

# TO CALCULATE EFFECTIVE PARTICLES OF LONG CHAIN POLYMER ON VELOCITY AND FLOW RATE USING MATHEMATICAL MODEL

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**ABSTRACT:** In this paper, theoretical analysis of effective particles of long-chain polymer on performance improvement of the human joint is presented. mathematical model depended on a laminar steady flow of synovial fluid that it contains on hyaluronic acid between articular cartilage in( hip- knee –ankle) joint, where Reynolds equation governing the fluid film pressure was derived and solved analytically. Quasi-statics equilibrium equation determined friction force during daily activities, the influence of film thickness and squeeze action, length of chain and speed on the squeeze film characteristics were discussed. It has been found that the effect of the decreased film thickness of particle leads to an increase in the load carry capacity, friction force and decreased flow rate. It is observed that the effect of length of a particle on pressure film between articular surface with sliding motion is to increase.

**Keyword:** Hyaluronic acid, length of chain, Quasi-statics equilibrium equation, Reynolds equation.

## INTRODUCTION:

Synovial fluid is a thick, viscous fluid found in the cavities of synovial human joints. It reduces friction and wear between the articular cartilage and other tissues in joints by lubricating and pillowing them during daily activities. Normal synovium contains synovial lining cells that are 1-3 cells deep [1]. The synovial fluid in the joint has many functions: works as a candidate, allowing nutrients to reach the tendons and cartilage, but prevents the passage of cells and harmful substances and eliminates harmful inflammatory proteins, which reduces pain and swelling when the joint is injured. Synovium lines all intracapsular frame except the contact areas of articular cartilage. The main difference between synovial fluid and other body fluids derived from plasma is the high content of hyaluronic acid in synovial fluid. The normal viscosity of synovial fluid is due to hyaluronic acid [2]. The hyaluronic acid see figure (1.1) is a substance in the body produced by the cells naturally and distributed to the rest of the tissues of the body, the natural person has a proportion of hyaluronic acid about 15 grams, It is responsible for the flexibility of the body and its ability to move to accomplish daily activities[3]. Hyaluronic acid has a tremendous ability to absorb water, absorbing up to 1000 times its weight from water. It attracts water like a sponge, holds it in the skin and works as a reservoir: it holds water and redistributes it as needed. Changes in the concentration of hyaluronate or lubricin in the synovial fluid will affect the overall lubrication and the amount of friction that is present [4]. Many experiments have confirmed that articular coefficients of friction in synovial joints are far lower than those created with manufactured lubricants [5] [8]. The lower the coefficient of friction, the lower the resistance to movement. Injections of hyaluronate and lubricin have been used successfully to alleviate symptoms of osteoarthritis in both hip and knee joints. In, This study, we show that important particle of the long-chain polymer of synovial fluid in support improves performance of synovial joints so determine squeezed action and sliding motion for particle [6]



Figure (1.1): particle of hyaluronic acid in joint human[7]

## 1.1 Module of the particle of long-chain polymer:

Mathematical module two was designed of particle of long-chain polymer, in this module inlet film thickness ( $h_i$ ) can expression  $h_{ib} + h_{ic} + T_p$  and outlet film thickness ( $h_o$ ) can expression  $h_{ob} + h_{oc} + T_p$ ,  $L_p$  represent length of particle, force effective on particle represent with  $F_b, F_c$  respectively, load-carrying capacity of particle represent with  $W_b, W_c$  respectively,  $P_a$  and  $P_d$  represent pressure inlet and pressure outlet, so present in this model sliding motion  $u_p$  and squeeze action  $v_p$  for particle in synovial fluid, shown in figure (1.2), mathematical module of hyaluronic acid differ from mathematical module of particle since values inlet pressure, outlet pressure, sliding motion, squeeze action known, while values pressure and velocity were unknown with respect to particle. Basic on Quasi-statics equilibrium equation will calculate the value of the pressure (inlet-outlet).

## 1.2 Quasi-statics equilibrium equation:

The force acting on the plane as a result of the entry into laminar incompressible, it was a one-dimensional flow of two of two directions  $F_x, F_y$  represents the

following:

$$\sum F_x = 0 \quad (1.1)$$

$$\sum F_x = (P_a - P_d) T_p \cos(\theta) + (F_c - F_b) \cos(\theta) + (W_c - W_b) \sin(\theta)$$

$$\sum F_y = 0 \quad (1.3)$$

$$\sum F_y = (P_a - P_d) T_p \sin(\theta) + (F_c - F_b) \sin(\theta) + (W_c - W_b) \cos(\theta) \quad (1.4)$$

Where the left sides in equations (1.2) and (1.4) are the sum of all force and the right sides are friction force and body weight. To find the values pressure (inlet-outlet) generated result to pass particle in through the gap between two bones, first calculate the value of the angle  $\theta$  mentioned in the figure (1.2) using the following formula

$$\sin \theta = \frac{\sqrt{(l_b)^2 - (L_p)^2}}{(L_p)} \quad (1.5)$$

This formula depended on the length of hyaluronic acid ( $l_b$ ) and particle ( $L_p$ ) using, values mentioned in an appendix (A), it was found that  $\theta = 0.020$  Therefore it was founded  $\sin \theta = 0.02$  and  $\cos \theta = 0.91$ . Substitute values load carrying capacity and friction force respectively  $W_c = 530$  N  $W_b = 525$  N  $F_c = 0.094$  N,  $F_b = 0.070$  N into equations (1.2) and (1.4), thus we obtain the following algebraic equation:

$$(P_a - P_d) 0.5 \times 10^{-6} \text{ mm} \cos \theta + 16 \times 10^{-3} \text{ N} \cos \theta + 5 \text{ N} \sin \theta = 0$$

$$(P_a - P_d) 0.5 \times 10^{-6} \text{ mm} \sin \theta + 16 \times 10^{-3} \text{ N} \sin \theta + 5 \text{ N} \cos \theta = 0$$

For  $0 < \theta < \frac{(1.6)}{\alpha}$  by combining equations we obtain the system of two equations for determination of two unknown, inlet pressure ( $P_a$ ) and outlet pressure ( $P_d$ ) and action simple calculation become equation (1.6) form

$$(P_a - P_d) = -2.5178 \text{ N } 10^5 / \text{mm} \quad (1.7)$$

$$(P_a - P_d) = -4.55 \text{ N } 10^5 / \text{mm}$$

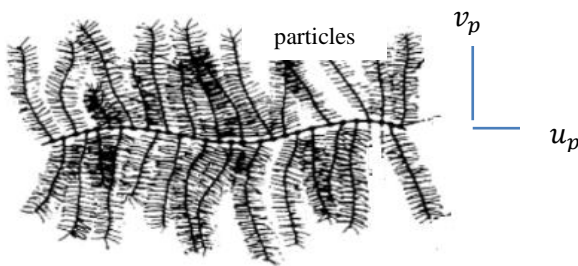
After simplifying equation (1.7) thus it was obtained the pressure inlet  $1.01325 \text{ N} \times 10^5 / \text{mm}$  and value pressure outlet  $4.54 \text{ N} \times 10^5 / \text{mm}$ .

### 1.3 sliding motion and squeeze action for particles:

Hyaluronic acid contains on smaller chains to include number of particles as shown in figure (1.3) rotational velocity of particle is ignored since particle like a long

cylinder each of the particles has sliding motion ( $u_p$ ) and squeeze action ( $v_p$ ), particles 1,2,3 (M+1) are assumed to be geometrically similar, it was calculus sliding motion and squeeze action depended on number of particles in each long chain of hyaluronic acid and using formula following :

$$u_p = \left( \frac{M+1-m}{M+1} \right) U, \quad v_p = \left( \frac{M+1-m}{M+1} \right) V \quad (1.8)$$



hyaluronic acid chain

Figure (1.2): illustrates calculate sliding motion and squeeze action depending on the number of particles

Table (1.1): sliding motion and squeeze action

Number of particles(m)	Squeeze action of particles $v_p$ (m/s)	The sliding motion of particles $u_p$ (m/s)
10	$25 \times 10^{-3}$	$56 \times 10^{-3}$
20	$17 \times 10^{-3}$	$38 \times 10^{-3}$
30	$93 \times 10^{-4}$	$20 \times 10^{-3}$
40	$85 \times 10^{-4}$	$18 \times 10^{-3}$

### 1.3.1 Pressure distribution of the particles:

Particles pass through a gap between two articular cartilage effect of Pressure distribution, boundary conditions for the pressure and film thickness in the synovial hip joint as given:

$$\left. \begin{aligned} P &= P_a, & x &= 0, & h &= h_i \\ P &= P_d, & x &= L_p, & h &= h_o \end{aligned} \right\} \quad (1.9)$$

Using the pressure equation

$$P = \frac{12\eta (U_1 - U_2)}{2} \frac{1}{(h_i - (h_i - h_o))^2} L + \frac{12\eta (V_2 - V_1)}{(h_i - (h_i - h_o))^3} \cdot \frac{L^2}{2} + \frac{12 A \eta}{(h_o - (h_i - h_o))^3} + 12\eta \frac{B}{(h_i - (h_i - h_o))^3} \quad (1.10)$$

and replace values pressure inlet  $P_i$  and pressure outlet  $P_o$

with  $P_a = 1.01325 \text{ N} \times 10^5 / \text{mm}$  and pressure outlet  $P_d = 4.54715 \text{ N} \times 10^5 / \text{mm}$  respectively and substitute values sliding motion ( $U$ ) and squeeze action ( $V$ ) with values that it was obtained becomes equation form following:

$$P = \frac{12 \eta (u_p)}{2} \cdot \frac{1}{(h_i - (h_i - h_o))^2} \cdot L_p + \frac{12 \eta (v_p)}{(h_i - (h_i - h_o))^3} \cdot \frac{L_p^2}{2} + \frac{12 \eta A}{(h_i - (h_i - h_o))^3} \cdot L_p + 12 \eta \frac{B}{(h_i - (h_i - h_o))^3} \quad (1.11)$$

Where

$$A = \frac{-h_i h_o}{h_i + h_o} \cdot u_p - \frac{h_i^2 h_o^2}{6 \eta L_p (h_i + h_o)} (P_a - P_d) - \frac{L_p h_i}{(h_i + h_o)} v_p$$

$$B = \frac{-L_p}{2(h_i^2 - h_o^2)} u_p + \frac{P_i h_i^2 - P_o h_i^2}{12 \eta (h_i^2 - h_o^2)} + \frac{L_p^2}{(h_i^2 - h_o^2)(h_i + h_o)} v_p \quad (1.12)$$

### 1.3.2 Load carrying capacity of the particles:

The applied load is carried by pressure generated within the fluid contain on particles to calculate load carrying capacity for particles  $W_t$  integration of the pressure distribution (1.11).

$$W_t = \int_0^{L_p} P \, dx \quad (1.13)$$

After integrating with respect to  $x$  it was getting

$$W_t = \frac{6\eta u_p L_p^2}{h_o^2} H_{w1} - \frac{12\eta v_p L_p^2}{h_o^3} H_{w2} + L_p (H_{w3} P_a + H_{w4} P_d) \quad (1.14)$$

Where  $W_t$  represent total load carrying capacity for the region (a,b,c,d) as seen figure (1.2) which varies from one region to another. Introducing the dimensionless load carrying capacity

$$W^* = \frac{W_t}{W} \quad (1.15)$$

Substitute load carry capacity with a particle ( $W_t$ ) and load carry capacity without particle (standard bearing) (

$$W^* = \frac{\frac{6\eta u_p L_p^2}{h_o^2} H_{w1} - \frac{12\eta v_p L_p^2}{h_o^3} H_{w2}}{\frac{6\eta (U_1 - U_2) L^2}{h_o^2} H_{w1} - \frac{12\eta (V_2 - V_1) L^2}{h_o^3} H_{w2}} + \frac{L_p (H_{w3} P_a + H_{w4} P_d)}{L_p (H_{w3} P_i + H_{w4} P_o)} \quad (1.16)$$

$W$ ) in the formula (1.15), it was obtained following form

### 1.3.3 Center of the pressure of the particles:

The location of the center of pressure  $x_{cp}$  indicates the location of the center of pressure in the x-direction. The expression for calculating the location is

$$W_t x_{cp} = \int_0^{L_p} P x \, dx \quad (1.17)$$

$$W_t x_{cp} = \frac{6\eta u_p L_p^3}{h_o^2} H_{wcp1} - \frac{12\eta v_p L_p^4}{h_o^3} H_{wcp2} - L_p^2 H_{wcp3} P_a \quad (1.18)$$

After that substitute, the load-carrying capacity of particles represents in equation (1.14) and the result of the integration of the equation (1.18). Therefore, the center of pressure can be written as:

$$x_{cp} = \frac{\frac{-0.24\eta u_p L_p^3}{h_o^2} + \frac{30.24\eta v_p L_p^4}{h_o^3}}{\frac{0.1602\eta u_p L_p^2}{h_o^2} - \frac{3.516\eta v_p L_p^2}{h_o^3}} + \frac{0.36477 L_p}{L_p (H_{w3} P_a + H_{w4} P_d)} \quad (1.19)$$

### 1.3.4 Friction force on the particle:

The friction forces on both surfaces of the particles after it get pressure (inlet and outlet) values.

Assume that Newtonian fluid

$$\tau = \eta \left( \frac{\partial u}{\partial y} \right) \quad (1.20)$$

Where the term  $\left( \frac{du}{dy} \right)$  velocity gradient of  $y$  is obtained from the velocity distribution

$$u = \frac{1}{2\eta} \frac{dp}{dx} y(y-h) + \frac{h-y}{h} U_1 + \frac{y}{h} U_2 \quad (1.21)$$

Now we calculate the first derivative of the function (1.21) with respect to  $y$ :

$$\frac{\partial u}{\partial y} = \frac{1}{\eta} \frac{dp}{dx} (y-h) + \frac{(U_2 - U_1)}{h} \quad (1.22)$$

(A) Friction force on down surface coordinate ( $y=0$ ) is given by

$$F_t = \int_0^{L_p} \tau_{y=0} \, dx \quad (1.23)$$

Substitute equation (1.20) and (1.22) in the formula

(1.23) and after integration for  $F_{y=0}$  was obtained the friction forces

$$F_t = \frac{-\eta u_p L_p}{h_o} \left\{ \frac{4}{n-1} L n n - \frac{6}{n+1} \right\} + \frac{6\eta v_p L_p^2}{h_o} \left\{ \frac{2}{n^2-1} - \frac{1}{(n-1)^2} L n n \right\} + \frac{nh_o}{n+1} (P_a - P_d) \quad (1.24)$$

(B) Friction force on up surface coordinate ( $y=h$ ) is given by

$$F_t = \int_0^{L_p} \tau_{y=h} \, dx \quad (1.25)$$

Substitute equation (1.20) and (1.22) in the formula

(1.25) and after integration for  $F_{y=h}$  was obtained the friction forces

$$F_t = \frac{\eta u_p L_p}{h_o} \left\{ \frac{2}{n-1} L n n - \frac{6}{n+1} \right\} + \frac{6\eta v_p L_p^2}{h_o} \left\{ \frac{2}{n^2-1} - \frac{1}{(n-1)^2} L n n \right\} - \frac{nh_o}{n+1} (P_a - P_d) \quad (1.26)$$

Where  $F_t$  represent totally friction force for region (a,b,c,d) as is seen figure (1.2) which varies from one region to another. Introducing the dimensionless friction force

$$F^* = \frac{F_t}{F} \quad (1.27)$$

Substitute friction force with a particle ( $F_t$ ) and friction force without particle (standard bearing) ( $F$ ) in the formula (1.27): it was obtained following form. For  $y=0$

$$F^* = \frac{\eta u_p L_p}{h_o} \left\{ \frac{2}{n-1} L n n - \frac{6}{n+1} \right\} - \frac{6\eta v_p L_p^2}{h_o} \left\{ \frac{2}{n^2-1} - \frac{1}{(n-1)^2} L n n \right\} - \frac{nh_o}{n+1} (P_a - P_d) / \frac{\eta (U_1 - U_2) L}{h_o} \left\{ \frac{2}{n-1} L n n - \frac{6}{n+1} \right\} - \frac{6\eta (V_2 - V_1) L^2}{h_o} \left\{ \frac{2}{n^2-1} - \frac{1}{(n-1)^2} L n n \right\} - \frac{nh_o}{n+1} (P_a - P_d) \quad (1.28)$$

it was obtained following form. For  $y=h$

$$F^* = \frac{-\eta u_p L_p}{h_o} \left\{ \frac{4}{n-1} L n n - \frac{6}{n+1} \right\} - \frac{6\eta v_p L_p^2}{h_o} \left\{ \frac{2}{n^2-1} - \frac{1}{(n-1)^2} L n n \right\} + \frac{nh_o}{n+1} (P_a - P_d) / \frac{-\eta (U_1 - U_2) L}{h_o} \left\{ \frac{4}{n-1} L n n - \frac{6}{n+1} \right\} - \frac{6\eta (V_2 - V_1) L^2}{h_o} \left\{ \frac{2}{n^2-1} - \frac{1}{(n-1)^2} L n n \right\} + \frac{nh_o}{n+1} (P_a - P_d) \quad (1.29)$$

Coefficient friction of particles has an expression

$$\mu_t = \frac{F_t}{W_t} \quad (1.30)$$

Substituting equations (1.14) and (1.26) in the above equation obtain for  $y=h$

$$\begin{aligned} \mu_t = & \frac{\eta u_p L_p}{h_o} \left\{ \frac{2}{n-1} L n n - \frac{6}{n+1} \right\} + \frac{6\eta v_p L_p^2}{h_o} \\ & \left\{ \frac{2}{n^2-1} - \frac{1}{(n-1)^2} L n n \right\} + \frac{n h_o}{n+1} (P_a - P_d) \\ & / \left( \frac{0.1602 \eta u_p L_p^2}{h_o^2} - \frac{3.522 \eta v_p L_p^2}{h_o^3} \right) \\ & + L_p (0.5875 P_a - 0.3125 P_d) \end{aligned} \quad (1.31)$$

### 1.3.5 Flow rate for particles:

The flow rate is defined as  $Q_x = \int_0^h u \, dy$ , integrate from

0 to  $h$  with respect to  $y$  and substituting for inlet and outlet pressure and squeeze action and sliding motion of particles, we get

$$\begin{aligned} Q_x = & h_o \frac{u_p}{2} n - h_o \frac{u_p}{2} n \frac{n-1}{n+1} + \frac{1}{6 \eta L} \frac{n^2 h_o^3}{n+1} \\ & (P_a - P_d) + \frac{L n n}{n+1} v_p \end{aligned} \quad (1.32)$$

It is clear from the equation (1.32) that the flow rate function is depended on pressure inlet and pressure outlet who is the biggest influence on the flow rate.

### 1.4 Numerical results:

In this section numerical results are displayed through figures (1.3) - (1.13) to show the effects of various parameters such as sliding motion and squeeze action for hyaluronic acid ( $U, V$ ), sliding motion and squeeze action for particles ( $u_p, v_p$ ), film thickness (inlet-outlet) ( $h_i, h_o$ ), length of hyaluronic acid ( $L$ ), length of particle ( $L_p$ ), viscosity ( $\eta$ ) on the acid ( $L$ ), length of particle ( $L_p$ ), viscosity ( $\eta$ ) on the pressure difference, load-carrying capacity, friction force and rate of flow. In order to estimate the quantitative effects of the various parameters in the results of the present analysis.

#### 1.4.1 Effective hyaluronic acid on performance synovial hip joint:

Figure (1.3) represent the variation of the pressure distribution ( $P$ ) as a function of length of particles ( $L_p$ ) for different values of squeeze-action parameters ( $v_p$ ) with using equation (1.11) and value ( $\eta = 2 \times 10^{-2}$  Ns/m<sup>2</sup>). It is interesting to notice the effect of the squeeze action on the pressure distribution and length of particles with decreasing the squeeze action, the pressure developed in synovial fluid will increase. The percentage rate of increase in pressure distribution was approximately 84% at ( $L_p = 24$ ). The effect of length parameter ( $L_p$ ) on the variation of pressure ( $P$ ) with film thickness outlet ( $h_o$ ) is shown in figure (1.4) with the

parametric ( $u_p = 55 \times 10^{-3}$  m/s). It should be noted that with increasing pressure the slight appearance of the gap results in increase in length of the particles. Using equation (1.14) as shown in figure (1.5) the difference in the thickness of particles in hyaluronic acid has a profound effect on the of load carry capacity ( $W_t$ ) that are developed. It should be pointed out that the thickness was decreased in the load-bearing area, in order to increase the area at which the load applied. The variation of the load-carrying capacity ( $W_t$ ) as a function of the length of the hyaluronic acid chain ( $L_p$ ) for different values of viscosity parameters are as shown in figure (1.6). It is observed load carry capacity decrease with decreasing values of viscosity maximum values of load carrying capacity 650 N at normal viscosity 0.01Ns/m<sup>2</sup>. In figure (1.7), the variation of friction force with respect to length of particle ( $L_p$ ) is shown for different values of thickness ( $T_p = 0.5 \times 10^{-3}$   $\mu$ m,  $0.4 \times 10^{-3}$   $\mu$ m) by fixed other parameters  $u_p = 0.055$  m/s,  $v_p = 0.035$  m/s, there is a nonlinear relation between friction force and length of particles rate of increase in friction force approximately 20% at  $T_p = 0.4 \times 10^{-3}$  mm. In figure (1.8), the variation of friction force with respect to the length of the particle ( $L_p$ ) is shown for the value of It is worth mentioning that an increase in friction force results for decreasing values of thickness the percentage sliding notion ( $u_p = 0.055$  m/s). So, there is a nonlinear relation between friction force and length of particles the percentage rate of increase in friction force approximately 25%. The variation coefficient of friction (COF) as a function of a thickness ( $T_p$ ) for different values of length ( $L_p$ ) is as shown in figure (1.9). It observed that increase in the coefficient of friction with decrease thickness particle, we see that the maximum (COF) is 82% that result in a length of particles is  $0.3 \times 10^{-3}$  mm figure depicts coefficient of friction with a length of particles ( $L_p$ ) for various values of viscosity ( $\eta = 0.01$ - $0.0025$  Ns/m<sup>2</sup>). Figures (1.10-1.11) show the variations of flow rate  $Q$  with sliding motion and length of particles  $L_p$ . under the influence of all parameter squeeze action, pressure inlet, pressure outlet, viscosity and thickness of particles. It is observed from the figures that the flow rate increases with increased thickness of particles and viscosity.

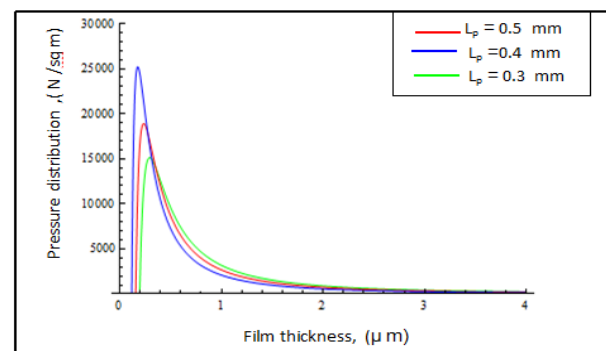


Figure (1.3): Show that variation hydrodynamic pressure distribution ( $P$ ) with length ( $L_p$ ) for different squeeze action ( $v_p$ ) ( $u_p = 0.056$  m/s)



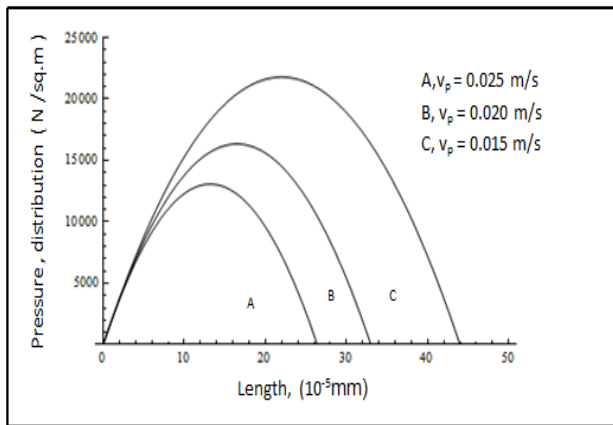


Figure (1.4): show that variation pressure distribution (P) with film thickness (h) for different length particles ( $L_p$ ). ( $u_b=0.056$  m/s,  $v_b=0.025$  m/s)

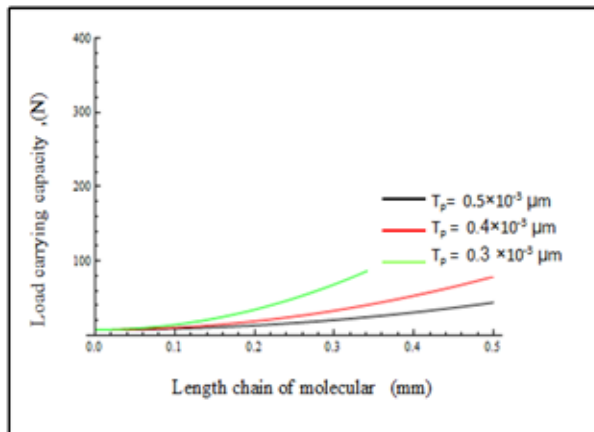


Figure (1.5): Show that variation load carry capacity W with a length of particle ( $L_p$ ) for different values film thickness ( $u_b=0.056$  m/s,  $v_b=0.025$  m/s)

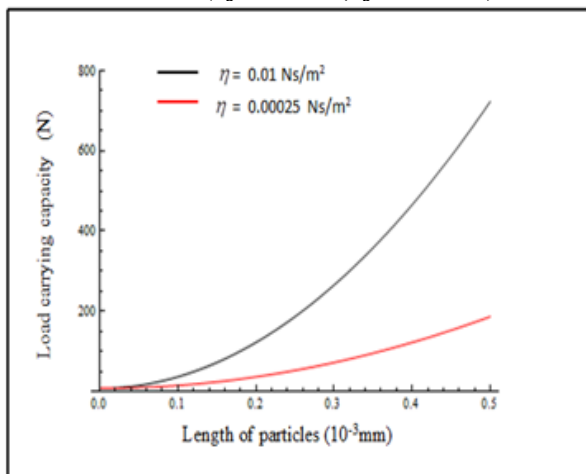


Figure (1.6) Show that variation load carry capacity W with length chain of molecular ( $L_p$ ) for different values film thickness ( $u_b=0.056$  m/s,  $v_b=0.025$  m/s)

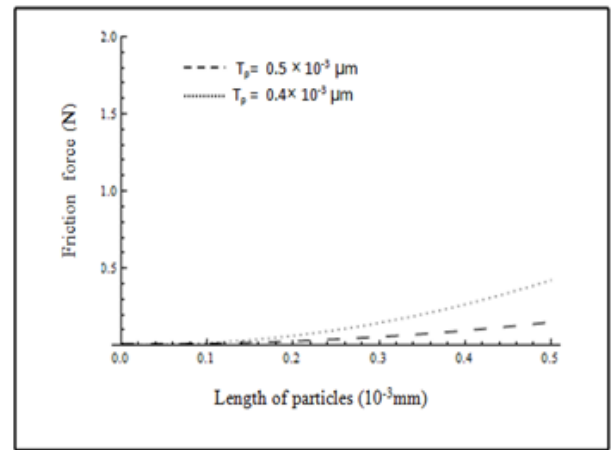


Figure (1.8) friction force versus length of particle ( $L_p$ ) for the different values thickness of particles ( $T_p$ ) ( $u_p=0.056$  m/s,  $v_p=0.025$  m/s)

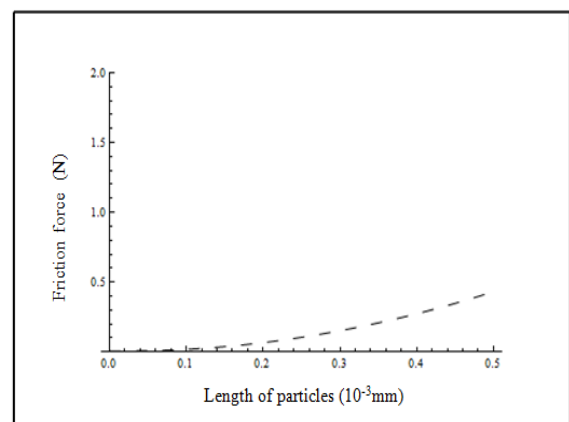


Figure (1.9) Show that friction force versus the length of particle ( $L_p$ ) sliding motion of particles ( $u_p=0.057$  m/s)

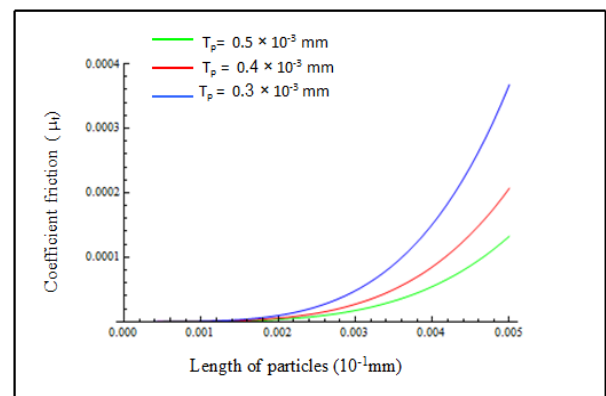
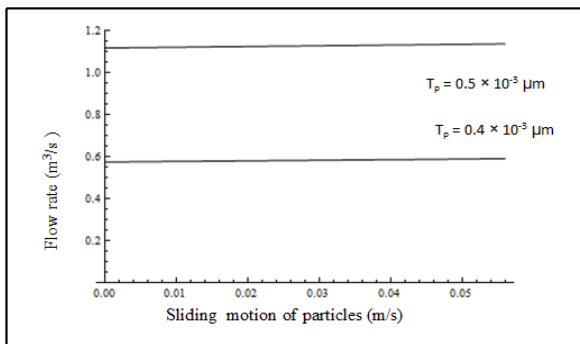
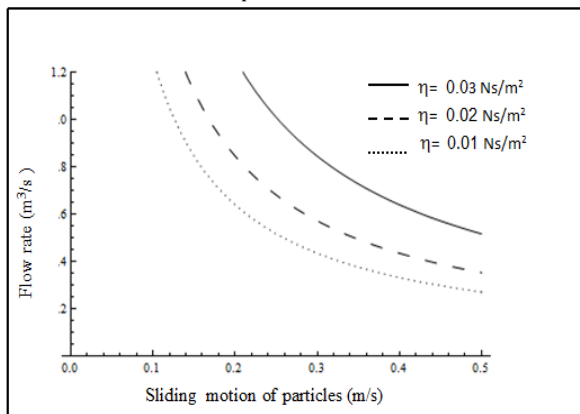


Figure (1.9): Show that coefficient of friction with a length of particle ( $L_p$ ) for different values thickness of particles ( $T_p$ ). ( $u_p=0.056$  m/s,  $v_p=0.025$  m/s)



**Figure (1.10):** Show that flow rate sliding motion of particle ( $L_p$ ) for different thickness particles  $T_p$ . ( $u_p=0.056$  m/s,  $v_p=0.025$  m/s)



**Figure (1.11):** Show that flow rate versus length of particles ( $L_p$ ) for different viscosity ( $u_p=0.056$  m/s,  $v_p=0.025$  m/s)

**Table (1.2):** relation between rate flow and squeeze action with different viscosity synovial fluid, thickness of particles ( $T_p=0.5 \times 10^{-3} \mu m$ ) the sliding motion

$\eta = 0.01 \text{ Ns/m}^2$		$\eta = 0.02 \text{ Ns/m}^2$	
Flow rate	Squeeze action	Flow rate	Squeeze action
1.0089	0.025	0.5172	0.025
1.0087	0.025	0.5170	0.025
1.0084	0.024	0.5158	0.024
1.0082	0.023	0.5155	0.023
1.0080	0.022	0.5153	0.022
1.0077	0.021	0.5151	0.021
1.0075	0.020	0.5158	0.020

### 1.5 CONCLUSIONS:

The effected particles of hyaluronic acid on the hydrodynamic lubrication between a two articular cartilage are presented in synovial human joint. The modified Reynolds equation, governing the hydrodynamic film pressure of particles is derived using the mathematical model describe long chin polymer exit in synovial fluid. According to the results obtained the following conclusions:

- 1) The effect particles of hyaluronic acid on increase pressure film with different squeeze action and length of hyaluronic acid with particle, Comparing between squeeze action and length, It was found squeeze has influenced more than length.

- 2) The effect particles of hyaluronic acid on increase load-carrying capacity of articular cartilage with the different film thickness in (hydrodynamic – squeeze elastohydrodynamic) and viscosity in healthy and disease.
- 3) The effective particles of film thickness and sliding motion on friction force between surface particulars that decreases with the presence of hyaluronic acid in synovial fluid in a healthy joint.
- 4) The effect particles of film thickness and sliding motion on increase flow rate synovial fluid between articular cartilage. This increase is attributed to the presence of hyaluronic acid.

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