FINITE ELEMENT ANALYSIS OF PLANE TRUSS STRUCTURES

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ABSTRACT: *The focus of this research is to calculate element equations; assemble them to form global equations, incorporate essential boundary conditions into the system and obtain the final reduced system of equations in terms of the unknowns, solve it for nodal values, compute reactions and verify overall equilibrium and also determine axial strains, axial stresses, and axial forces in different elements of the truss by using finite element Analysis. We use five bar plane trusses. Also five bar truss problem with an inclined support with multipoint constraint due to inclined support, Lagrange multiplier used with global equations and Truss Supporting a Rigid Plate to use the penalty function approach and we choose the penalty parameter equal to l0^5. The results of the trusses problems were obtained by using MATLAB which demonstrate the effectiveness, applicability of results.*

Keywords: plane trusses, finite element analysis, trusses with an inclined support, trusses supporting rigid plate, penalty function.

INTRODUCTION

Application of physical principles, such as mass balance, energy conservation, and equilibrium, naturally leads many engineering analysis situations into differential equations. Methods have been developed for obtaining exact solutions for various classes of differential equations. However, these methods do not apply to many practical problems because either their governing differential equations do not fall into these classes or they involve complex geometries. Finding analytical solutions that also satisfy boundary conditions specified over arbitrary two- and three-dimensional regions becomes a very difficult task. Numerical methods are therefore widely used for solution of practical problems in all branches of engineering [3].

The finite element method is one of the numerical methods for obtaining approximate solution of ordinary and partial differential equations. It is especially powerful when dealing with boundary conditions defined over complex geometries that are common in practical applications. Other numerical methods such as finite difference and boundary element methods [1, 4] may be competitive or even superior to the finite element method for certain classes of problems. However, because of its versatility in handling arbitrary domains and availability of sophisticated commercial finite element software, over the last few decades, the finite element method has become the preferred method for solution of many practical problems [2, 7].

Many structural systems used in practice consist of long slender members of various shapes. Common examples are roof trusses, bridge supports, crane booms, and antenna towers. Structural systems that are arranged so that each member primarily resists axial forces only are usually known as trusses. Long slender members that are subjected to loading normal to their longitudinal axis must resist bending and shear forces and are called beams. A structural frame consists of members that must resist both bending and axial forces. A truss is a structure in which members are arranged in such a way that they are subjected to axial loads only. The joints in trusses are considered pinned. Plane trusses where all members are assumed to be in the xy-plane are considered [5-7].

MATERIAL AND METHODS

A plane truss element is an axial deformation element oriented arbitrarily in a two dimensional space.

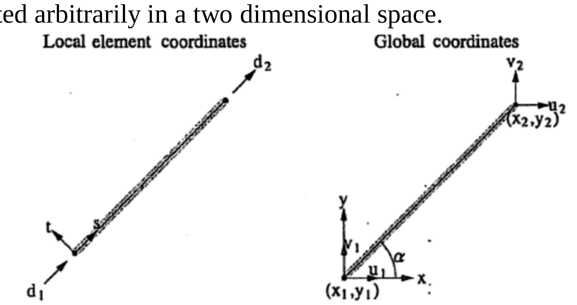

Figure 1. Local and global coordinates for an axially loaded bar

In a local coordinates that runs along its axis ($0 \le s \le L$), the element is exactly the same as the two-node axial deformation element developed. Thus in terms of *s* the assumed displacement over the element is

$$u(s) = \begin{pmatrix} \frac{L-S}{L} & \frac{S}{L} \end{pmatrix} \begin{pmatrix} d_1 \\ d_2 \end{pmatrix} \qquad 0 \le s \le L$$

and the element equations in the local coordinate system are

$$\frac{EA}{L} \begin{pmatrix} 1 & -1 \\ -1 & 1 \end{pmatrix} \begin{pmatrix} d_1 \\ d_2 \end{pmatrix} = \begin{pmatrix} p_1 \\ p_2 \end{pmatrix} \Rightarrow k_1 d_1 = r_1$$

Local degree of freedom: $d_l = \begin{pmatrix} d_1 \\ d_2 \end{pmatrix}$

Local applied modal forces: $r_l = \begin{pmatrix} p_1 \\ p_2 \end{pmatrix}$

Where, E= Elastic modulus of the material
A= Area of cross section of the element
L= Length of the element

d_1 and d_2 are the displacements along the axis of the element and p_1 and p_2 are possible axial loads applied at the bar ends. To assemble element equations, we must refer all elements to one common reference coordinate system. Thus we define a global *x-y* coordinate system and locate all elements with respect to this system. The components of the axial displacements in the global coordinate system are the *x* displacements denoted by *u* and *y* displacements denoted by v. Each node thus has two degrees of freedom in the global coordinate system [9, 11]. The possible applied loads at the element ends are also decomposed into their *x* and *y* components. Thus in the global coordinates we have

Nodal degrees of freedoms: $d = \begin{bmatrix} u_1 \\ v_1 \\ u_2 \\ v_2 \end{bmatrix}$

Applied nodal forces: $r = \begin{bmatrix} F_{x1} \\ F_{y1} \\ F_{x2} \\ F_{y2} \end{bmatrix}$

$$u_1 = d_1 \cos\alpha = d_1 l_s$$

$$v_1 = d_1 \sin\alpha = d_1 m_s$$

$$\Rightarrow \begin{pmatrix} u_1 \\ v_1 \end{pmatrix} = \begin{pmatrix} l_s \\ m_s \end{pmatrix} d_1$$

where l_s is the cosine of the angle between the element *s* axis and the global x-axis and m_s is the cosine of the angle between the element *s* axis and the global y-axis.

Element length $L = \sqrt{\left(x_2 - x_1\right)^2 + \left(y_2 - y_1\right)^2}$

Direction cosines $l_s = \cos\alpha = \dfrac{x_2 - x_1}{L}$

$$m_s = \sin\alpha = \frac{y_2 - y_1}{L}$$

Transformation from global to local degrees of freedom

$$u_1 l_s + v_1 m_s = d_1$$

The same relationship holds for the degrees of freedom at other nodes. Thus the transformation between the global and the local degrees of freedom can be written as follows:

Global to local: $\begin{pmatrix} d_1 \\ d_2 \end{pmatrix} = \begin{pmatrix} l_s & m_s & 0 & 0 \\ 0 & 0 & l_s & m_s \end{pmatrix} \begin{pmatrix} u_1 \\ v_1 \\ u_2 \\ v_2 \end{pmatrix} \Rightarrow d_l = Td$

Local to global: $\begin{pmatrix} u_1 \\ v_1 \\ u_2 \\ v_2 \end{pmatrix} = \begin{pmatrix} l_s & 0 \\ m_s & 0 \\ 0 & l_s \\ 0 & m_s \end{pmatrix} \begin{pmatrix} d_1 \\ d_2 \end{pmatrix}$

$$\Rightarrow d = T^T d_l$$

Using this transformation, the element equations in the local coordinates can be connected to the global coordinate system as follows:

$$k_l d_l = r_l \Rightarrow k_l T d = r_l$$

Multiplying both sides by T^T ,we get

$$T^T k_l T d = T^T r_l \quad \Rightarrow \quad kd = r \text{ ,}$$

where $k = T^T K_l T$ and $r = T^T r_l$

Carrying out matrix multiplication, we get the following element equations

$$\frac{EA}{L}\begin{bmatrix} l_s^2 & l_s m_s & -l_s^2 & -l_s m_s \\ l_s m_s & m_s^2 & -l_s m_s & -m_s^2 \\ -l_s^2 & -l_s m_s & l_s^2 & l_s m_s \\ -l_s m_s & -m_s^2 & l_s m_s & m_s^2 \end{bmatrix}\begin{bmatrix} u_1 \\ v_1 \\ u_2 \\ v_2 \end{bmatrix} = \begin{bmatrix} F_{x1} \\ F_{y1} \\ F_{x2} \\ F_{y2} \end{bmatrix}$$

Add the concentrated loads directly to the global equations at the start of the assembly.

Thus in the element equations the right hand side, load vector is taken as all zeros

$$\frac{EA}{L}\begin{bmatrix} l_s^2 & l_s m_s & -l_s^2 & -l_s m_s \\ l_s m_s & m_s^2 & -l_s m_s & -m_s^2 \\ -l_s^2 & -l_s m_s & l_s^2 & l_s m_s \\ -l_s m_s & -m_s^2 & l_s m_s & m_s^2 \end{bmatrix}\begin{bmatrix} u_1 \\ v_1 \\ u_2 \\ v_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

The assembly and solution procedures have been discussed in five-bar truss problem. After computing nodal displacements for each element, the element solution is computed by first transforming the nodal displacements back to the local coordinates [10-12] as follows:

Axial displacement: $\begin{pmatrix} d_1 \\ d_2 \end{pmatrix} = \begin{pmatrix} l_s & m_s & 0 & 0 \\ 0 & 0 & l_s & m_s \end{pmatrix} \begin{bmatrix} u_1 \\ v_1 \\ u_2 \\ v_2 \end{bmatrix}$

The axial displacement at any point along the element is calculated as:

$$u(s) = \begin{pmatrix} \frac{L-S}{L} & \frac{S}{L} \end{pmatrix} \begin{pmatrix} d_1 \\ d_2 \end{pmatrix} \quad , \qquad 0 \le S \le L$$

The axial strain is simply the first derivative of the axial displacement, giving constant strain over the element as follows:

$$\varepsilon = \frac{du}{ds} = \begin{pmatrix} -\frac{1}{L} & \frac{1}{L} \end{pmatrix} \begin{pmatrix} d_1 \\ d_1 \end{pmatrix} = \frac{1}{L}\left(-d_1 + d_2\right)$$

The axial stress is $\sigma = E\varepsilon$ and axial force in the element is $F = \sigma A$. The sign convention used in these equations assumes that the tension is positive and the compression is negative.

NUMERICAL ILLUSTRATIONS

FIVE BAR TRUSS PROBLEM

Construct element equations; assemble them to form global equations for the five bar plane truss [8, 15] shown in figure 2. Incorporate essential boundary conditions into the system and obtain the final reduced system of equations in terms of the unknowns. Solve for nodal values. Compute reactions and verify overall equilibrium. Determine axial strains, axial stresses, and axial forces in different elements of the truss. The area of cross section for elements 1 and 2 is *40* cm^2, for elements 3 and 4 is *30* cm^2 and for element 5 is *20* cm^2. The first four elements are made of a material with *E=200* GPa and the last one with *E=70* GPa. The applied load *P=150* kN.

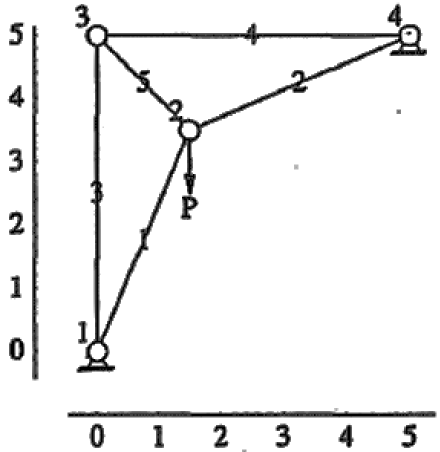

Figure 2. Five-bar plane truss

Each node in the model has two- displacement degrees of freedom. They are identified by the letters *u* and *v* with a subscript indicating the corresponding node number and are shown in Figure 3. without considering the specified zero displacements at the supports, the model has a total of eight degrees of freedom. Thus the global equations will be a system of eight equations in eight unknowns.

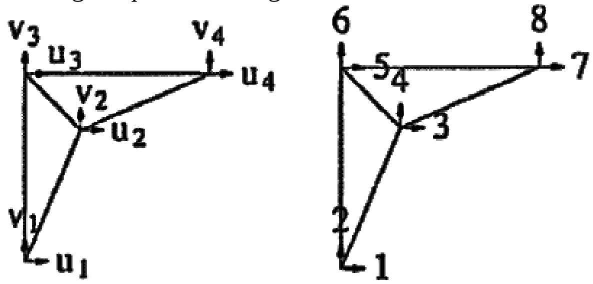

Figure 3. Five-bar plane truss finite element model

The load at node 2 is (0, -150 kN).

Table 1. Specific nodal loads:

Node	Degrees Of Freedom	Value
2	u_2	0
	v_2	-150000

The global equations at the start of the element gathering process are

$$\begin{bmatrix} 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0 \end{bmatrix}\begin{bmatrix} u_1\\ v_1\\ u_2\\ v_2\\ u_3\\ v_3\\ u_4\\ v_4 \end{bmatrix}=\begin{bmatrix} 0\\ 0\\ 0\\ -150000\\ 0\\ 0\\ 0\\ 0 \end{bmatrix}$$

The equations for element 1 are as follows

E=200000 MPa

A=4000 mm

Table 2. 1Equations for element

Element Node	Global Node number	x	y
1	1	0	0
2	2	1500	3500

Length of the element: L = 3807.89

Direction Cosines: $l_s = 0.393919$, $m_s = 0.919144$

Element Equations: Substituting into the truss element equations, we get

$$\begin{bmatrix} 32600 & 76067 & -32600 & -76067\\ 76067 & 177489 & -76067 & -177489\\ -32600 & -76067 & 32600 & 76067\\ -76067 & -177489 & 76067 & 177489 \end{bmatrix}\begin{bmatrix} u_1\\ v_1\\ u_2\\ v_2 \end{bmatrix}=\begin{bmatrix} 0\\ 0\\ 0\\ 0 \end{bmatrix}$$

The element contributes to [1,2,3,4] global degree of freedom,

Its global vector: $\begin{bmatrix} 1\\ 2\\ 3\\ 4 \end{bmatrix}$

Its global matrix: $\begin{bmatrix} [1,1] & [1,2] & [1,3] & [1,4]\\ [2,1] & [2,2] & [2,3] & [2,4]\\ [3,1] & [3,2] & [3,3] & [3,4]\\ [4,1] & [4,2] & [4,3] & [4,4] \end{bmatrix}$

These values are plane truss stiffness matrix

$$\begin{bmatrix} 32600 & 76067 & -32600 & -76067 & 0&0&0&0\\ 76067 & 177489 & -76067 & -177489 & 0&0&0&0\\ -32600 & -76067 & 32600 & 76067 & 0&0&0&0\\ -76067 & -177489 & 76067 & 177489 & 0&0&0&0\\ 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0 \end{bmatrix}\begin{bmatrix} u_1\\ v_1\\ u_2\\ v_2\\ u_3\\ v_3\\ u_4\\ v_4 \end{bmatrix}=\begin{bmatrix} 0\\ 0\\ 0\\ -150000\\ 0\\ 0\\ 0\\ 0 \end{bmatrix}$$

Global Equations: We proceed above steps till to find the global equations.

$$\begin{bmatrix} 32600 & 76067 & -32600 & -76067 & 0 & 0 & 0 & 0\\ 76067 & 297489 & -76067 & -177489 & 0 & -120000 & 0 & 0\\ -32600 & -76067 & 540073 & -177850 & -329984 & 329984 & -17789 & -76067\\ -76067 & -177489 & -177850 & 540073 & 329984 & -329984 & -76067 & -32600\\ 0 & 0 & -329984 & 329984 & 449984 & -329984 & -120000 & 0\\ 0 & -120000 & 329984 & -329984 & -329984 & 449984 & 0 & 0\\ 0 & 0 & -17789 & -76067 & -120000 & 0 & 297489 & 76067\\ 0 & 0 & -76067 & -32600 & 0 & 0 & 76067 & 32600 \end{bmatrix}\begin{bmatrix} u_1\\ v_1\\ u_2\\ v_2\\ u_3\\ v_3\\ u_4\\ v_4 \end{bmatrix}=\begin{bmatrix} 0\\ 0\\ 0\\ -150000\\ 0\\ 0\\ 0\\ 0 \end{bmatrix}$$

Boundary Conditions: Nodes 1 and 4 of the truss have pin supports.

Table 3. The essential boundary conditions are

Node	Degrees Of Freedom	Value
1	u_1 v_1	0 0
4	u_4 v_4	0 0

After deleting equations (1, 2, 7, 8), we have

$$\begin{bmatrix} -32600 & -76067 & 540073 & -177850 & -329984 & 329984 & -17789 & -76067 \\ -76067 & -177489 & -177850 & 540073 & 329984 & -329984 & -76067 & -32600 \\ 0 & 0 & -329984 & 329984 & 449984 & -329984 & -120000 & 0 \\ 0 & -120000 & 329984 & -329984 & -329984 & 449984 & 0 & 0 \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ u_2 \\ v_2 \\ u_3 \\ v_3 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} 0 \\ -150000 \\ 0 \\ 0 \end{bmatrix}$$

Since all entries in columns (1,2,7, 8) are multiplied by zero nodal values, they can be removed. The final global system of equations is thus as follows:

$$\begin{bmatrix} 540073 & -177850 & -329984 & 329984 \\ -177850 & 540073 & 329984 & -329984 \\ -329984 & 329984 & 449984 & -329984 \\ 329984 & -329984 & -329984 & 449984 \end{bmatrix} \begin{bmatrix} u_2 \\ v_2 \\ u_3 \\ v_3 \end{bmatrix} = \begin{bmatrix} 0 \\ -150000 \\ 0 \\ 0 \end{bmatrix}$$

Nodal Values: Solving the final system of global equations, we get the following nodal values:

u_2 = 0.538954, v_2 = -0.953061, u_3 = 0.264704, v_3 = -0.264704

Table 4. Complete Nodal Values

	u	***v***
1	0	0
2	0.538954	-0.953061
3	0.264704	-0.264704
4	0	0

Computation of Reactions: Equation numbers of degree of freedom with specified values: (1,2,7, 8) Extracting equations (1,2,7, 8) from the global system, Substituting the nodal values and rearranging we have

$$\begin{bmatrix} R_1 \\ R_2 \\ R_3 \\ R_4 \end{bmatrix} = \begin{bmatrix} 32600.2 & 76067.2 & -32600.2 & -76067.2 & 0 & 0 & 0 & 0 \\ 76067.2 & 297490 & -76067.2 & -177490 & 0 & -120000 & 0 & 0 \\ 0 & 0 & -177490 & -76067.2 & -120000 & 0 & 297490 & 76067.2 \\ 0 & 0 & -76067.2 & -320600.2 & 0 & 0 & 76067.2 & 32600.2 \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ 0.538954 \\ -0.953061 \\ 0.264704 \\ -0.264704 \\ 0 \\ 0 \end{bmatrix}$$

Table 5. Reactions

Lebel	Degree of Freedom	Reaction
R_1	u_1	54926.7
R_2	v_1	159927
R_3	u_4	-54926.7
R_4	v_4	-9926.67

Sum of reactions:

dof:	*u*	0
dof:	*v*	*150000.*

There is no applied load in the *x* direction. Since the sum of reactions in the horizontal direction is zero, the equilibrium is satisfied in this direction. There is an applied load of 150000 in the -*y* direction which balances with the sum of reactions in the *y* direction, indicating that equilibrium is satisfied in this direction as well. Hence the solution satisfies the overall equilibrium.

The displacements are in inches, loads in pounds, and stresses in pounds per inch squared.

The solution for element 1 is as follows:

Element nodal coordinates:

First node (node #1): (0,0);

Second node (node #2): (1500.,3500.)

x_1 =0; y_1 =0; x_2 =1500.; y_2 =3500.

Length of the element: L = 3807.89;

Direction Cosines: $l_s = 0.393919$,

$m_s = 0.919145$

Global to local transformation matrix

$$T = \begin{pmatrix} 0.393919 & 0.919145 & 0 & 0 \\ 0 & 0 & 0.393919 & 0.919145 \end{pmatrix}$$

Element nodal displacements in global coordinates,

$$d = \begin{bmatrix} u_1 \\ v_1 \\ u_2 \\ v_2 \end{bmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0.538954 \\ -0.953061 \end{pmatrix}$$

Element nodal displacements in local coordinates,

$$d_l = Td = \begin{pmatrix} 0 \\ -0.663697 \end{pmatrix}$$

Axial displacements at element ends, d1=0 , d2= -0.663697

Axial strain = -0.000174'295

Axial stress = -34.8591;

Axial force = -139436

For any other element the calculations follow exactly the same pattern.

Table 6. Solution summary of five bar truss problem

	Stress	**Axial force**
1	-34.8591	-139436
2	-6.29994	-25199.8
3	-10.5881	-31764.4
4	-10.5881	-31764.4
5	22.4608	44921.7

FIVE BAR TRUSS PROBLEM WITH AN INCLINED SUPPORT

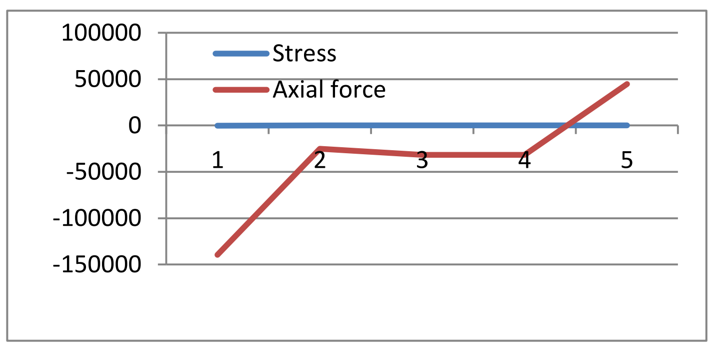

Figure 4. Graph of five bar trusses for stress and axial force

Consider the five-bar pin-jointed structure shown in 4. All members have the same cross-sectional area and are of the same material, E =70 GPa, and A =10^{-3} m^2. The load P =20 kN. The dimensions in meters are shown in the figure.

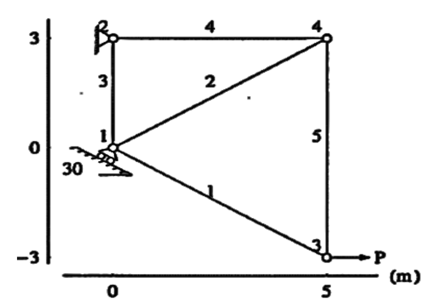

Figure 5. Five bar truss problem with an inclined support

Determine axial strains, axial stresses, and axial forces in different elements of the truss. The global system of equations is as follows

$$\begin{pmatrix} 17654.3 & 0 & 0 & 0 & -8827.13 & 5296.28 & -8827.13 & -5296.28 \\ 0 & 29688.9 & 0 & -23333.3 & 5296.28 & -3177.77 & -5296.28 & -3177.77 \\ 0 & 0 & 14000 & 0 & 0 & 0 & -14000 & 0 \\ 0 & -2333.3 & 0 & 2.3333.3 & 0 & 0 & 0 & 0 \\ -8827.13 & 5296.28 & 0 & 0 & 8827.13 & -5296.28 & 0 & 0 \\ 5296.28 & -3177.77 & 0 & 0 & -5296.28 & 14844.4 & 0 & -11666.7 \\ -8827.13 & -5296.28 & -14000 & 0 & 0 & 0 & 22827.1 & 5296.28 \\ -5296.28 & -3177.77 & 0 & 0 & 0 & -11666.7 & 5296.28 & 14844.4 \end{pmatrix} \begin{bmatrix} u_1 \\ v_1 \\ u_2 \\ v_2 \\ u_3 \\ v_3 \\ u_4 \\ v_4 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 20000 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

The multipoint constraint due to inclined support at node 1 is $u_1 \sin(\pi/6) + v_1 \cos(\pi/6) = 0$.

The augmented global equations with the Lagrange multiplier are as follows:

$$\begin{pmatrix} 17654.3 & 0 & -8827.13 & 5296.28 & -8827.13 & -5296.28 & 0.5 \\ 0 & 29688.9 & 5296.28 & -3177.77 & -5296.28 & -3177.77 & 0.866 \\ -8827.13 & 5296.28 & 8827.13 & -5296.28 & 0 & 0 & 0 \\ 5296.28 & -3177.77 & -5296.28 & 14844.4 & 0 & -11666.7 & 0 \\ -8827.13 & -5296.28 & 0 & 0 & 22827.1 & 5296.28 & 0 \\ -5296.28 & -31.77.77 & 0 & -11666.7 & 5296.28 & 14844.4 & 0 \\ 0.5 & 0.866 & 0 & 0 & 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} u_1 \\ v_1 \\ u_3 \\ v_3 \\ u_4 \\ v_4 \\ \lambda \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 20000 \\ 0 \\ 0 \\ 0 \\ 0 \end{pmatrix}$$

Solving the final system of global equations, we get $u_1 = 5.14286$, $v_1 = -2.96923$, $u_3 = 16.8629$, $v_3 = 12.788$, $u_4 = -1.42857$, $v_4 = 11.7594$, $\lambda = 80000$.

Ina similar manner, we can compute the solutions over the remaining elements:

Table 7. Solution summary of five bar truss problem with an inclined support [14]

	Stress	Axial force
1	23.3238	23323.8
2	23.3238	23323.8
3	69.282	69282
4	-20	-20000
5	-12	-12000

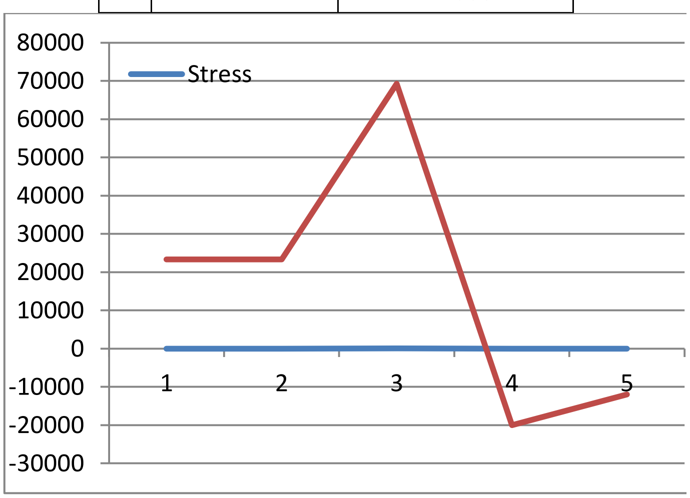

Figure 6. Graph of five bar truss problem with an inclined support for stress and axial force

TRUSS SUPPORTING A RIGID PLATE

A plane truss is designed to support a rigid triangular plate [13] as shown in Figure 6. All members have the same cross-sectional area A = 1 in2 and are of the same material, E = 29,000 ksi. The load P = 20 kips. The dimensions in ft are shown in the figure. Note there is no connection between the diagonal Determine axial strains, axial stresses, and axial forces in different elements of the truss members where they cross each other.

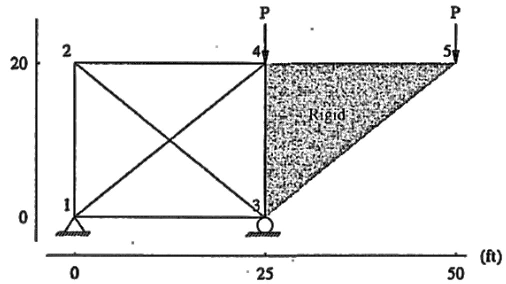

Figure 7. Truss supporting a rigid triangular plate

$$\begin{pmatrix} 142.693 & -36.8215 & -46.0268 & 36.8215 & -96.6667 & 0 & 0 & 0 \\ -36.8215 & 150.29 & 36.8215 & -29.4572 & 0 & 0 & 0 & 0 \\ -46.0268 & 36.8215 & 142.693 & -36.8215 & 0 & 0 & 0 & 0 \\ 36.8215 & -29.4572 & -36.8215 & 150.29 & 0 & -120.833 & 0 & 0 \\ -96.6667 & 0 & 0 & 0 & 142.693 & 36.8215 & 0 & 0 \\ 0 & 0 & 0 & -120.833 & 36.8215 & 150.29 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} u_2 \\ v_2 \\ u_3 \\ v_3 \\ u_4 \\ v_4 \\ u_5 \\ v_5 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ -20 \\ 0 \\ -20 \end{pmatrix}$$

To use the penalty function approach [16], we choose the penalty parameter μ equal to 10^5 times the largest number in the global K matrix, incorporating the constraints into the global equations with this value of μ.

Table 8. Solution summary of truss supporting a rigid triangular plate

	Strain	Stress / Axial Force
1	0.000318525	9.23722
2	-0.000463913	-13.4535
3	0.000594098	17.2289
4	0.000398156	11.5465
5	-0.00050989	-14.7868
6	8.52877×10^{-9}	0.000247334

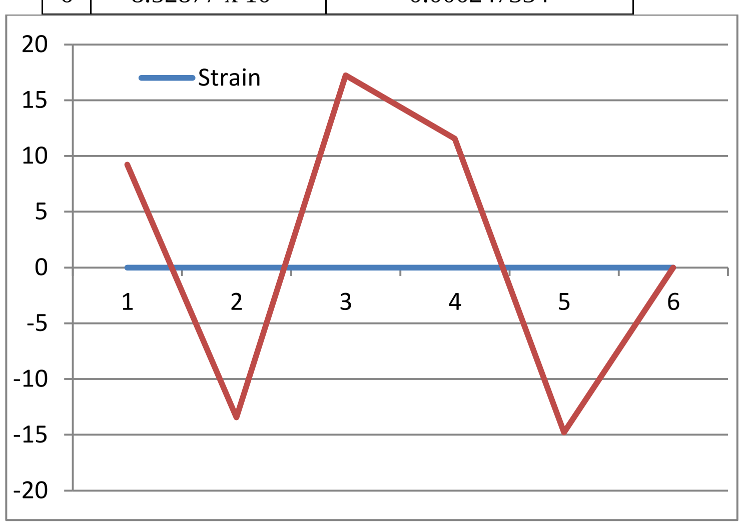

Figure 8. Graph of truss supporting a rigid triangular plate for strain and axial force

CONCLUSION

The Lagrange multiplier method of imposing constraints has two drawbacks. First, it requires adding new rows and columns to the global system of equations that for large systems may be inefficient. Second, the resulting system has zeros on the diagonal corresponding to the constraint equations. Some simple equation solvers that assume nonzero diagonal terms may not work for this system. Another standard technique of imposing constraints, the so-called penalty function approach, does not have these two drawbacks. The performance of the method depends on the value chosen for the penalty parameter μ. Large values, say of the order of $\mu = 10^{10}$, give accurate solutions; General rule of thumb is to set μ equal to 10^5 times the largest number in the global K matrix. The technique of penalty function approach in finite element analysis is batter then Lagrange multiplier method. We used five bar plane trusses, five bar truss problem with an inclined and Truss Supporting a rigid plate, by using the finite element method we calculated element equations, global equations, incorporate essential boundary conditions into the system and obtain the final reduced system of equations in terms of the unknowns, calculated nodal values, compute reactions and verify overall equilibrium and also determine axial strains, axial stresses, and axial forces in different elements of the truss.

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