REGULAR AG-GROUPOIDS IN TERMS OF HESITANT FUZZY SETS

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ABSTRACT: *In the present paper, we have studied the concept of hesitant fuzzy ideals (right, left), hesitant fuzzy bi-ideals, hesitant fuzzy interior ideals and hesitant fuzzy quasi-ideals in term of regular AG-groupoids.*

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1. INTRODUCTION

An AG-groupoid is also called LA-semigroup. An AG-groupoid is the speculation of commutative semigroup which was initially presented by Kazim and Naseeruddin in 1972. An AG-groupoid G is said to be regular if for each y in G there exists some z in G such that y = (yz)y. If G is regular AG-groupoid, then it is easy to see that G =G^2 [10, 11, 19, 20]. The theory of a fuzzy set of a set was first time introduced by Zadeh [31]. The concept of fuzzy sets in the form of groups was first introduced by Rosenfelt in 1971. The fuzzy semigroups were first studied by Kuroki in 1979. Different approaches have been presented later. The fuzzification of LA-semigroups were considered by Zanib in [33] and then by many others, for detail see [3, 4, 5, 13, 15 17, 25, 27, 30] and [1, 2, 6, 8, 14, 22, 23, 24, 26, 28, 29]. A hesitant fuzzy set abbreviated by (HFSs) can be defined in the term of a function that at the point when connected to a set X again put back a subset of [0, 1]. Hesitant fuzzy set is a useful generalization of the fuzzy set. The hesitant fuzzy set pernicious the membrane unit of a component to a set to be spoken to by an arrangement of conceivable qualities space 0 and 1. Similar to the situations of the hesitant fuzzy set where a decision maker may hesitate between several possible values as the membership degree when evaluating an alternative, in a qualitative circumstance, a decision maker may hesitate between several terms to assess a linguistic variable. This hesitant fuzzy set hypothesis has been used in many functional issues. Hesitant fuzzy set hypothesis has been connected to numerous down to earth issues, basically in the range of choice making. For more detail on hesitant fuzzy sets see [9, 21]. Gulistan et al. studied hesitant fuzzy AG-groupoids [7].

In this present paper, we have studied the concept of hesitant fuzzy ideals (right, left), hesitant fuzzy bi-ideals, hesitant fuzzy interior ideals and hesitant fuzzy quasi ideals in term of regular AG-groupoids.

2. Preliminaries

In this section, we give some basic definitions and results, which will be used in our main section. Throughout the paper G stands for an AG-groupoid.

Definition 2.1: [10, 11] A groupoid G is said to be an Abel Grassmann's groupoid , abbreviated an AG-groupoid if all the elements of G satisfy left invertive law.

Definition 2.2: [10, 11] In G the average (medial) law holds, such that

$(mn)(op) = (po)(nm),\ \forall\ m,\ n,\ o,\ p \in G.$

Definition 2.3: [10,11] If $\phi \neq \mathrm{B} \subseteq G,$ then B is said to be a sub AG-groupoid of $G,$ if $\mathrm{B} \circ \mathrm{B} \subseteq \mathrm{B}.$

Definition 2.4: [10,11] Let G be an AG-groupoid and $\phi \neq \mathrm{B}r \subseteq G\ (\phi \neq \mathrm{B}l \subseteq G)$ is called a right and left ideal of G if $\mathrm{B}r \circ G \subseteq \mathrm{B}r$ ($G \circ \mathrm{B}l \subseteq \mathrm{B}l$) and it is called an ideal if it isboth left and a right ideal of G .

Definition 2.5: [10,11] $\phi \neq \mathrm{B}_b \subseteq G$ a sub AG-groupoid of G is called a bi-ideal of G if $(\mathrm{B}_b \circ G) \circ \mathrm{B}_b \subseteq \mathrm{B}_b.$

Definition 2.6: [10,11] $\phi \neq \mathrm{B}_q \subseteq G$ is called a quasi-ideal of G if $(G \circ \mathrm{B}_q) \cap (\mathrm{B}_q \circ G) \subseteq \mathrm{B}_q.$

Definition 2.7: [10,11] $\phi \neq \mathrm{B} \subseteq G$ is called an interior ideal of G if $(G \circ \mathrm{B}) \circ G \subseteq \mathrm{B}.$

Lemma 2.8: [10, 11] Let G be an regular AG-groupoid. Then every left ideal is a right ideal in G.

Lemma 2.9: [10, 11] Let G be a regular AG-groupoid and let $\mathrm{B}r$ be a right ideal and $\mathrm{B}l$ be a left ideal of G. Then, $\mathrm{B}r \circ \mathrm{B}l = \mathrm{B}r \cap \mathrm{B}l.$

Lemma 2.10: [10,11] Let G be a regular AG-groupoid and let $\mathrm{B}r$ be a right ideal of G. Then $\mathrm{B}r = \mathrm{B}r^2.$

Theorem 2.11: [32] Let G ba an AG-groupid with left identity e such that (xe)G=xG for all x∈G. Then the following are equivalent:

(1) G is regular.
(2) $H_1 \cap H_2 = H_1H_2$ for every right ideals H_1 and left ideals H_2 of G.
(3) A=(AG)A for every quasi ideal A of G.

Theorem 2.12: [32] Let G ba an AG-groupid with left identity e such that (xe)G=xG for all x∈G. Then the following are equivalent:

(1) G is regular.
(2) A=(AG)A for every quasi ideal A of G .
(3) A=(AG)A for every generalized bi- ideal A of G.

For more detail see [32].

Definition 2.13: [32] A fuzzy subset μ of an AG-groupoid G is a function of G into the closed unit interval [0, 1] that is, $\mu : G \to [0,1]$.

Lemma 2.14: [32] Let G be a regular AG-groupoid. Then every fuzzy left ideal is a fuzzy right ideal in G.

Lemma 2.15: [32] Let G be a regular AG-groupoid and let f be a fuzzy right ideal and g be a fuzzy left ìdeal of G. Then, $f \circ g = f \cap g$.

Lemma 2.16: [32] Let f be a fuzzy right ìdeal of G. Then $f = f^2$.

Theorem 2.17: [32] Let G ba an AG-groupid with left identity e such that (xe)G=xG for all x∈G. Then the following are equivalent:

(1) G is regular,

(2) f = (f G) f for every fuzzy quasi ideal f of G.

(3) f = (f G) f for every fuzzy generalized bi-ideal f of G.

Definition 2.18: [7]: Let H be a hesitant fuzzy set of AG-groupoid G. Then we have

$$H = H(a), H_a^b = H(a) \cap H(b),$$

$$H_a^b[c] = \big(H(a) \cap H(b)\big) \cap H(c).$$

$H_a = H_b \Leftrightarrow H_a \subseteq H_b,\ \ H_b \subseteq H_a$, for all $a,b \in G$.

Definition 2.19: [7]: Let H_1 and H_2 be two hesitant fuzzy sets of G . Then hesitant union $H_1 \cup H_2$ and hesitant intersection $H_1 \cap H_2$ of H_1 and H_2 are define to be hesitant fuzzy sets of G as follow:

$$H_1 \cup H_2 : \mathrm{G} \to P([0,1]),$$

$$\mathrm{a} \to H_{1a} \cup H_{2a} = \max\{H_{1a},\ H_{2a}\}$$

$$H_1 \cap H_2 : \mathrm{G} \to P([0,1]),$$

$$\mathrm{a} \to H_{1a} \cap H_{2a} = \min\{H_{1a},\ H_{2a}\}.$$

Definition 2.20: [7]**:** A hesitant fuzzy set H of an AG-groupoid G is called a hesitant fuzzy sub AG-groupoid of G if it satisfies: $H_{ab} \supseteq H_a^b = H_a \cap H_b,\ \forall\ a,b \in G.$

Definition 2.21: [7]**:** A hesitant fuzzy set H of G is called a hesitant fuzzy left (right) ideal of G if it satisfies:

$$H_{ab} \supseteq H_a (H_{ab} \supseteq H_b),\ \forall\ a,b \in G.$$

Definition 2.22:[7]**:** Let H_1 and H_2 be two hesitant fuzzy sets of G . Then their product defined as follows:

$$(H_1 \circ H_2)_a = \begin{cases} \bigcup\limits_{a=bc} \{H_{1b} \cap H_{1c}\} \\ \phi \quad otherwise. \end{cases}$$

Definition 2.23: [7]**:** A hesitant fuzzy subset H of an AG-groupoid G is called a hesitant fuzzy bi-ideal of G if $H_{(\mathrm{xy})\mathrm{z}} \supseteq H_x \cap H_z,\ for\ all\ x,y,z \in G.$

Definition 2.24: [7]**:** A hesitant fuzzy subset H of an AG-groupoid G is called a hesitant fuzzy interior ideal of G if $H_{(\mathrm{xa})\mathrm{y}} \supseteq H_a,\ for\ all\ a,b,c \in G.$

Definition 2.25: [7]**:** A hesitant fuzzy set H of G is called a hesitant fuzzy quasi ideal of G if the following condition is valid:

$$(H \circ G) \cap (G \circ H) \subseteq H.$$

3.Hesitant Regular Abel-Grassmann's Groupoids

In this section, we characterize regular AG-groupoids by hesitant fuzzy left ideal, hesitant fuzzy right ideal, hesitant fuzzy bi-ideal, hesitant fuzzy interior ideal, hesitant fuzzy quasi-ideal and hesitant fuzzy generalized bi-ideal.

Lemma 3.1: In an AG-groupoid G, with left ìdentity e, for every hesitant fuzzy left ideal H of G we have $(GoH) = H$.

Proof: Since $H \subseteq GoH$. Now for any $x \in G$,

$$(G \circ H)_x = \bigcup_{x=ab} \{G_a \cap H_b\}$$

$$= \bigcup_{x=ab} \{[0,1] \cap H_b\} \subseteq \bigcup_{x=ab} H_b = H_x.$$

Thus $(G \circ H) = H$.

Theorem 3.2: For any $\phi \neq A \subseteq G$, the following are equivalent:

(a) A is a left ideal of G.

(b) The characteristic hesitant fuzzy set $[H_C]$ of G is a hesitant fuzzy left ideal on G.

Proof: Straightforward.

Corollary 3.3: Let $\phi \neq A \subseteq G$. *T*hen the following are equivalent:

(a) A is a two-sided ideal of G.

(b) The set $[H_A]$ on G is a hesitant fuzzy two-sided ideal of G.

Proof. Straightforward.

Lemma 3.4: Let $\phi \neq A_1,\ \mathrm{A}_2 \in G$ and let $[H_{A_1}]$ and $[H_{A_2}]$ be characteristic hesitant fuzzy sets of G. Then the following properties hold:

(a) $\left[H_{A_1}\right]\cap\left[H_{A_2}\right]=\left[H_{A_1\cap A_2}\right].$

(b) $\left[H_{A_1}\right]\circ\left[H_{A_2}\right]=\left[H_{A_1A_2}\right].$

Proof. (a) If $a\in A_1\cap A_2\Rightarrow a\in A_1$ and $a\in A_2,\ \forall$ a $\in G.$ Then

$$\left(\left[H_{A_1}\right]\cap\left[H_{A_2}\right]\right)_a=\left[H_{A_1}\right]_a\cap\left[H_{A_2}\right]_a=[0,1]=\left[H_{A_1\cap A_2}\right]_a.$$

If $a\notin A_1\cap A_2$, then $a\notin A_1$ and $a\notin A_2$.

So we have

$$\left(\left[H_{A_1}\right]\cap\left[H_{A_2}\right]\right)_a=\left(\left[H_{A_1}\right]\right)_a\cap\left(\left[H_{A_2}\right]\right)_a=\phi=\left(\left[H_{A_1\cap A_2}\right]\right)_a.$$

Therefore $\left[H_{A_1}\right]\cap\left[H_{A_2}\right]=\left[H_{A_1\cap A_2}\right].$

(b) If $a\in G$, assume that, $a\in A_1A_2$.

Then there exist $b\in A_1$ and $c\in A_{2,}$ such that $a=bc.$

$$\left(\left[H_{A_1}\right]\circ\left[H_{A_2}\right]\right)_a=\bigcup_{a=bc}\left\{\left(\left[H_{A_1}\right]\right)_b\left(\left[H_{A_2}\right]\right)_c\right\}$$

$$\supseteq\left[H_{A_1}\right](b)\cap\left[H_{A_2}\right](c)=[0,1].$$

$$\Rightarrow\left(\left[H_{A_1}\right]\circ\left[H_{A_2}\right]\right)_a=[0,1].$$

If $a\neq bc,\ \forall\ b,c\in G,$ then $\left(\left[H_{A_1}\right]\circ\left[H_{A_2}\right]\right)_a=\phi=\left(\left[H_{A_1A_2}\right]\right)_a.$

So in any case, we have $\left[H_{A_1}\right]\circ\left[H_{A_2}\right]=\left[H_{A_1A_2}\right].$

Theorem 3.5: Let H be a hesitant fuzzy set of G and $\left[H_G\right]$ be an identity hesitant fuzzy set of G. Then the following are equivalent:

(a) H is a hesitant fuzzy right ideal of G.

(b) $H\circ\left[H_G\right]\subseteq H.$

Proof. Suppose that H is a hesitant fuzzy right ideal of G. Let $u\in G$, such that $u=vw.$ Then we have

$$\left(H\circ\left[H_G\right]\right)_u=\bigcup_{o=vw}\left\{H_v\cap\left(\left[H_G\right]\right)_w\right\}\subseteq\bigcup_{o=vw}\left\{H_v\cap[0,1]\right\}=H_u.$$

Otherwise, we have,

$$\left(H\circ\left[H_G\right]\right)_u=\phi\subseteq H_u\Rightarrow\left(H\circ\left[H_G\right]\right)_u\subseteq H_u.$$

Conversely, suppose that,

$H\circ\left[H_G\right]\subseteq H_u.$ Let $u,v\in G,$ we have

$$H_{uv}\supseteq\left(H\circ\left[H_G\right]\right)_{uv}\supseteq H_u\cap\left(\left[H_G\right]\right)_v$$
$$=H_u\cap[0,1]=H_u\Rightarrow H_{uv}\supseteq H_u.$$

Which show that H is a hesitant fuzzy right ideal on G.

Corollary 3.6: Let H be a hesitant fuzzy two-sided ideal of G and $\left[H_G\right]$ be an identity hesitant fuzzy set of G, then the following are equivalent:

(a) H is a hesitant fuzzy two-sided on G.

(b) $\left[H_G\right]\circ H\subseteq H$ and $H\circ\left[H_G\right]\subseteq H.$

Theorem 3.7: Let H_1 and H_2 be hesitant fuzzy set of G. If H_1 is a hesitant fuzzy left ideal of G, then so is the hesitant fuzzy product $H_1\circ H_2$.

Proof. Let $a,b\in G.$ If $b=yz,\ \forall\ c,d\in G,$ then $ab=a(yz)=(ay)z.$

$$\left(H_1\circ H_2\right)_b=\bigcup_{b=yz}\left\{H_{1y}\cap H_{2z}\right\}\subseteq\bigcup_{ab=(ay)z}\left\{H_{1ay}\cap H_{2z}\right\}$$
$$\subseteq\bigcup_{ab=yz}\left\{H_{1y}\cap H_{2e}\right\}=\left(H_1\circ H_2\right)_{ab}.$$

If $b\neq yz$, then

$$\left(H_1\circ H_2\right)_b=\phi\subseteq\left(H_1\circ H_2\right)_{ab}.$$

Thus,

$$\left(H_1\circ H_2\right)_{ab}\supseteq\left(H_1\circ H_2\right)_b,\ \forall\ a,b\in G.$$

Corollary 3.8: Let H_1 and H_2 be hesitant fuzzy sets of G. If H_1 and H_2 are two hesitant fuzzy left and right ideals respectively, then there product, $H_1\circ H_2$ is also a hesitant fuzzy two-sided ideal of G.

Theorem 3.9: Let H_1 and H_2 is a hesitant fuzzy right ideal and left ideal respectively of G. Then $H_1\circ H_2\subseteq H_1\cap H_2$.

G. Then $H_1\circ H_2\subseteq H_1\cap H_2$.

Proof. If $a=bc,\ \forall\ a,b,c\in G.$ Then

$$\left(H_1\circ H_2\right)_a=\bigcup_{a=bc}\left\{H_{1b}\cap H_{2c}\right\}\subseteq\bigcup_{a=bc}\left\{H_{1bc}\cap H_{2bc}\right\}$$
$$=H_{1a}\cap H_{2a}=\left(H_1\cap H_2\right)_a.$$

So in any case, we have $H_1\circ H_2\subseteq H_1\cap H_2$.

Lemma 3.10: If H is a hesitant fuzzy left ideal in an AG-groupoid G with left identity e, then H being hesitant fuzzy interior ideal is a hesitant fuzzy bi- ideal of G.

Proof. Since H is a hesitant fuzzy left ideal in G,

$$H_{xy}\supseteq H_y,\ \forall\ \mathrm{x,y}\in\mathrm{G}.$$

As e is left identity in G.

So $H_{xy}=H_{(ex)y}\supseteq H_x.$ Which

Implies that $H_{xy} \supseteq H_x \cap H_y, \ \forall$ x,y $\in$ G.

Thus H is a hesitant fuzzy sub AG-groupoid. Consider, for any x, y and z in G. Then

$$H_{(xy)z} = H_{(x(ey))z} = H_{(e(xy))z} \supseteq H_{xy} = H_{(ex)y} \supseteq H_x.$$

also

$$H_{(xy)z} = H_{(zy)x} = H_{(z(ey))x}$$

$$= H_{(e(zy))x} \supseteq_{zy} = H_{(ez)y} \supseteq H_z.$$

Thus,

$$H_{(xy)z} \supseteq H_x \cap H_z, \ x, y \in G.$$

Lemma 3.11: If H_1 and H_2 is a hesitant fuzzy quasi-ideal of G. Then their product $H_1 \circ H_2$ is a hesitant fuzzy bi-ideal of G.

Proof. We want to show that $(H_1 \cap H_2) \circ (H_1 \cap H_2) \subseteq H_1 \circ H_2$. If H is a hesitant fuzzy quasi-ideal, then $H_1 \circ G \circ H_2 \subseteq H_1$. Then

$$(H_1 \circ H_2) \circ (H_1 \circ H_2) = ((H_1 \circ H_2) \circ H_1) \circ H_2$$

$$\subseteq ((H_1 G_2) H_1) H_2 \subseteq H_1 \circ H_2$$

$$\Rightarrow (H_1 \circ H_2) \circ (H_1 \circ H_2) \subseteq H_1 \circ H_2.$$

Theorem 3.12: Every hesitant fuzzy two-sided ideal of a regular AG-groupoid G is Idempotent.

Proof. Let H be a hesitant fuzzy two-sided ideal of G and we want to show that $H = H \circ H$. Since G is regular, so we must have, $H \circ G \circ H = H$. As every hesitant fuzzy two-sided ideal of G is a hesitant fuzzy generalised bi-ideal. So $H = (H \circ G \circ H) \subseteq H \circ H$. And also we know that $H \circ H \subseteq H \circ G \subseteq H$. Thus $H = H \circ H$.

Lemma 3.13: Let G be a regular AG-groupoid. Then every hesitant fuzzy right ideal is a hesitant fuzzy left ideal of G.

Proof. Straightforward.

Theorem 3.14: Let H is a hesitant fuzzy right ideal and β is a hesitant fuzzy set of a regular AG-groupoid G. Then $H \cap \beta \subseteq H \circ \beta$.

Proof. Let $t \in G$, then $\exists$ r $\in H$, such that $(tr)t = t$, since G is regular. Then

$$(H \circ \beta)_t = \bigcup_{t=no} \{H_n \cap \beta_o\} = \bigcup_{(rt)t=no} \{H_n \cap \beta_o\}$$

$$\supseteq H_t \cap \beta_t \supseteq H_t \cap \beta_t, \text{ as } H_{rt} \supseteq H_t$$

$$= (H \cap \beta)_t.$$

Thus $H \cap \beta \subseteq H \circ \beta$.

Corollary 3.15: Let H_r be a hesitant fuzzy right ideal and H_l is a hesitant fuzzy left ideal of a regular AG-groupoid G. Then $H_r \cap H_l = H_r \circ H_l$.

Lemma 3.16: Let H be a hesitant fuzzy subset of a regular AG-groupoid G. Then the following are identical:

(a) H is a hesitant fuzzy ideal of G.

(b) H is a hesitant fuzzy interior ideal of G.

Proof. $(a) \Rightarrow (b)$: Suppose H is a hesitant fuzzy ideal of G. Then we want to show that H is a hesitant fuzzy interior ideal of G.

$$((G \circ H) \circ G)_t = \bigcup_{t=no} \{((G \circ H)_n \cap G)_o\}$$

$$= \bigcup_{t=no} \left\{ \bigcup_{n=pq} \{G_p \cap H_q\} \cap G_o \right\}$$

$$= \bigcup_{t=no} \left\{ \bigcup_{n=pq} \{[0,1] \cap H_q\} \cap [0,1] \right\}$$

$$= \bigcup_{t=(pq)o} H_q \subseteq H_t.$$

$$\Rightarrow G \circ H \circ G \subseteq H.$$

Hence H is a hesitant fuzzy interior ideal of G.

$(b) \Rightarrow (a)$: Let H be a hesitant fuzzy interior ideal of G. Then we want to show that H is a hesitant fuzzy ideal of G . Let $a, b \in G$. Then there exists $n, y \in G$, such that $a = (an)a$ and $b = (by)b$.

$H_{ab} = H_{((an)a)b} = H_{((b)a)an} \supseteq H_a$. Because H is a hesitant fuzzy interior ideal. Thus H is a hesitant fuzzy right ideal of G. $H_{ab} = H_{a(bo)b} \supseteq H_b$. Because H is a hesitant fuzzy interior ideal. Thus H is hesitant fuzzy left ideal of G.

Theorem 3.17: If G be an AG-groupoid, then the following are identical:

(a) G is regular.

(b) $H = HoGoH$ for each hesitant fuzzy generalized bi-ideal H of G.

(c) $H = HoGoH$ for each hesitant fuzzy bi-ideal H of G.

(d) $H = HoGoH$ for each hesitant fuzzy quasi-ideal H of G.

Proof. Straightforward.

Theorem 3.18: Let G be an AG-groupoid such that *(xe)G* = *xG* for all *x*∈*G*. Then the following are equivalent:

(a) G is regular.

(b) $H_1 \cap H_2 = (H_1 o H_2) o H_1$ for each hesitant fuzzy quasi ideal H_1 and every hesitant fuzzy two sided ideal

H_2 of G.

(c) $H_1 \cap H_2 = (H_1 o H_2) o H_1$ for each hesitant fuzzy quasi ideal H_1 and every hesitant fuzzy interior ideal H_2 of G.

(d) $H_1 \cap H_2 = (H_1 o H_2) o H_1$ for each hesitant fuzzy bi ideal H_1 and every hesitant fuzzy two sided ideal H_2 of G.

(e) $H_1 \cap H_2 = (H_1 o H_2) o H_1$ for each hesitant fuzzy bi ideal H_1 and every hesitant fuzzy interior ideal H_2 of G.

(f) $H_1 \cap H_2 = (H_1 o H_2) o H_1$ for each hesitant fuzzy generalized bi ideal H_1 and every hesitant fuzzy two sided ideal H_2 of G.

Proof: Straightforward.

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