SEQUENCES OF MATHEMATICAL TASKS FOR INTENSIFYING PROCEDURAL FLUENCY IN INTEGER OPERATION

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ABSTRACT: *This study explored the effects of sequencing students' engagement in mathematical tasks on their level of procedural fluency. The teacher randomly selected two intact classes from a stream of Grade 7 classes. These two classes were randomly assigned as the experimental group and the control group. The teacher employed the conceptual-to-procedural approach in teaching the experimental group while the procedural-to-conceptual approach in teaching the control group. Quantitative quasi-experimental pretest-posttest non-equivalent control group design was employed. Quantitative method was used to examine the students' level of procedural fluency on integer operation. Results showed that conceptual-to-procedural instruction influenced the Grade 7 students' procedural fluency on integer operation more than the procedural-to-conceptual instruction. Thus, the researchers recommend that mathematics teachers may practice the conceptual-to-procedural sequence in teaching mathematics to strengthen the foundation of mathematical concepts, and in turn, develop procedural fluency. Future studies on the effectiveness of these sequences of mathematical tasks are needed to support these research findings.*

Keywords: mathematical tasks engagement, procedural fluency, mathematical tasks

1. INTRODUCTION

Students struggle in secondary level mathematics because they lack the foundation skills. One of these essential pre-requisite skills needed for success in higher mathematics is the operations of integers. The concepts of integers mark the shift from arithmetic to algebra [1]. If students fell behind, then they are unable to understand the new lesson due to the sequential nature of Mathematics [2].

Building the foundation of mathematical skill is critical. To do so, the method of teaching mathematics should involve far more than memorizing procedures and applying algorithms [3]. Curriculum reforms state that the students must acquire both procedural fluency and conceptual understanding [4]. Procedural fluency and conceptual understanding are two very important strands in building students' mathematical proficiency [5]. Conceptual understanding happens when mathematical concepts, operations, and relations are internalized by the students. Procedural fluency, on the other hand, is the skill shown as students carry out procedures with efficiency, flexibility, appropriateness and most of all accuracy [6].

How to teach mathematics so that the students will become proficient in both procedural fluency and conceptual understanding became a debate among mathematics educators [8]. Mathematics education researchers believe that the sequence of mathematical task matters. Some said that conceptual-to-procedural ordering of instruction is ideal than procedural-to-conceptual ordering [7], [8]. Procedural instruction or practice problem solving does not always support conceptual knowledge growth [9]. However, others believed that procedural instruction support gains in conceptual knowledge if proper sequencing of procedural instruction is provided, that is procedural lessons are designed to encourage detection of underlying concepts [10]. However, a study comparing these different ways of sequencing conceptual and procedural instructions is limited [8].

This study adds empirical evidence on the effectiveness of both sequences of instruction. Although, both procedural fluency and conceptual understanding are important in mathematics proficiency, in this study only procedural fluency was measured. The researchers compared the effect of the two sequences of mathematical task, conceptual-to-procedural and procedural-to-conceptual on procedural fluency of integer operations.

2. METHOD

2.1 Research Design

This study used a quantitative quasi-experimental pretest-posttest non-equivalent control group. This method was used to examine the students' level of procedural fluency on integer operation. The design of this study is illustrated below.

$$\begin{array}{ccc} O_1 & X_1 & O_2 \\ \hdashline O_1 & X_2 & O_2 \end{array}$$

In this study, pretest which is designed to measure students' level of procedural fluency on integer operations was given to both groups. Then the treatments were given. The experimental group was given the task engagement X_1, from conceptual understanding to procedural fluency while the control group was exposed to task engagement X_2, from procedural fluency to conceptual understanding. After the treatment, posttest was administered to both groups.

2.2 Participants

The study was conducted at Binuangan National High School, one of the 108 public high schools in the Division of Misamis Oriental, located in the northern part of Mindanao, Philippines. The participants were the two intact classes of Grade 7 students enrolled in the school year 2018-2019. The two classes were randomly selected from a stream of three intact classes of Grade 7 level. One section was assigned randomly as the experimental group and the other as the control group.

The academic abilities of the participants in each group were assumed to be heterogeneous as it is a policy of the school to have each class a heterogeneous group of students. Some of them were graduates from private elementary school while others were from public elementary schools, from nearby municipalities. Some of them are honors while others are not. Moreover, the participants were assumed to bear similar characteristics in regional or ethnic groups as they are all inhabitants of the place by birth.

The experimental group was composed of 28 participants, 16 males, and 12 females while the control group consisted of 27 participants, 16 males, and 11 females. These groups are handled by the researcher.

2.3 Research Instrument

The 10-item researcher-made procedural fluency test was used to collect data for analysis in this study. This instrument shows evidence of validity and reliability.

At first, the researchers made a 15-item procedural fluency test on integer operation based on the Table of Specification (TOS). A group of experts examined each item of this test for face and content validity. The experts made some corrections and suggestions. After which, the researchers modified the instrument and administered the revised test to the grade 8 students of this school so that an item analysis and reliability test can be calculated. Since the test is open-ended, the researchers requested two mathematics teachers, teaching grade 7 mathematics to rate the students' answers to avoid bias. Inter-rater reliability using Pearson Correlation was computed (pretest r = .91, p = .001; posttest r = .97, p= .001) and found out that there was a consistency of the ratings. Hence, the average score of the two raters was the final score of the students. Students' responses on this test were scored using the holistic rubrics as shown below:

0 if there is no attempt done to solve the problem;
1 if an attempt is made but nothing is correct;
2 if procedures are in the right direction but not executed properly and there are errors in computation;
3 if the students are able to arrive at the correct answer, however difficulty in carrying out the procedure is evident;
4 If the procedures are executed flexibly, completely and accurately with the correct answer.

The reliability of this 10-item pretest-posttest instrument was calculated using a Cronbach alpha and yielded a value of 0.849 which signifies that the test was valid and reliable.

2.4 Data Collection

Data were collected by means of a 10-item open-ended pretest-posttest on procedural fluency. The pretest was administered to both groups, and test papers were retrieved so as to reduce the familiarity effect with test items. The treatment process took place on the day after the pretest. Each group met four times a week and each meeting lasted for 1 hour.

Students in the control group were taught using procedural-to-conceptual approach. This approach starts with presenting the rules of performing the operations of integers and then followed by practice exercise by the group. When the rules have mastered the procedures, the mathematical concepts behind the operations of integers were modeled using a real-life scenario of models of integer operation. Students were encouraged to discuss and complete the exercise.

On the other hand, students in the experimental group were taught by the same teacher using conceptual-to-procedural approach. This approach starts with presenting a real-life scenario or models of integer operations to teach the concepts. This was followed by an exercise composed of a real-life scenario to be answered by the group. The students' answers were subsequently presented before their classmates. The students were also encouraged to help each other to complete the task. When the concepts were understood, the teacher then introduced the rules of performing the operations. The post-tests were administered right after the end of the unit.

2.5 Data Analysis

In order to examine the effect of task engagement in sequence on grade 7 students' procedural fluency, data were subjected to descriptive and inferential statistical analyses. For descriptive analysis, the mean and standard deviation of the pretests and posttests were calculated for both experimental and control groups. For inferential analysis at .05 level, t-test was computed to investigate the statistically significant difference in procedural fluency between pre-test mean scores and between post-test mean scores of the groups. To investigate statistically significant gains due to treatment conditions, the one-way analysis of covariance (ANCOVA) was calculated, the pretests were analyzed as covariates for the dependent variables. Eta squared values were calculated to determine the effect sizes.

3. RESULTS AND DISCUSSION

Pairwise comparisons of the mean scores of the control group and experimental group in pretests and posttests on procedural fluency were computed. As shown in Table 1.

Table 1. Descriptive statistics of the procedural knowledge of the experimental and control group

Groups	N	Pretest		Posttest		Mean Difference
		Mean (M)	SD	Mean (M)	SD	
Control	27	7.8	1.83	18.8	4.95	10.98
Experimental	28	7.9	1.47	24.6	6.61	16.68

The descriptive analysis of the differences between the mean scores of the pretests and the corresponding posttests showed general improvements in both experimental and control groups. The control group got 7.8 in the pretest and 18.8 in the posttest. The experimental group got 7.9 in the pretest and 24.6 in the posttest. However, the experimental group had higher posttest mean score than the control group. Also, the difference in mean score over the time interval of treatment for the experimental group was relatively higher than that for the control group, (16.68 vs 10.98). The mean results above indicate that harnessing the mathematical concepts behind the operations of integers is better when compared to developing the procedural fluency before the concepts. Furthermore, the results indicate that presenting the mathematical concepts and developed it profoundly is beneficial and perceived to be an advantage to successfully improve their achievement scores. The researcher observed this in the class while following the conceptual-procedural sequence. In this case, students tend to be more active in the class and tend to possess a sound understanding of the concepts behind the fundamental operations of integers. Thus, executing the procedures would not be very challenging for the students in the experimental group. This is true because comprehension of procedural mathematics is a knowledge which focuses on step-by-step procedures and skills without clear reference to mathematical ideas. Often, mere procedural skills do not provide readily applicable methods for solving math problem. A knowledge that involved a thorough understanding of underlying and foundational concepts behind the algorithms performed in mathematics is what we called comprehension in conceptual mathematics [11].

In terms of the difference between groups, an independent t-test was computed. The summary of the results is shown in Table 2.

Table 2. Summary of independent t-test results comparing the experimental and control group.

	t	df	Sig. (2-tailed)	Hedges' g
Pretest	-.34	53	.738	.06
Posttest	-3.70	53	.001	.99

Prior to treatment, the independent t-test result based on pretest scores (t(53) = -.34, p =.74, g=.06) unveiled that there was no significant difference between groups in their procedural fluency. This denotes that there were no pre-existing variations of students' knowledge of integer operations, and thus, no possible treats from the study design [12]. Hedges'g was calculated to compute the effect size because the sample size is not equal. Hedges'g provides a measure of effect size weighted according to the relative size of each sample [13]. This finding indicates a good point for departure for understanding the context, result pattern and treatment effect because the two groups were comparable prior to the treatment [14]. As a result, any significant difference in the post-test mean score of the procedural fluency between groups may be credited largely to the treatment effect.

With regards to the post-test scores, the independent t-test at .05 level indicated that scores were significantly higher (t(53)=-3.70, p=.001, g=.99) for the experimental group (M=24.6, SD=6.61) than for the control group (M=18.8, SD=4.95). The magnitude of difference in the posttest means between groups was large. If the Hedges'g is 0.8 and above, then the effect size is large [15].

Furthermore, the one-way analysis of covariance (ANCOVA) was calculated, the pretests were analyzed as covariates for the posttest. Table 3 displays the summary of one-way ANCOVA results.

Table 3. One-way ANCOVA summary

Source	Type III Sum of Squares	df	Mean Square	F	Sig.	Partial Eta Squared (n_p^2)
Groups	484	1	484	14.6	.001	.22
Error	1730	52	33.3			
Total	2214	55				

The F-ratio showed a statistically significant difference (F(1,52)=14.6, p=.001) in gains between groups on their posttest mean scores. Also, the proportion of variance in the gains between groups that is explained by the methods of teaching is large (n_p^2=.22). If the partial eta squared is .138 and above, then the effect size is large [15].

In summary, both descriptive and inferential analyses revealed that conceptual to procedural instruction influenced the grade 7 students' procedural fluency on integer operation more than the procedural to conceptual instruction. The proportion of variance in the dependent variable that is explained by the independent variable is large. This is perhaps in conceptual-to-procedural instruction, the teacher saw to it that concepts were fully understood by the students before teaching them with procedures because conceptual understanding forms the foundation, and is essential, for developing procedural fluency [1, 16, 17]. Most of the time, in conceptual-to-procedural instruction, the bulk of the time was spent for conceptual development which supports the claim of [18], [19] that conceptual knowledge should be developed over an extended period of time before procedures are taught and practiced. However, the outcome of this study supports the other finding that when teachers spent considerable time developing conceptual knowledge before introducing and practicing procedures, students gained comparable procedural knowledge compared to usual classroom instruction, which focused on procedural knowledge and often included little concept instruction [8, 20, 21].

4. CONCLUSION AND RECOMMENDATION

Achieving fluency in mathematical procedures such as the operations of integers is fundamental to students' mathematical development [22]. The usual way to develop procedural fluency is to practice repetitive exercises, but the results above showed that harnessing the concepts first is better than harnessing the procedures first. Thus, the researchers recommend that mathematics teachers may practice the conceptual-to-procedural sequence in teaching mathematics to strengthen the foundation of mathematical concepts, and in turn, develop procedural fluency. In order to support these research findings, future studies on the effectiveness of this sequence of mathematical tasks are needed.

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