppose first there exists a prime period two solutions

$..,\phi,\psi,\phi,\psi,...$

If k,s,t,l and m are even then

$z_n=z_{n-k}=z_{n-s}=z_{n-t}=z_{n-l}=z_{n-m}=\phi$ and $z_{n+1}=z_{n-p}=\psi$

From Eq.(1.1)

$$\psi=\alpha_0\phi+\alpha_1\phi+\alpha_2\phi+\frac{b_0\phi+b_1\phi+b_2\phi+b_3\psi}{b_4\phi+b_5\phi+b_6\phi+b_7\psi}$$
$$\phi=\alpha_0\psi+\alpha_1\psi+\alpha_2\psi+\frac{b_0\psi+b_1\psi+b_2\psi+b_3\phi}{b_4\psi+b_5\psi+b_6\psi+b_7\phi}$$

On simplifying

$$(b_4+b_5+b_6)\phi\psi+b_7\psi^2=(\alpha_0\phi+\alpha_1\phi+\alpha_2\phi)(b_4\phi+b_5\phi+b_6\phi+b_7\psi)+(b_0\phi+b_1\phi+b_2\phi+b_3\psi)$$
$$(b_4+b_5+b_6)\phi\psi+b_7\phi^2=(\alpha_0\psi+\alpha_1\psi+\alpha_2\psi)(b_4\psi+b_5\psi+b_6\psi+b_7\phi)+(b_0\psi+b_1\psi+b_2\psi+b_3\phi)$$

Then

$$(b_4+b_5+b_6)\phi\psi+b_7\psi^2=(\alpha_0+\alpha_1+\alpha_2)(b_4+b_5+b_6)\phi^2+(\alpha_0+\alpha_1+\alpha_2)b_7\phi\psi+(b_0\phi+b_1\phi+b_2\phi+b_3\psi)$$
$$(b_4+b_5+b_6)\phi\psi+b_7\phi^2=(\alpha_0+\alpha_1+\alpha_2)(b_4+b_5+b_6)\psi^2+(\alpha_0+\alpha_1+\alpha_2)b_7\phi\psi+(b_0\psi+b_1\psi+b_2\psi+b_3\phi)$$

On subtracting

$$b_7(\psi^2-\phi^2)=(\alpha_0+\alpha_1+\alpha_2)(b_4+b_5+b_6)(\phi^2-\psi^2)+(b_0+b_1+b_2)(\phi-\psi)+b_3(\psi-\phi)$$
$$\psi+\phi=\frac{b_3-(b_0+b_1+b_2)}{b_7+(\alpha_0+\alpha_1+\alpha_2)(b_4+b_5+b_6)} \qquad (1.5)$$

Adding

$$2\phi\psi[((b_4+b_5+b_6)-(\alpha_0+\alpha_1+\alpha_2)b_7]=[(\alpha_0+\alpha_1+\alpha_2)(b_4+b_5+b_6)-b_7](\phi^2+\psi^2)+(b_0+b_1+b_2+b_3)(\phi+\psi)$$
$$2\phi\psi[(b_4+b_5+b_6)-(\alpha_0+\alpha_1+\alpha_2)b_7]=[(\alpha_0+\alpha_1+\alpha_2)(b_4+b_5+b_6)-b_7][(\phi+\psi)^2-2\phi\psi]+(b_0+b_1+b_2+b_3)(\phi+\psi)$$
$$2\phi\psi[(b_4+b_5+b_6)-(\alpha_0+\alpha_1+\alpha_2)b_7+(\alpha_0+\alpha_1+\alpha_2)(b_4+b_5+b_6)-b_7]=[(\alpha_0+\alpha_1+\alpha_2)(b_4+b_5+b_6)-b_7]$$
$$\left[\frac{b_3-(b_0+b_1+b_2)}{b_7+(\alpha_0+\alpha_1+\alpha_2)(b_4+b_5+b_6)}\right]^2+(b_0+b_1+b_2+b_3)\left[\frac{b_3-(b_0+b_1+b_2)}{b_7+(\alpha_0+\alpha_1+\alpha_2)(b_4+b_5+b_6)}\right]$$
$$2\phi\psi\left[\{(b_4+b_5+b_6)-b_7\}\{1+(\alpha_0+\alpha_1+\alpha_2)\}\right]=\frac{\begin{matrix}[(\alpha_0+\alpha_1+\alpha_2)(b_4+b_5+a_6)-b_7][b_3-(b_0+b_1+b_2)]^2+\\ [b_7+(\alpha_0+\alpha_1+\alpha_2)(b_4+b_5+b_6)](b_0+b_1+b_2+b_3)\\ [b_3-(b_0+b_1+b_2)]\end{matrix}}{[b_7+(\alpha_0+\alpha_1+\alpha_2)(b_4+b_5+b_6)]^2}$$
$$=\frac{[b_3-(b_0+b_1+b_2)]\left\{\begin{matrix}((\alpha_0+\alpha_1+\alpha_2)(b_4+b_5+b_6)-b_7)(b_3-b_0-b_1-b_2)+\\ (b_7+(\alpha_0+\alpha_1+\alpha_2)(b_4+b_5+b_6))(b_0+b_1+b_2+b_3)\end{matrix}\right\}}{[b_7+(\alpha_0+\alpha_1+\alpha_2)(b_4+b_5+b_6)]^2}$$
$$=\frac{[b_3-(b_0+b_1+b_2)]\left\{\begin{matrix}(\alpha_0+\alpha_1+\alpha_2)(b_4+b_5+b_6)(b_3-b_0-b_1-b_2)-b_7(b_3-b_0-b_1-b_2)+\\ b_7(b_3+b_0+b_1+b_2)+(\alpha_0+\alpha_1+\alpha_2)(b_4+b_5+b_6)(b_3+b_0+b_1+b_2)\end{matrix}\right\}}{[b_7+(\alpha_0+\alpha_1+\alpha_2)(b_4+b_5+b_6)]^2}$$

$$= \frac{2\left[b_3-(b_0+b_1+b_2)\right]\left\{b_3(\alpha_0+\alpha_1+\alpha_2)(b_4+b_5+b_6)+b_7(b_0+b_1+b_2)\right\}}{\left[b_7+(\alpha_0+\alpha_1+\alpha_2)(b_4+b_5+b_6)\right]^2}$$

$$\phi\psi = \frac{\left[b_3-(b_0+b_1+b_2)\right]\left\{b_3(\alpha_0+\alpha_1+\alpha_2)(b_4+b_5+b_6)+b_7(b_0+b_1+b_2)\right\}}{\left[\left\{(b_4+b_5+b_6)-b_7\right\}\left\{1+\alpha_0+\alpha_1+\alpha_2\right\}\right]\left[b_7+(\alpha_0+\alpha_1+\alpha_2)(b_4+b_5+b_6)\right]^2} \quad (1.6)$$

Let ϕ,ψ be the roots of quadratic equation which are positive, real and distinct.

$$t^2-(\phi+\psi)t+\phi\psi=0$$

$$t^2-\left[\frac{b_3-(b_0+b_1+b_2)}{b_7+(\alpha_0+\alpha_1+\alpha_2)(b_4+b_5+b_6)}\right]t+$$

$$\frac{\left[b_3-(b_0+b_1+b_2)\right]\left\{b_3(\alpha_0+\alpha_1+\alpha_2)(b_4+b_5+b_6)+b_7(b_0+b_1+b_2)\right\}}{\left[\left\{(b_4+b_5+b_6)-b_7\right\}\left\{1+(\alpha_0+\alpha_1+\alpha_2)\right\}\right]\left[b_7+(\alpha_0+\alpha_1+\alpha_2)(b_4+b_5+b_6)\right]^2}=0 \quad (1.7)$$

Thus, discriminant is

$$\left[\frac{b_3-(b_0+b_1+b_2)}{b_7+(\alpha_0+\alpha_1+\alpha_2)(b_4+b_5+b_6)}\right]^2-$$

$$4\frac{\left[b_3-(b_0+b_1+b_2)\right]\left\{b_3(\alpha_0+\alpha_1+\alpha_2)(b_4+b_5+b_6)+b_7(b_0+b_1+b_2)\right\}}{\left[\left\{(b_4+b_5+b_6)-b_7\right\}\left\{1+(\alpha_0+\alpha_1+\alpha_2)\right\}\right]\left[b_7+(\alpha_0+\alpha_1+\alpha_2)(b_4+b_5+b_6)\right]^2}>0$$

$$[b_3-(b_0+b_1+b_2)]\left\{\left[b_3-(b_0+b_1+b_2)\right]\left[(b_4+b_5+b_6)-b_7\right]\right\}$$

$$\left\{1+(\alpha_0+\alpha_1+\alpha_2)\right\}-4\{b_3(b_4+b_5+b_6)(\alpha_0+\alpha_1+\alpha_2)+b_7(b_0+b_1+b_2)\}>0$$

Hence inequality (1.3) holds. Now suppose inequality (1.3) is true. Now we will show that Eq. (1.1) has prime period two solutions.

Let

$$\phi=\frac{b_3-A+\eta}{2(b_7+GH)} \text{ and } \psi=\frac{b_3-A-\eta}{2(b_7+GH)}$$

Where $\eta=\sqrt{(b_3-A)^2-\dfrac{4(b_3-A)(b_3GH+b_7A)}{(H-b_7)(1+G)}}$, $A=b_0+b_1+b_2$, $G=\alpha_0+\alpha_1+\alpha_2$

and $H=b_4+b_5+b_6$.

Now set

$$z_{-k}=\psi, z_{-s}=\psi, z_{-t}=\psi, z_{-l}=\psi, z_{-m}=\psi, z_{-p}=\phi,\ldots, z_{-3}=\phi, z_{-2}=\psi,$$

$$z_{-1}=\phi, z_0=\psi.$$

We will prove $z_1=z_{-1}=\phi$ and $z_2=z_0=\psi$.

From Eq.(1.1)

$$z_1=\alpha_0\psi+\alpha_1\psi+\alpha_2\psi+\frac{b_0\psi+b_1\psi+b_2\psi+b_3\phi}{b_4\psi+b_5\psi+b_6\psi+b_7\phi}$$

$$=G\psi+\frac{(b_0+b_1+b_2)\psi+b_3\phi}{(b_4+b_5+b_6)\psi+b_7\phi}$$

$$=G\psi+\frac{A\left(\dfrac{b_3-A-\eta}{2(b_7+GH)}\right)+b_3\left(\dfrac{b_3-A+\eta}{2(b_7+GH)}\right)}{H\left(\dfrac{b_3-A-\eta}{2(b_7+GH)}\right)+b_7\left(\dfrac{b_3-A+\eta}{2(b_7+GH)}\right)}$$

Dividing numerator and denominator by $2(b_7+GH)$

$$=G\psi+\frac{A(b_3-A-\eta)+b_3(b_3-A+\eta)}{H(b_3-A-\eta)+b_7(b_3-A+\eta)}$$

$$= G\psi + \frac{(A+b_3)(b_3-A)-A\eta+b_3\eta}{(H+b_7)(b_3-A)-H\eta+b_7\eta}$$

$$= G\psi + \frac{(b_3-A)(A+b_3+\eta)}{(H+b_7)(b_3-A)+\eta(b_7-H)}$$

Multiplying and dividing by $(H+b_7)(b_3-A)-\eta(b_7-H)$

$$= G\psi + \frac{(b_3-A)(A+b_3+\eta)}{(H+b_7)(b_3-A)+\eta(b_7-H)} \times \frac{(H+b_7)(b_3-A)-\eta(b_7-H)}{(H+b_7)(b_3-A)-\eta(b_7-H)}$$

$$= G\psi + \frac{(b_3-A)^2(H+b_7)(A+b_3+\eta)-(b_3-A)(A+b_3+\eta)\eta(b_7-H)}{(H+b_7)^2(b_3-A)^2-\eta^2(b_7-H)^2}$$

$$= G\psi + \frac{(b_3-A)\{(b_3-A)(H+b_7)(A+b_3+\eta)-(A+b_3+\eta)\eta(b_7-H)\}}{(H+b_7)^2(b_3-A)^2-\left[\left(b_3-A\right)^2-\frac{4\left(b_3-A\right)(b_3GH+b_7A)}{(H-b_7)(1+G)}\right](b_7-H)^2}$$

$$= G\psi + \frac{(b_3-A)\{(b_3{}^2-A^2)(H+b_7)+\eta(b_3-A)(H+b_7)-\eta(A+b_3)(b_7-H)-\eta^2(b_7-H)\}}{(b_3-A)^2 4Hb_7+\frac{4\left(b_3-A\right)(H-b_7)(b_3GH+b_7A)}{(1+G)}}$$

$$= G\psi + \frac{(b_3-A)\left\{(b_3{}^2-A^2)(H+b_7)+\eta(2Hb_3-2Ab_7)-\left[\left(b_3-A\right)^2-\frac{4\left(b_3-A\right)(b_3GH+b_7A)}{(H-b_7)(1+G)}\right](b_7-H)\right\}}{(b_3-A)\left\{\frac{4Hb_7(b_3-A)(1+G)+4(H-b_7)(b_3GH+b_7A)}{(1+G)}\right\}}$$

$$= G\psi + \frac{(b_3-A)\left\{(b_3{}^2-A^2)(H+b_7)+\eta(2Hb_3-2Ab_7)-\left[\left(b_3-A\right)^2-\frac{4\left(b_3-A\right)(b_3GH+b_7A)}{(H-b_7)(1+G)}\right](b_7-H)\right\}}{(b_3-A)\left\{\frac{4(b_7+HG)(Hb_3-b_7A)}{(1+G)}\right\}}$$

$$= G\psi + \frac{(b_3-A)\left\{(b_3{}^2-A^2)(H+b_7)+\eta(2Hb_3-2Ab_7)-\left(b_3-A\right)^2(b_7-H)-\frac{4\left(b_3-A\right)(b_3GH+b_7A)}{(1+G)}\right\}}{((b_3-A)\left\{\frac{4(b_7+HG)(Hb_3-b_7A)}{(1+G)}\right\}}$$

$$= G\psi + \frac{(b_3-A)\left\{2(b_3-A)(b_3H+Ab_7)+2\eta(Hb_3-Ab_7)-\frac{4\left(b_3-A\right)(b_3GH+b_7A)}{(1+G)}\right\}}{(b_3-A)\left\{\frac{4(b_7+HG)(Hb_3-b_7A)}{(1+G)}\right\}}$$

$$= G\psi + \frac{\left\{2(b_3-A)(b_3H+Ab_7)(1+G)+2\eta(Hb_3-Ab_7)(1+G)-4\left(b_3-A\right)(b_3GH+b_7A)\right\}}{\left\{4(b_7+HG)(Hb_3-b_7A)\right\}}$$

$$= G\psi + \frac{\left[(b_3-A)\left\{b_3H+b_3HG+Ab_7+Ab_7G-2b_3GH-2b_7A\right\}\right]+\eta(Hb_3-Ab_7)(1+G)}{\left\{2(b_7+HG)(Hb_3-b_7A)\right\}}$$

$$= G\left(\frac{b_3-A-\eta}{2(b_7+GH)}\right)+\frac{(b_3-A)(1-G)(b_3H-Ab_7)+\eta(Hb_3-Ab_7)(1+G)}{\left\{2(b_7+HG)(Hb_3-b_7A)\right\}}$$

$$= G\left(\frac{b_3 - A - \eta}{2(b_7 + GH)}\right) + \frac{(b_3 - A)(1-G) + \eta(1+G)}{\{2(b_7 + HG)\}}$$

$$= \frac{G(b_3 - A) - \eta G - G(b_3 - A) + (b_3 - A) + \eta + \eta G}{2(b_7 + HG)} = \frac{b_3 - A + \eta}{2(b_7 + GH)} = \phi$$

In the same way ,its quite easy to prove that $z_2 = \psi$ .By mathematical induction for all $n \geq -\mu$, we get $z_{2n} = \psi$ and $z_{2n+1} = \phi$.Thus eq.(1.1) has prime period two solution which are distinct roots of eq.(1.7).

$..,\phi,\psi,\phi,\psi,...$

Theorem 5

Eq.(1.1) has no prime period two solutions if k, s, t, l, m, p are even and

$(\alpha_0 + \alpha_1 + \alpha_2) \neq 1.$

Proof: Proof is same as previous and therefore omitted.

2.4 GRAPHICAL EXAMPLES:

Here we give some numerical examples to confirm the obtained results. These examples give some different types of solutions of eq.(1.1)

Example 1:

$k = 5, s = 4, t = 2, l = 3, m = 0, p = 1,$ and values to coefficients stated in expression (A) in order $0.1, 0.2, 0, 2, 0.5, 0.6, 0.1, 0.3, 1, 0.2, 0.5.$ and the initial conditions taken in order $0.2, 0.7, 0.5, 2.1, 1.15, 0.4, 0.1, 0.3$.(See fig 1.1)

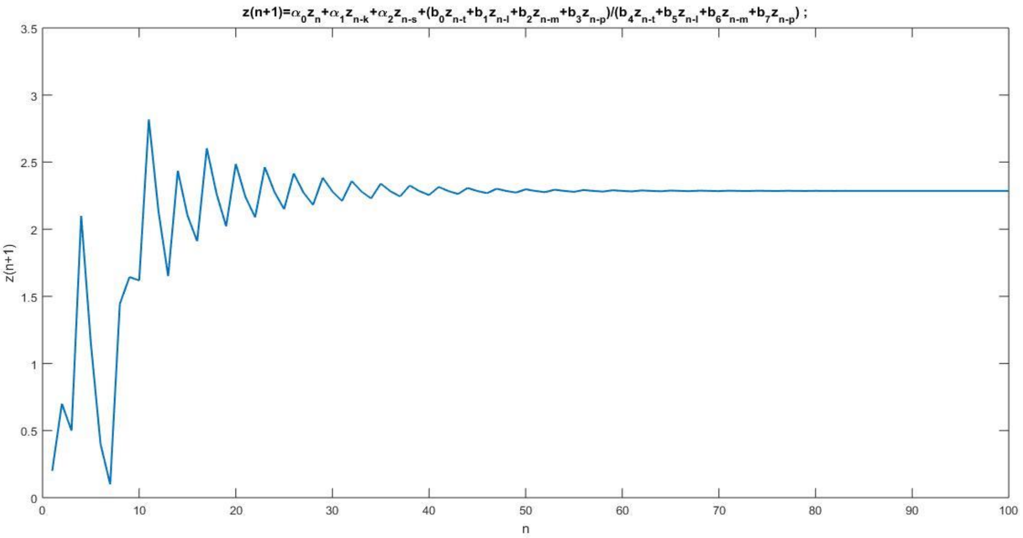

Fig 1.1:Behavior of $z_{n+1} = \alpha_0 z_n + \alpha_1 z_{n-k} + \alpha_2 z_{n-s} + \dfrac{b_0 z_{n-t} + b_1 z_{n-l} + b_2 z_{n-m} + b_3 z_{n-p}}{b_4 z_{n-t} + b_5 z_{n-l} + b_6 z_{n-m} + b_7 z_{n-p}}$

Example 2:

$k = 5, s = 4, t = 4, l = 0, m = 0, p = 1,$ and values to coefficients stated in expression (A) $0.9, 0.2, 0.23, 2, 0.5, 0.06, 0.15, 0.3, 1.5, 0.2, 0.25,$ and the initial conditions taken in order $1.22, 0.7, 0, 2.1, 1.15, 1.4, 0.1, 0.$

(See fig 1. 2)

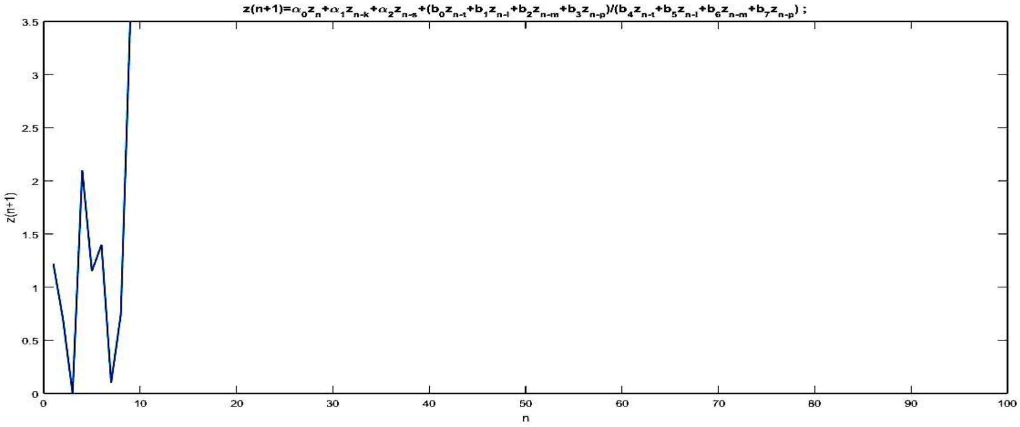

Fig 1.2:Behavior of $z_{n+1} = \alpha_0 z_n + \alpha_1 z_{n-k} + \alpha_2 z_{n-s} + \dfrac{b_0 z_{n-t} + b_1 z_{n-l} + b_2 z_{n-m} + b_3 z_{n-p}}{b_4 z_{n-t} + b_5 z_{n-l} + b_6 z_{n-m} + b_7 z_{n-p}}$

Example 3:

$k = 5, s = 4, t = 2, l = 3, m = 0, p = 1,$ and values to coefficients stated in expression (A) $0.1, 0.13, 0.15, 1.5, 0.06, 0.2, 1.1, 0.3, 1, 0.2, 0.5,$ and the initial conditions taken in order $0.2, 0.7, 0.5, 2.1, 1.15, 0.4, 0.1, 0.3.$

(See fig 1. 3)

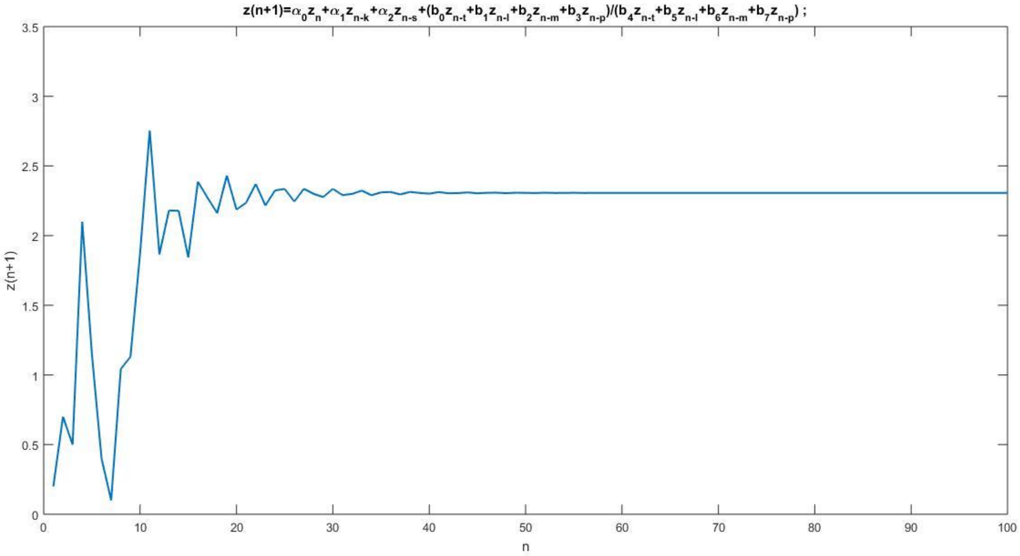

Fig 1.3:Behavior of $z_{n+1} = \alpha_0 z_n + \alpha_1 z_{n-k} + \alpha_2 z_{n-s} + \dfrac{b_0 z_{n-t} + b_1 z_{n-l} + b_2 z_{n-m} + b_3 z_{n-p}}{b_4 z_{n-t} + b_5 z_{n-l} + b_6 z_{n-m} + b_7 z_{n-p}}$

Example 4:

$k = 5, s = 4, t = 2, l = 3, m = 0, p = 1,$ and values to coefficients stated in expression (A) $0.1, 0.03, 0.05, 1.02, 0.18, 0.02, 0.1, 0.3, 1, 0.2, 0.05,$ and the initial conditions taken in order $0.2, 0.7, 0.5, 2.1, 1.15, 0.4, 0.1, 0.3$.

(See fig1.4)

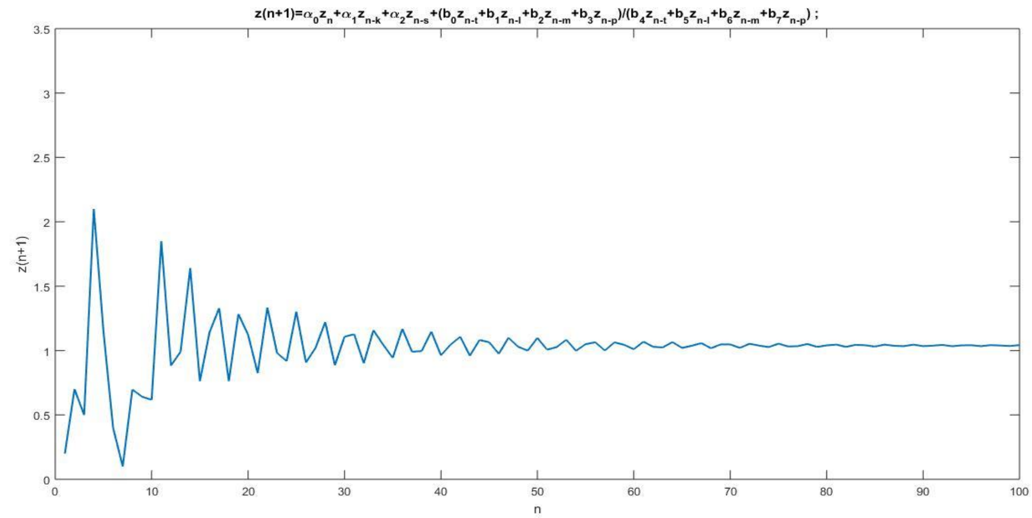

Fig 1.4:Behavior of $z_{n+1} = \alpha_0 z_n + \alpha_1 z_{n-k} + \alpha_2 z_{n-s} + \dfrac{b_0 z_{n-t} + b_1 z_{n-l} + b_2 z_{n-m} + b_3 z_{n-p}}{b_4 z_{n-t} + b_5 z_{n-l} + b_6 z_{n-m} + b_7 z_{n-p}}$

Example 5:
$k = 4, s = 4, t = 4, l = 4, m = 4,\ p = 5,$ and values to coefficients stated in expression (A) $0.03, 0.04, 0.02, 0.01, 0.55, 0.01, 6, 0.01, 0.1, 0.02, 0.5,$ and the initial conditions taken in order $0.5, 1.5, 0.5, 1.5, 0.5, 1.5, 0.5, 1.5$.
(See fig1.5)

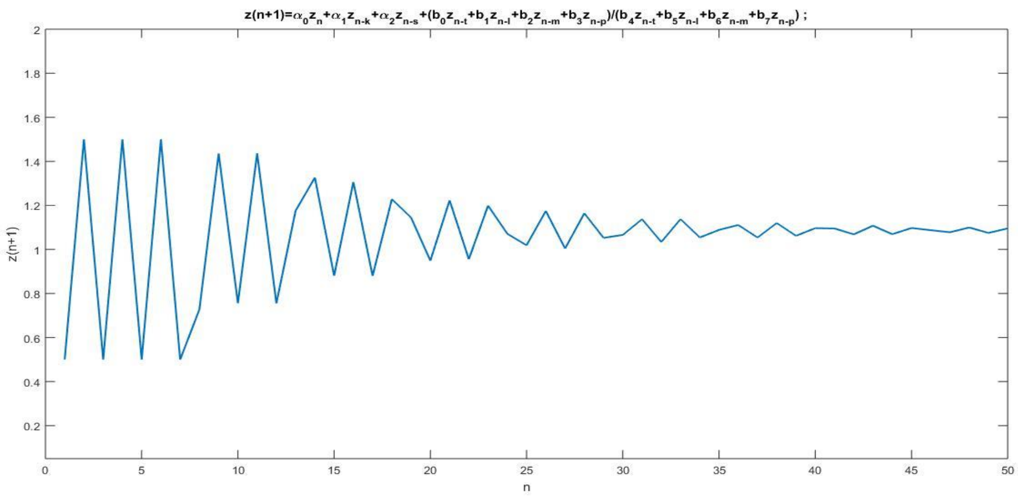

Fig 1.5:Behavior of $z_{n+1} = \alpha_0 z_n + \alpha_1 z_{n-k} + \alpha_2 z_{n-s} + \dfrac{b_0 z_{n-t} + b_1 z_{n-l} + b_2 z_{n-m} + b_3 z_{n-p}}{b_4 z_{n-t} + b_5 z_{n-l} + b_6 z_{n-m} + b_7 z_{n-p}}$

Example 6:
$k = 4, s = 4, t = 4, l = 4, m = 4,\ p = 5,$ and values to coefficients stated in expression (A) $0.03, 0.04, 0.02, 0.01, 0.5, 0.2, 2, 0.2, 0.1, 0.12, 0.5,$ and the initial conditions taken in order $0.5, 1.5, 0.1, 1.2, 0.5, 2.5, 0.2, 0.5$.
(See fig 1. 6)

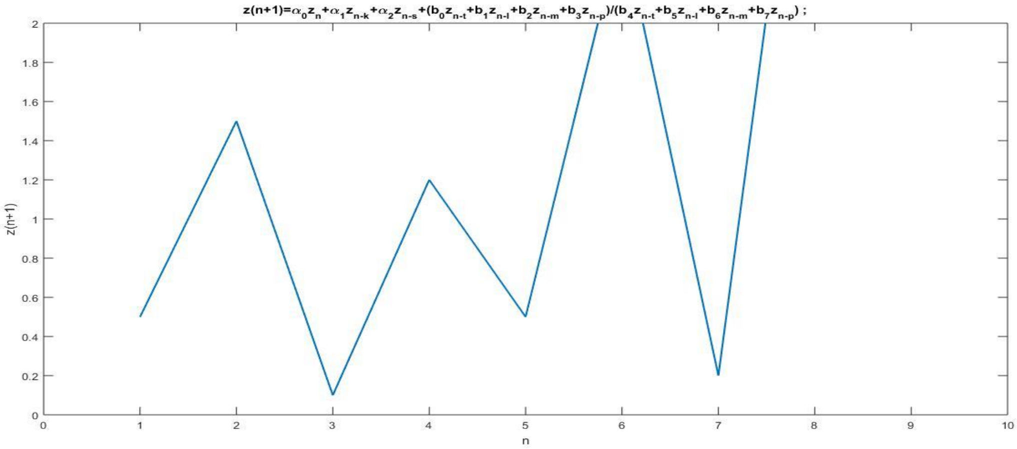

Fig1.6:Behavior of $z_{n+1} = \alpha_0 z_n + \alpha_1 z_{n-k} + \alpha_2 z_{n-s} + \frac{b_0 z_{n-t} + b_1 z_{n-l} + b_2 z_{n-m} + b_3 z_{n-p}}{b_4 z_{n-t} + b_5 z_{n-l} + b_6 z_{n-m} + b_7 z_{n-p}}$

3. CONCLUSION

We studied the global stability , bounded behavior and forms of solutions of $z_{n+1} = \alpha_0 z_n + \alpha_1 z_{n-k} + \alpha_2 z_{n-s} + \frac{b_0 z_{n-t} + b_1 z_{n-l} + b_2 z_{n-m} + b_3 z_{n-p}}{b_4 z_{n-t} + b_5 z_{n-l} + b_6 z_{n-m} + b_7 z_{n-p}}$ with non-negative initial conditions $z_{-\mu}, z_{-\mu+1}, ..., z_{-1}, z_0$ where $\mu = \max\{k, s, t, l, m, p\}$ and the coefficients $\alpha_0, \alpha_1, \alpha_2, b_0, b_1, b_2, b_3, b_4, b_5, b_6, b_7 \in R^+$ .It is concluded that if $\alpha_0 + \alpha_1 + \alpha_2 < 1$, then the unique equilibrium point is $\bar{z} = \frac{(b_0 + b_1 + b_2 + b_3)}{(b_4 + b_5 + b_6 + b_7)(1 - \alpha_0 - \alpha_1 - \alpha_2)}$ .Using conditions $\alpha_0 + \alpha_1 + \alpha_2 \neq 1$ and $b_0 + b_1 + b_3 \neq b_2$ .We see that $M = m$ .Hence $\bar{z}$ is a global attractor of eq.(1.1). Every solution of equation (1.1) is bounded if $\alpha_0 + \alpha_1 + \alpha_2 < 1$ .Every solution of (1.1) is unbounded if $\alpha_0 > 1$ or $\alpha_1 > 1$ or $\alpha_2 > 1$ .If k, s, t, l, m are even and p is odd then eq.(1.1) has a prime period two solutions iff

$$[b_3 - (b_0 + b_1 + b_2)]\left\{[b_3 - (b_0 + b_1 + b_2)][(b_4 + b_5 + b_6) - b_7]\right\}$$
$$\left\{1 + (\alpha_0 + \alpha_1 + \alpha_2)\right\} - 4\{b_3(b_4 + b_5 + b_6)(\alpha_0 + \alpha_1 + \alpha_2) + b_7(b_0 + b_1 + b_2)\} > 0$$

In the end we obtained solution of four different types of Eq. (1.1) and gave numerical examples of each case by assigning different initial values by using Matlab.

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