ANALYSIS OF BOUNDARY LAYER FLOW OVER A STRETCHING SHEET FOR AN EYRING-POWELL FLUID BY OHAM

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ABSTRACT. *This work is presented to the study of boundary layer flow over a stretching sheet for Eyring-Powell, non-Newtonian fluid. A second-order semi-analytical model for estimating flow of the fluid is used. Optimal Homotopy Asymptotic Method (OHAM) is used to get a semi-analytical solution. The impact of relevant non-Newtonian fluid parameters* M *and* κ *on the velocity is examined through graphical results.*

Key Words: Eyring-Powell model, Optimal Homotopy Asymptotic Method, Non-Newtonian fluid, Semi-analytical solution, Stretching sheet

1. INTRODUCTION

The boundary layer viscous float caused by an extending surface moving with a specific speed in a generally inactive unsolidified medium frequently happens in numerous designing processes. Such streams have promising applications in industry, for instance, consistent extending of plastic movies, counterfeit strands, metal turning, metal expulsion, constant throwing, glass blowing, the expulsion of a grafix plastics from a kick bucket, the illustration of plastic movies, and some more. During the manufacturing processes of these sheets, the liquefy problems from a cut and is eventually extended to achieve the substantial thickness. During the process, the mechanical properties of very last items severely rely upon the cooling and extending rates [1].

In 1970, the spearheading work of Crane for the stream of an incompressible thick liquid past, an extending sheet has turned into an exemplary in the writing [2]. It concedes a precise scientific arrangement. What's more, it has created many related issues, each consolidating another impact and as yet giving a definite arrangement. In 1977, Gupta and Gupta analyzed the extending stream subject to suction or infusion [3]. In 1981, Brady and Acrivos investigated the stream inside an extending channel or cylinder, while the stream outside an extending tube was examined by Wang [4-5]. Another research done by Wang stretched out the stream examination to a three-dimensional axisymmetric extending surface [6]. In 1990, the shaky stream initiated by an extending film has likewise been talked about by Wang, Usha and Sridharan [7-8].

All the previously mentioned investigations are bound to the stream of Newtonian liquids. As a general rule, a large portion of the fluids utilized in modern applications are non-Newtonian in nature, particularly in polymer preparing and synthetic designing procedures. In 1976, Siddappa and Khapate inspected the stream of a second-request liquid over an extending sheet [9]. Afterward, Rajagopal *et al*., investigated the stream of viscoelastic second-request liquid past an extending surface by unraveling the limit layer conditions numerically for little estimations of the viscoelastic parameter [10]. Dandapat and Gupta expanded the issue in Rajagopal *et al.* with warmth exchange investigation and gave the precise explanatory arrangement of the self-comparative stream represented by nonlinear conditions [11]. Cortell [12], expanded crafted by Dandapat and Gupta [13] to inspect the impacts of an incompressible second-grade liquid over an extending sheet by thinking about the impact of warmth exchange. Eldabe *et al*. and Zueco and Beg [14[examined non-Newtonian liquid stream under the impact of couple worries between two parallel plates utilizing the Eyring-Powell demonstrate [15]. In all respects as of late, Abd El-Aziz contemplated the impact of variable liquid properties and thick dissemination on constrained convection of viscoelastic fluids in a slim film over an extending surface [16]. Marinca *et. al*. presented another strategy known as the Optimal Homotopy Asymptotic Method (OHAM) [17]. The upside of OHAM is in the worked in intermingling criteria like HAM yet progressively adaptable. In arrangement of papers Marinca *et al.* have connected this strategy effectively to get the arrangement of right now vital issues in science, and have likewise demonstrated its adequacy, speculation and unwavering quality [18-19]. Sarwar *et. al*., solves the fractional order problems to expand the OHAM [20-23].

The primary point of the present examination is to consider the boundary layer stream over an extending sheet for another non-Newtonian liquid, specifically the Eyring-Powell show [24]. The article is sorted out as pursues: the following area contains the scientific definition of the issue, at that point the numerical outcomes and discourse are exhibited, and lastly we have incorporated some finishing up comments.

2. MATHMATICAL FORMULATION

Consider incompressible flow of an Eyring-Powell non-Newtonian fluid. A Cartesian system of coordinate with the x-axis along the stretching sheet as given in Figure 1., is considered. The fluid occupies the positive half plane. In order to stretch the surface, equal and opposite forces are applied. u is assumed to be stretching velocity of the sheet.

The proposition of rate procedures is utilized to determine the Eyring-Powell to demonstrate for depicting the shear of the fluid.

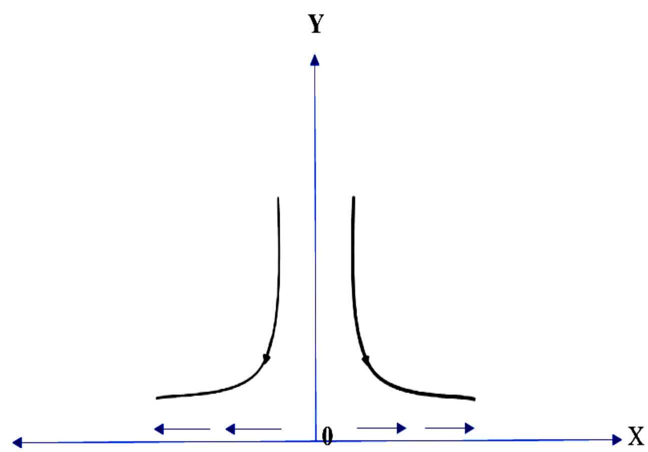

Figure 1: Geometry of Physical Problem for stream of Eyring-Powell model.

At times, this model envisages the viscous conduct of polymer arrangements. The stress tensor in the described model is:

$$T = \left\{ \mu + \frac{1}{\beta \gamma} \sinh^{-1}\left(\frac{1}{c}\gamma\right) \right\} A_1, \quad (1)$$

and

$$\dot{\gamma} = \sqrt{\frac{1}{2}\, tr\,(A_1^2)} \quad (2)$$

where μ is the dynamic viscosity of the liquid, β and c are the liquid parameters of the Eyring Powell model, and c has the element of (time)-1. We should had to take the second one order of estimation of the $\sinh^{-1}$ as given below:

$$\sinh^{-1}\left(\frac{1}{c}\dot{\gamma}\right) \cong \frac{\dot{\gamma}}{c} - \frac{\dot{\gamma}^3}{6c^3}, \quad with \ \frac{\gamma^5}{c^5} <<< 1. \quad (3)$$

The derived model from the above equation will be:

$$(1+M)\digamma''' - \digamma\digamma'' - M\lambda \digamma''^2 \digamma''' = 0, \quad (4)$$

$$\digamma(0) = 0, \ \digamma'(0) = 1, \ \digamma'(\infty) = 0. \quad (5)$$

The resident frictional drag coefficient (skin-friction coefficient) on the superficial of the extending sheet is

$$C_\digamma = \frac{\tau_{xy}|_{y=0}}{\rho (ax)^2}, \quad (6)$$

And the dimensionless coefficient of the skin-friction model can be:

$$Re_{x^{1/2}} C_\digamma = \left\{ (1+M)\, \digamma''(0) - \frac{\lambda}{3} M \digamma''(0) \right\}, \quad (7)$$

where Re_x is a local Reynolds integer and $C_\digamma$ is frictional drag coefficient/skin-friction coefficient:

$$\begin{aligned} \digamma = \frac{1}{1088640} \xi (&-181440(-6+\xi) \\ &- \frac{504\xi(-1215+360\xi-30\xi^2+2\xi^3)C_1}{1+M} \\ &+ \frac{1}{(1+M)^2}\xi(-504(1+M)(-1215+360\xi \\ &-30\xi^2+2\xi^3)C_1 + (1211598-181440\xi \\ &-35910\xi^2+5796\xi^3-504\xi^4-24\xi^5 \\ &+\xi^6+56M(-9+\kappa)(-1215+360\xi \\ &-30\xi^2+2\xi^3))C_1^2 \\ &-504(1+M)(-1215 \\ &+360\xi-30\xi^2+2\xi^3)C_2)) \end{aligned}$$

3. RESULTS AND DISCUSSIONS

In this section, the nonlinear model with the boundary conditions [32] has been analyzed semi-analytically using OHAM. The details of the method can be found in [25-31].

The effects of physical parameters on the velocity field $\digamma$ and the skin-friction coefficient $Re_x^{1/2}\, C_\digamma$ are observed in Figures 2–4.

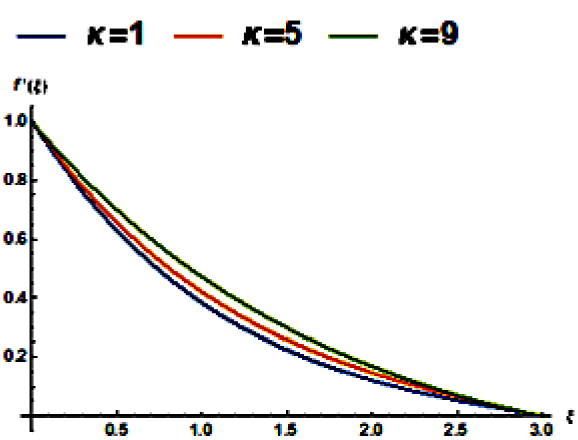

Figure 2: Velocity profiles of $\digamma'$ vs. ξ for different values of κ and keeping $M = 0.4$.

Figure 2 shows the impacts of the material parameter κ on the velocity profile $\digamma'$ making the parameter $M = 0.4$ static. It is found that the velocity $\digamma'$ is decreased by increasing the values of the parameter κ.

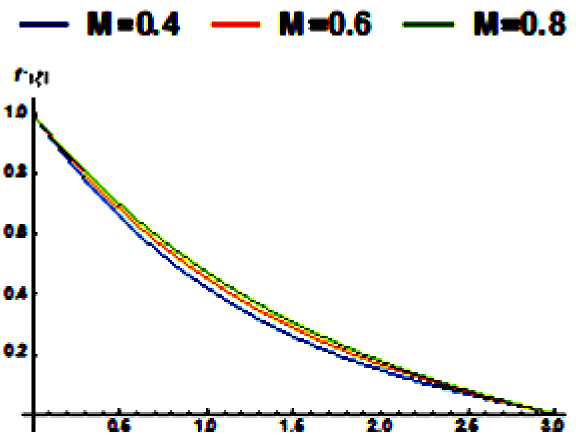

Figure 3: Velocity profiles of $\digamma'$ vs. ξ for different values of M and keeping $\kappa = 5$.

Figure 3 illustrates the variations of the fluid material parameter M on the velocity profile $\digamma'$ keeping $\kappa = 5$ fixed. It clearly validates that the velocity $\digamma'$ is a decreasing function of the parameter κ.

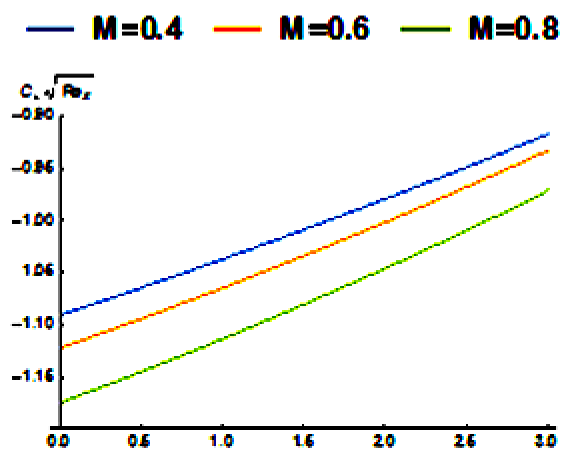

Figure 4: Variations of the skin-friction coefficient $Re_x^{1/2}\, C_\digamma$ against the parameter κ for different values of M.

It is also clearly noted in Figure 4 that the skin-friction coefficient $Re_x^{1/2}\, C_\digamma$ increases with an increase in M

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5. CONCLUSIONS

In this article, Eyring-Powell fluid flow over a stretching sheet is examined. A semi-analytical solution is developed using OHAM. The magnitude of velocity is decreased for the increasing values of physical parameters M and κ. The skin- friction parameter is also examined. The results are reflected for the parameters M and κ through graphs.

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