INTUITIONISTIC FUZZY TOPOLOGY: FUZZY μ-STRONG SEMI CONTINUITY AND FUZZY μ-STRONG SEMI RETRACTS

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ABSTRACT *The concept of a fuzzy retract was introduced by Rodabaugh in 1981 and The concept of a intuitionistic fuzzy topology (IFT) was introduced by Coker 1997. The aim of this paper is to introduce a new concepts of fuzzy of Intuitionistic fuzzy μ-strongly semi open set of a nonempty set X and define an Intuitionistic fuzzy μ-strong semi continuity and Intuitionistic fuzzy μ-strongly semi retract. Also we prove that the product and the graph of two Intuitionistic fuzzy μ-strong semi continuity are Intuitionistic fuzzy μ-strong semi continuity . The concept of Intuitionistic fuzzy μ-strongly semi retract are introduced , the relations between these new concepts are discussed.*

Keywords IF$_\mu$- strongly semiopen, IF$_\mu$- strongly semi continuous, IF$_\mu$-retract and IF$_\mu$- neighbourhood retract, IF$_\mu$ – strongly semiretract

1-INTRODUCTION

The notions of Intuitionistic fuzzy retracts are introduced by Hanafy and khalaf [6]. In [4,5] weaker forms of Intuitionistic fuzzy continuity between of Intuitionistic fuzzy topological space are introduced. In this work we introduced and explain in section 2 a new notions of Intuitionistic fuzzy open sets (IF$_\mu$- open sets) are studied. in section 3 many results of IF$_\mu$- strongly semi continuous are obtained, finely in section 4 we define IF$_\mu$-retract and IF$_\mu$- neighborhood retract Finley in section 5 IF$_\mu$- strongly semi retract as applications of IF$_\mu$- strongly semi continuous. The relations between all these concepts are discussed.

Definition 1.1 [1] Let X be a nonempty set. An IF-set A is an object of the form $A=\{x,\mu_A(x),\ v_A(x): x\in X\}$. where the functions $\mu_A: X\to[0,1]$ and $v_A: X\to[0,1]$ denote respectively, the degree of membership function (namely $\mu_A(x)$ and the degree of non-membership function (namely $v_A(x)$ of A, $0\le\mu_A(x)+v_A(x)\le 1$, for each $x\in X$). An IF-set $A=\{x,\mu_A(x),\ v_A(x): x\in X\}$ can be written in the form A= $\{x,\mu_A,v_A\}$

Definition 1.2 [1] Let $A=\{x,\mu_A,v_A\}$, $B=\{x,\mu_B,v_B\}$ $A=\{x,\mu_A,v_A\}(t\in J)$. be IF -set on X and $f: X\to Y$ a function Then,

(i) $A=\{x,\mu_A,v_A\}$

(ii) $A\le B \Leftrightarrow$ for each $x\in X[\mu_A\le\mu_B$ and $v_A\ \ge v_B]$

(iii) $A=B\ \Leftrightarrow A\le B$ and $B\le A$

(iv) $\wedge A=\{x,\wedge\mu_A,\vee\ v_A\}$ [7]

(v) $\vee A=\{x,\vee\mu_A,\ \wedge v_A\}$ [7]

Definition 1.3 [6] Let A be an IF-set of an IF-ts (X,δ). Then A is called :

(i) An IF-regular open (IF-ro, for short) set if $A=int(cl(A))$

(ii) An IF-semi open (IF-so, for short) set if $A\le cl(int(A))$

(iii) An IF-preopen (IF-po, for short) set if $A\le int(cl(A))$

(iv) An IF-strongly semi open (IF-so, for short)set if $A\le int(cl(int(A)))$

(v) An IF-semi-preopen (IF-spo, for short) set if $A\le cl(int(cl(A)))$

Their complements are called IF-semi closed, IF-pre closed, IF-strongly semi closed and IF-semi-pre closed sets

Definition 1.3 [1] Let X and Y be two nonempty sets and $f: X\to Y$ be a function

(i) If $B=\ \{y,\mu_A(y),v_A(y): y\in Y\}$ is an IFS in Y, then the pre image of B under f(denoted by $f^{\leftarrow}(B)$) is defined by $f^{\leftarrow}(B)=\{x,f^{\leftarrow}(\mu_s(x),f^{\leftarrow}(v_s(x): x\in X\}$

(ii) If $A=\{x,\lambda_A(y),v_A(y): x\in X\}$ is an IFS in X, then the Image of A under f(denoted $f(A)$) is defined by $f(A)=\{x,f(\lambda_A)(y),(f((v_\lambda(y))')': y\in Y\}$

Definition 1.4 [6] Let (X,δ) be a IF –ts, and $A\subset X$, Then, the F-subspace (A,δ_A) is called a IF – retract (for short, IFR) of (X,δ) if there exists a IF-continuous mapping $r:(X,\delta)\ \to\ (A,\delta_A)$ such that $r(a)=a$ for all $a\in A$. In this case r is called an IF-retraction

Definition 1.5 [6] Let (X,δ) be a IF –ts .Then (A,δ_A) is said to be IF-Neighborhood retract (IF – nbd R) of (X,δ) if (A,δ_A) is a IFR of (Y,δ_Y) , Such that $A\subset Y\subset X$, $1_Y\ \in\delta$.

Definition 1.6 [6] Let (X,δ) be a IF –ts , and $A\subset X$, then the IF - subspace (A,δ_A) is called a IF - semi retract (for short, IFSR) (resp. IF – pre retract , IF – strongly semi retract and IF –semi pre retract) (IFPR , IFSSR, IFSPR) of (X,δ) if there exists a IF-semicontinuous (resp. IF-pre continuous, IF –strongly Semi continuous, IF-semi pre continuous) mapping $r:(X,\delta)\ \to\ (A,\delta_A)$ such that $r(\mathrm{a})=\mathrm{a}\ \ \forall\ \mathrm{a}\in A$. In this case, f is called an IF-semi retraction (resp., -IF pre retraction, IF-strongly semi retraction, IF- semi pre retraction)

Definition 1.7 [6] Let (X,δ) be an IF –ts .Then (A,δ_A) is said to be an IF- neighborhood semi retract , (for short, IF-nbd SR) (resp. IF-nbd pre retract , IF-nbd strongly semi retract, IF-nbd semi pre retract.) (for short, IF-nbd PR, IF-nbd SSR, IF-nbd SPR) of (X,δ) . (A,δ_A) is IFSR (resp. IFPR,IFSSR,IFSPR.) of (Y,δ_Y) , such that $A\subset Y\subset X$, $1_Y\ \in\ \delta$

2. IF$_\mu$- semiopen, IF$_\mu$- preopen , IF$_\mu$- strongly semiopen and IF$_\mu$- semi preopen sets

Definition 2.1 Let (X,δ) be a IF-ts , $\mu\in\mathrm{IFS}(X)$, $\nu\in\mathcal{A}_\mu$. Then ν is called

(i) a IF$_\mu$-semi open (briefly , IF$_\mu$so) set if there exist $\lambda\in\delta\mu$. such that $\lambda\le\nu\le Cl_\mu(\lambda)$ (or, $\nu\le Cl_\mu(Int_\mu(\nu))$.

(ii) a IF$_\mu$-preopen (briefly , IF$_\mu$po) set if $\nu\le\mathrm{Int}_\mu\left(Cl_\mu(\nu)\right)$.

(iii) a IF$_\mu$-regular open (briefly , IF$_\mu$ro) set if $\nu\le\mathrm{Int}_\mu(Cl_\mu(\nu))$.

(iv) a IF_μ- strongly semi open (briefly , IF_μsso) set if there exists $\lambda \in \delta\mu$ such that $\lambda \le \nu \le \mathrm{Int}_\mu(\mathrm{Cl}_\mu(\lambda))$. (or , $\nu \le \mathrm{Int}_\mu\left(\mathrm{Cl}_\mu(\mathrm{Int}_\mu(\nu))\right)$

(v) a IF_μ-semi preopen (briefly , IF_μspo) set if there exists a IF_μ- preopen set λ such that $\lambda \le \nu \le \mathrm{Cl}_\mu(\lambda)$(or, $\nu \le \mathrm{Cl}_\mu(\mathrm{Int}_\mu(\mathrm{Cl}_\mu(\nu))$

Their complements are called IF_μ-semi closed (briefly, IF_μsc) , IF_μ-pre closed (briefly , IF_μpc) , IF_μ-regular closed (briefly, IF_μ rc) , IF_μ-strongly semi closed (briefly , IF_μ ssc) , IF_μ-semipre closed (briefly, IF_μspc) set . $\mathrm{IF}_\mu\mathrm{SO}$, $\mathrm{IF}_\mu\mathrm{PO}$, $\mathrm{IF}_\mu\mathrm{RO}$, $\mathrm{IF}_\mu\mathrm{SSO}$ and $\mathrm{IF}_\mu\mathrm{SPO}$ (resp. $\mathrm{IF}_\mu\mathrm{SC}$, $\mathrm{IF}_\mu\mathrm{PC}$, $\mathrm{IF}_\mu\mathrm{RC}$, $\mathrm{IF}_\mu\mathrm{SSC}$, $\mathrm{IF}_\mu\mathrm{SPC}$) will always denote the family of IF_μ-semi open, IF_μ-preopen, IF_μ-regular open , IF_μ- strongly semi open, IF_μ- semi preopen (resp. IF_μ- semi closed, IF_μ- preclosed, IF_μ- regular closed , IF_μ- strongly semiclosed, IF_μ- semi preclosed) sets

Remark 2.1 The implications between these different notions of IF- sets are given by the following diagram.

IF_μro (IF_μ rc)
$\Downarrow \Longrightarrow$ IF_μso (IF_μsc)
IF_μo (IF_μ c) $\Longrightarrow$ IF_μsso (IF_μ ssc)
$\Longrightarrow$ IF_μspo (IF_μspc)
$\Longrightarrow$ IF_μpo (IF_μpc)

But the converse need not to be true, in general as shown by the following examples

Example 2.1 Let $\mathrm{X} = \mathrm{Y} = [0,1]$, Let (X, δ) IF-ts where , $\delta = \left\{ 0 ,1, C_{0.7,0.3}\, C_{\alpha,\beta} : 0 \le \alpha \le \frac{1}{2} ,\ 0 \le \beta \le \frac{1}{2} \right\}$, $\mu = C_{0.5,0.5}$ .Then $C_{0.1,0.3}$ is an IF_μ- semi preopen set but not IF_μ-preopen set, $C_{0.1,0.5}$ is an IF_μ- semi open set but not IF_μ-strongly semi open set, $C_{0.2,0.6}$ is an IF_μ- semi preopen set but not IF_μ- semi open set,

Example 2.2 Let $\mathrm{X} = \{\mathrm{a}, \mathrm{b}\}$ $\delta = \{ \underline{0}, \underline{1}, \lambda \}$ and , $\mu \in \mathrm{IFS(X)}$ are defined by,

$$\lambda = \langle x , \left(\frac{a}{0.2}, \frac{b}{0.3}\right) , \left(\frac{a}{0.4}, \frac{b}{0.5}\right) \rangle$$
$$\mu = \langle x , \left(\frac{a}{0.1}, \frac{b}{0.7}\right) , \left(\frac{a}{0.8}, \frac{b}{0.3}\right) \rangle$$
$$\theta = \langle x , \left(\frac{a}{0.3}, \frac{b}{0.2}\right) , \left(\frac{a}{0.7}, \frac{b}{0.7}\right) \rangle$$

θ is an IF_μ- preopen set but not IF_μ- strongly semi open set .

Example 2.3 Let $\mathrm{X} = \{\mathrm{a}, \mathrm{b}\}$ $\delta = \{ \underline{0}, \underline{1}, \lambda \}$ and, $\mu \in \mathrm{IFS(X)}$ are defined by ,

$$\lambda_1 = \langle x , \left(\frac{a}{0.2}, \frac{b}{0.5}\right) , \left(\frac{a}{0.3}, \frac{b}{0.2}\right) \rangle$$
$$\mu = \langle x , \left(\frac{a}{0.5}, \frac{b}{0.7}\right) , \left(\frac{a}{0.5}, \frac{b}{0.1}\right) \rangle$$
$$\lambda_2 = \langle x , \left(\frac{a}{0.1}, \frac{b}{0.4}\right) , \left(\frac{a}{0.2}, \frac{b}{0.2}\right) \rangle$$
$$\theta = \langle x , \left(\frac{a}{0.3}, \frac{b}{0.6}\right) , \left(\frac{a}{0.4}, \frac{b}{0.1}\right) \rangle$$

θ is an IF_μ- strongly semi open set but not IF_μ- open set . , λ_1 IF_μ- open set but not IF_μ-regular open set.

Theorem 2.1

(a) The μ-closure of a IF_μ- preopen set is a IF_μ-regular closed set ,

(b) The μ-interior of a $\mathrm{IF}_\mu -$ preclosed set is a IF_μ- regular open set

Proof . It is obvious

Theorem 2.2

(i) The intersection of two IF_μ- regular open sets is IF_μ- regular open ,

(ii) The union of two IF_μ- regular closed sets is a IF_μ- regular closed

Proof . It is obvious.

Proposition 2.1

(i) The intersection of any IF_μ-semi closed sets is also IF_μ-semi closed .

(ii) Any union of any IF_μ-semi open sets is also IF_μ-semi open

Proof . It is obvious

Theorem 2.3

(i) Arbitrary union of IF_μ-strongly semi open sets is IF_μ- strongly semi open

(ii) Arbitrary intersection of IF_μ-strongly semi closed sets is IF_μ-strongly semi closed (iii) Arbitrary union (intersection) of IF_μ- semi preopen (IF_μ- semi preclosed) sets is IF_μ- semi preopen (IF_μ- preclosed)

Proof . It is obvious

Remark 2.2 Let $\nu_1 \in \mathcal{A}_\lambda$ be a F_λ-closed set and $\nu_2 \in \mathcal{A}_\mu$ be a IF_μ- closed set . Then $\nu_1 \times \nu_2$ need not be a $\mathrm{F}_{\lambda\times\mu}$- closed set.

Example 2.4 Let λ_1 and λ be IF – sets on $\mathrm{X} = \{\mathrm{a}, \mathrm{b}\}$, defined by

$$\lambda_1 = \langle x , \left(\frac{a}{0.5}, \frac{b}{0.5}\right) , \left(\frac{a}{0.5}, \frac{b}{0.5}\right) \rangle$$
$$\lambda = \langle x , \left(\frac{a}{0.2}, \frac{b}{0.2}\right) , \left(\frac{a}{0.4}, \frac{b}{0.8}\right) \rangle$$
$$\nu_1 = \langle x , \left(\frac{a}{0.7}, \frac{b}{0.8}\right) , \left(\frac{a}{0.2}, \frac{b}{0.2}\right) \rangle$$

and $\delta = \{ \underline{0}, \underline{1}, \lambda_1 \}$ .Then ν_1 is a IF_λ- closed set . Let λ_2 and μ be IF – sets on $\mathrm{Y} = \{x, y\}$, defined by ,

$$\lambda_2 = \langle x , \left(\frac{a}{0.3}, \frac{b}{0.1}\right) , \left(\frac{a}{0.3}, \frac{b}{0.2}\right) \rangle$$
$$\mu = \langle x , \left(\frac{a}{0.1}, \frac{b}{0.6}\right) , \left(\frac{a}{0.2}, \frac{b}{0.2}\right) \rangle$$
$$\nu_2 = \langle x , \left(\frac{a}{0.3}, \frac{b}{0.2}\right) , \left(\frac{a}{0.1}, \frac{b}{0.1}\right) \rangle$$

and $\gamma = \{ \underline{0}, \underline{1}, \lambda_2 \}$. Then ν_2 is a IF_μ- closed set. But - $\nu_1 \times \nu_2$ is not a $\mathrm{IF}_{\lambda\times\mu}$ – closed set .

Remark 2.2 An IF_μ- semiopen set and a IF_μ- preopen set are independent concepts.

Example 2.5 Let λ_1, λ_2 and μ be IF- sets on $\mathrm{X} = \{\mathrm{a}, \mathrm{b}\}$, defined by

$$\lambda_1 = \langle x , \left(\frac{a}{0.5}, \frac{b}{0.3}\right) , \left(\frac{a}{0.2}, \frac{b}{0.2}\right) \rangle$$
$$\lambda_2 = \langle x , \left(\frac{a}{0.6}, \frac{b}{0.7}\right) , \left(\frac{a}{0.3}, \frac{b}{0.3}\right) \rangle$$
$$\mu = \langle x , \left(\frac{a}{0.8}, \frac{b}{0.8}\right) , \left(\frac{a}{0.2}, \frac{b}{0.2}\right) \rangle$$
$$\nu = \langle x , \left(\frac{a}{0.3}, \frac{b}{0.6}\right) , \left(\frac{a}{0.4}, \frac{b}{0.4}\right) \rangle$$

and $\delta = \{ \underline{0}, \underline{1}, \lambda_1, \lambda_2 \}$. Then ν is a IF_μ- preopen set, but not IF_μ- semiopen set .

Example 2.6 Let λ and μ be IF- sets on $X = \{a, b\}$, defined by

$$\lambda = \langle x, \left(\frac{a}{0.2}, \frac{b}{0.2}\right), \left(\frac{a}{0.6}, \frac{b}{0.6}\right)\rangle$$

$$\alpha = \langle x, \left(\frac{a}{0.2}, \frac{b}{0.2}\right), \left(\frac{a}{0.5}, \frac{b}{0.5}\right)\rangle$$

$$\mu = \langle x, \left(\frac{a}{0.2}, \frac{b}{0.2}\right), \left(\frac{a}{0.4}, \frac{b}{0.4}\right)\rangle$$

and $\delta = \{\underline{0}, \underline{1}, \lambda\}$. Then α is a IF_μ- semiopen set, but not IF_μ- preopen set .

Theorem 2.4. Let (X, δ) be a F-ts, $\mu \in I^X, \nu \in \mathcal{A}_\mu$. Then the following are equivalent .

(i) ν is a IF_μ- semi closed set ,

(ii) $(\mu - \nu)$ is a IF_μ- semiopen

(iii) $\mathrm{Int}_\mu(\mathrm{Cl}_\mu(\nu)) \leq \nu$

(iv) $\mathrm{Cl}_\mu(\mathrm{Int}_\mu(\mu - \nu)) \geq \mu - \nu$

Proof . It is obvious

3. IF_μ-semi continuous, IF_μ- precontinuous , IF_μ- strongly semi continuous and IF_μ- semi precontinuous mappings

Definition 3.1 Let $f : (X, \delta) \longrightarrow (Y, \gamma)$ be a mapping from a IF-ts(X, δ) to another IF-ts (Y, γ), $\mu \in \mathrm{IFS}(X)$. Then , f is called :

(i) a IF_μ-semicontinuous (briefly , IF_μ sc) mapping if for each $\nu \in \gamma_f \longrightarrow (\mu)$, we have $\mu \wedge f^{\leftarrow}(\nu) \in IF_\mu SO$.

(ii) a IF_μ- precontinuous (briefly , IF_μ pc) mapping if for each $\nu \in \gamma_f \longrightarrow (\mu)$, we have $\mu \wedge f^{\leftarrow}(\nu) \in IF_\mu PO$.

(iii) a IF_μ- strongly semi continuous (briefly , IF_μ ssc) mapping if for each $\nu \in \gamma_f \longrightarrow (\mu)$, we have $\mu \wedge f^{\leftarrow}(\nu) \in IF_\mu$ SSO .

(iv) a IF_μ- semi precontinuous (briefly , IF_μ spc) mapping if for each $\nu \in \gamma_f \longrightarrow (\mu)$, we have $\mu \wedge f^{\leftarrow}(\nu) \in IF_\mu SPO$

Remark 3.1 The implications between these different concepts are given by the following diagram

$\Rightarrow$ IF_μpc $\Rightarrow$

IFc $\Rightarrow$ IF_μc $\Rightarrow$ IF_μ ssc

IF_μsp

$\Rightarrow$ IF_μsc $\Rightarrow$

Example 3.1 Let $X = Y = [0.1]$, Let (X, δ) and (Y, η) be two IF-ts's where,

$\delta = \left\{0, 1, C_{0.7,0.3}\, C_{\alpha,\beta} : 0 \leq \alpha \leq \frac{1}{2}, 0 \leq \beta \leq \frac{1}{2}\right\}$ and $\eta = \{\underline{0}, \underline{1}, C_{0.0,0.5}\}$, $\mu = C_{0.3,0.3}$ $f(x) = x$. Then $f : (X, \delta) \longrightarrow (Y, \eta)$ is an IF_μ –continuous but not IF- continuous mapping .

Example 3.2 Let $X = Y = [0.1]$, Let (X, δ) and (Y, η) be two IF-ts's where, $\delta = \left\{0, 1, C_{0.7,0.3}\, C_{\alpha,\beta} : 0 \leq \alpha \leq \frac{1}{2}, 0 \leq \beta \leq \frac{1}{2}\right\}$ and

$\eta = \{\underline{0}, \underline{1}, C_{0.5,0.4}, C_{0.1,0.2}, C_{0.5,0.2}, C_{0.1,0.4}, C_{0.5,0.5}\}$,

$f(x) = x$. Then $f : (X, \delta) \longrightarrow (Y, \eta)$ is an IF_μ – semi precontinuous but not IF_μ- precontinuous mapping , also , IF_μ – semi continuous but not IF_μ –strongly semi continuous mapping .

Example 3.3 Let $X = Y = [0.1]$, Let (X, δ) and (Y, η) be two IF-ts's where, $\delta = \left\{0, 1, C_{0.7,0.3}, C_{\alpha,\beta} : 0 \leq \alpha \leq \frac{1}{2}, 0 \leq \beta \leq \frac{1}{2}\right\}$ and $\eta = \{\underline{0}, \underline{1}, C_{0.0,0.7}\}$, $\mu = C_{0.5,0.5}$ $f(x) = x$. Then $f : (X, \delta) \longrightarrow (Y, \eta)$ is an IF_μ –strongly semi continuous but not IF_μ –continuous mapping, also, IF_μ –semi precontinuous but not IF_μ – semicontinuous mapping .

Example 3.4 Let $X = Y = [0.1]$, Let (X, δ) and (Y, η) be two IF-ts's where,

$\delta = \left\{0, 1, C_{0.1,0.2}, C_{0.0,0.5}, C_{0.25,0.2}, C_{\alpha,\beta} : 0 \leq \alpha \leq \frac{1}{4}, 0 \leq \beta \leq \frac{1}{2}\right\}$ and

$\eta = \{\underline{0}, \underline{1}, C_{0.1,0.5}, C_{0.3,0.2}\}$, $\mu = C_{0.2,0.2}$, $f(x) = x$. Then, $f : (X, \delta) \longrightarrow (Y, \eta)$ is an IF_μ- precontinuous but not IF_μ- strongly semi continuous mapping .

Theorem 3.1 Let $f : (X, \delta) \longrightarrow (Y, \gamma)$ be an IF_μ- strongly semi continuous mapping . The following statements are equivalent:

(i) The inverse image of each IF_μ-closed set is IF_μ- strongly semi closed

(ii) $\left(\mathrm{cl}\left(\mathrm{int}\left(\mathrm{cl}\left(f^{\leftarrow}(\alpha)\right)\right)\right)\right) \leq \left(f^{\leftarrow}(\mathrm{cl}\,\alpha)\right)$. for each $\alpha \in IF_\mu$-open set of Y

(iii) $f\left(\mathrm{cl}\left(\mathrm{int}\left(\mathrm{cl}\left((\beta)\right)\right)\right)\right) \leq \mathrm{cl}(f(\beta))$. for each $\beta \in IF_\mu$-open set of X

Proof (i) $\Rightarrow$ (ii) Since $\mathrm{cl}\alpha$ is IF_μ- closed set of Y , $f^{\leftarrow}(\mathrm{cl}\,\alpha)$ is IF_μ- strongly semi closed and hence

$\left(\mathrm{cl}\left(\mathrm{int}\left(\mathrm{cl}\left(f^{\leftarrow}(\alpha)\right)\right)\right)\right) \leq \left(\mathrm{cl}\left(\mathrm{int}\left(\mathrm{cl}\left(f^{\leftarrow}(\mathrm{cl}\alpha)\right)\right)\right)\right) \leq \left(f^{\leftarrow}(\mathrm{cl}\alpha)\right)$

(ii) $\Rightarrow$ (iii) Let $\beta \in IF_\mu$ –open set of X . and $\alpha = f(\beta)$. By (ii) $\left(\mathrm{cl}\left(\mathrm{int}(\mathrm{cl}(\beta))\right)\right) \leq \left(\mathrm{cl}\left(\mathrm{int}\left(\mathrm{cl}\left(f^{\leftarrow}(\alpha)\right)\right)\right)\right) \leq \left(f^{\leftarrow}(\mathrm{cl}\,\alpha)\right)$. Thus $f\left(\mathrm{cl}\left(\mathrm{int}\left(\mathrm{cl}\left((\beta)\right)\right)\right)\right) \leq f(f^{\leftarrow}(\mathrm{cl}\,\alpha)) \leq cl(\alpha) = cl(f(\beta))$

(iii) $\Rightarrow$ (i) Let α be an IF_μ – closed set of Y , and $f^{\rightarrow}(\alpha) = \beta$. By (iii) $f(cl(\mathrm{int}\,(\mathrm{cl}((\beta))))) \leq \mathrm{cl}(f(\beta)) \leq \mathrm{cl}\ \alpha = \alpha$. Hence, $\mathrm{cl}\,(\mathrm{int}\,(\mathrm{cl}\,((\beta)))) \leq (f^{\leftarrow}(\alpha))$, i .e. , $(\mathrm{cl}\,(\mathrm{int}\,(\mathrm{cl}\,(f^{\leftarrow}(\alpha))))) \leq (f^{\leftarrow}(\alpha))$ and $f^{\leftarrow}(\alpha)$ IF_μ- strongly semiclosed.

Definition 3.2 .Let (X, δ) and (Y, γ) be IF-ts's, $\lambda \in \mathrm{IFS}(X)$, $\mu \in \mathrm{IFS}(Y)$ Then we define $\delta_\lambda \times \gamma_\mu$ as follows $\delta_\lambda \times \gamma_\mu = \{\eta \times \zeta : \eta \in \delta_\lambda, \zeta \in \gamma_\mu\}$

Lemma 2.1 Let (X, δ) and (Y, γ) . be IF-ts's, $\lambda \in$ IFS(X), $\mu \in$ IFS(Y)

Then, $\delta_\lambda \times \gamma_\mu = (\delta \times \gamma)_{\lambda \times \mu}$

Proof Follows directly from definition (3.2)

Theorem 3.2 (Coker 1996) Let (X, δ) , (Y, γ) and (Z, ρ) be IF-ts's, $\mu \in \mathrm{IFS}(X)$,

f: X $\longrightarrow$ Y and g : Y $\longrightarrow$ Z be mappings . If f is IF_μ-

continuous and g is IF$_{f\rightarrow(\mu)}$- continuous , then gf is IF$_{\mu}$- continuous

Theorem 3.3 Let (X_1, δ_1), (X_2, δ_2), (Y_1, γ_1) and (Y_2, γ_2) be IF-ts's, $\lambda \in$ IFS(X_1) and $\mu \in$ IFS(X_2). Then , $f_1 : X_1 \longrightarrow Y_2$ is IF$_{\lambda}$-continuous and $f_2 : X_2 \longrightarrow Y_2$ is IF$_{\mu}$-continuous iff the product . $f_1 \times f_2 : X_1 \times X_2 \longrightarrow Y_1 \times Y_2$ is IF$_{\lambda\times\mu}$continuous .

Proof Let $\eta \in (\gamma_1 \times \gamma_2)_{(f_1 \times f_2)} \longrightarrow_{(\lambda\times\mu)}$, i .e . , $\eta = (f_1 \times f_2)^{\rightarrow}(\lambda \times \mu) \wedge (\vee (\lambda_\alpha \times \mu_\beta))$, where λ_α' s and μ_β's are IF- open sets of (Y_1, γ_1) and (Y_2, γ_2), respectively , we want to show that $(\lambda \times \mu) \wedge (f_1 \times f_2)^{\leftarrow}(\eta) = (\lambda \times \mu) \wedge (f_1 \times f_2)^{\leftarrow}((f_1 \times f_2)^{\rightarrow}(\lambda \times \mu) \wedge (\vee (\lambda_\alpha \times \mu_\beta))) \in (\delta_{1\times}\delta_2)_{\lambda\times\mu}$. Since $f_1 : (X_1, \delta_1) \longrightarrow (Y_1, \gamma_1)$ is IF$_{\lambda}$ – continuous , $f_1^{\rightarrow}(\lambda) \wedge \lambda_\alpha \in (\gamma_1)_{f_1^{\rightarrow}(\lambda)}$, then $\lambda \wedge f^{\leftarrow}(f_1^{\rightarrow}(\lambda) \wedge \lambda_\alpha) = \lambda \wedge f^{\leftarrow}(\lambda_\alpha) \in (\delta_1)_\lambda$. Also , since $f_2 : (X_2, \delta_2) \longrightarrow (Y_2, \gamma_2)$, is IF- continuous , $\mu_\beta \wedge f_2^{\rightarrow}(\mu) \in (\gamma_2)_{f_2^{\rightarrow}(\mu)}$, then $\mu \wedge f^{\leftarrow}(f_2^{\rightarrow}(\mu) \wedge \mu_\beta = \mu \wedge f^{\leftarrow}(\mu_\beta) \in (\delta_2)_\mu$. By using lemma (2.1). We get $(\lambda \wedge f^{\leftarrow}(\lambda_\alpha)) \times (\mu \wedge f^{\leftarrow}(\mu_\beta)) = (\lambda \times \mu) \wedge (f^{\leftarrow}(\lambda_\alpha) \times f^{\leftarrow}(\mu_\beta)) = (\lambda \times \mu) \wedge (f_1 \times f_2)^{\leftarrow}(\lambda_\alpha \times \mu_\beta) \in (\delta_{1\times}\delta_2)_{\lambda\times\mu}$, hence, $(\lambda \times \mu) \wedge (f_1 \times f_2)^{\leftarrow}(\eta) \in (\delta_{1\times}\delta_2)_{\lambda\times\mu}$

Conversely , Let $\nu \in (\gamma_1)_{f_1^{\rightarrow}(\lambda)}$, i. e. $\nu = f_1^{\rightarrow}(\lambda) \wedge \zeta$ where $\zeta \in \gamma_1$, $\zeta \times 1 \in \gamma_1 \times \gamma_2$ and $(\zeta \times 1) \wedge (f_1 \times f_2)^{\rightarrow}(\lambda \times \mu) \in (\gamma_1 \times \gamma_2)_{(f_1 \times f_2)^{\rightarrow}(\lambda\times\mu)}$, since $f_1 \times f_2 : X_1 \times X_2 \longrightarrow Y_1 \times Y_2$ is a IF$_{\lambda\times\mu}$- continuous , we have $(\lambda \times \mu) \wedge (f_1 \times f_2)^{\leftarrow}((\zeta \times 1) \wedge (f_1 \times f_2)^{\rightarrow}(\lambda \times \mu)) = (\lambda \times \mu) \wedge (f_1 \times f_2)^{\leftarrow}(\zeta \times 1) = (\lambda \times \mu) \wedge (f^{\leftarrow}(\zeta) \wedge 1) = (\lambda \wedge f^{\leftarrow}(\zeta)) \times \mu \in (\delta_{1\times}\delta_2)_{(\lambda\times\mu)} = (\delta_1)_\lambda \times (\delta_2)_\mu$, i . e , $\lambda \wedge f^{\leftarrow}(\zeta) = \lambda \wedge f^{\leftarrow}(f_1^{\rightarrow}(\lambda) \wedge \zeta) = \lambda \wedge f^{\leftarrow}(\nu) \in (\delta_1)_\lambda$.

Hence , f_1 is a IF$_{\lambda}$- continuous . The proof with respect to f_2 in the same fashion .

Theorem 3.4 Let (X, δ), (Y, γ) be IF-ts's . and $f : (X, \delta) \longrightarrow (Y, \gamma)$ be a mapping . Then , the graph $g : (X, \delta) \longrightarrow (X \times Y, \theta)$ of f is IF$_{\mu}$- continuous iff f is IF$_{\mu}$- continuous , where θ is the F – product topology generated by δ and γ

Proof suppose the graph $g : (X, \delta) \longrightarrow (X \times Y, \theta)$ is IF$_{\mu}$- continuous . Let $\nu \in \gamma_{f\rightarrow(\mu)}$, i.e. , $\nu = f^{\rightarrow}(\mu) \wedge \eta$ where $\eta \in \gamma$, we want to show that , $\mu \wedge f^{\leftarrow}(f_1^{\rightarrow}(\mu) \wedge \eta) \in \delta_\mu$. since $\underline{1} \times \eta \in \theta$, $g^{\rightarrow}(\mu) \wedge (\underline{1} \times \eta) \in \theta_{g\rightarrow(\mu)}$, then $\mu \wedge g^{\leftarrow}(g^{\rightarrow}(\mu) \wedge (\underline{1} \times \eta)) = \mu \wedge g^{\leftarrow}(\underline{1} \times \eta) = \mu \wedge (\underline{1} \wedge f^{\leftarrow}(\eta)) = \mu \wedge f^{\leftarrow}(\eta) = \mu \wedge f^{\leftarrow}(f^{\rightarrow}(\mu) \wedge \eta) \in \delta_\mu$. so $f^{\rightarrow}$ is IF$_{\mu}$- continuous

Conversely , suppose f is IF$_{\mu}$- continuous , let $\xi \in \theta_{g\rightarrow(\mu)}$, i . e . $\xi = g \rightarrow(\mu) \wedge (\vee (\lambda_\alpha \times \mu_\beta))$, where $\lambda\alpha$'s and μ_β' s are F- open set of δ and γ respectively. Now $\mu \wedge g^{\leftarrow}(\xi) = \mu \wedge g^{\leftarrow}(g^{\rightarrow}(\mu) \wedge (\vee (\lambda_\alpha \times \mu_\beta))) = \mu \wedge g^{\leftarrow}(\vee (\lambda_\alpha \times \mu_\beta)) = \vee (\mu \wedge (\lambda_\alpha \wedge f^{\leftarrow}(\mu_\beta)) = \vee (\lambda_\alpha \wedge (\mu \wedge (f^{\leftarrow}(\mu_\beta)) = \vee (\lambda_\alpha \wedge \mu \wedge f^{\leftarrow}(f^{\rightarrow}(\mu) \wedge \mu_\beta)) \in \delta_\mu$. So $g \rightarrow$ is IF$_{\mu}$- continuous

Proposition 3.1 Let $\mu \in I^X$, $f : (X, \delta) \longrightarrow (X, \gamma)$ be an injective and IF$_{\mu}$- continuous mapping. Then for each $\nu \in \gamma^c_{f\rightarrow(\mu)}$ we have $\mu - (\mu \wedge f^{\leftarrow}(\nu))$ is IF$_{\mu}$-semi open (res . , IF$_{\mu}$-preopen , IF$_{\mu}$-strongly semi open , IF$_{\mu}$-semi preopen) set

Proof . It is obvious

Proposition 3.2 Let (X, δ), (Y, γ) be F-ts's , $\mu \in I^X$ and $f : X \longrightarrow Y$ be a bijective map. Then , if f is a IF$_{\mu}$-homeomorphism , then f is a IF$_{\mu}$-semi continuous (resp. IF$_{\mu}$-precontinuous , IF$_{\mu}$-strongly semi continuous , IF$_{\mu}$- semi pre continuous) mappings

Proof . It is obvious

Theorem 3.5 Let (X, δ), (Y, γ) be IF-ts's , $\mu \in$ IFS(X) , If $f : (X, \delta) \longrightarrow (Y, \gamma)$. is IF$_{\mu}$- semi continuous and IF$_{\mu}$- precontinuous , then f is IF$_{\mu}$-strongly semi- continuous

Proof The proof is simple and hence omitted

Remark 3.2 a IF$_{\mu}$- semicontinuity and IF$_{\mu}$-precontinuity are independent concepts .

Example 3.5 Let $X = \{a, b\}$, $Y = \{x, y\}$, $\delta = \{\underline{0}, \underline{1}, \lambda_1\}$ and $\gamma = \{\underline{0}, \underline{1}, \lambda_2\}$. λ_1, $\mu \in$ IFS(X) and $\lambda_2 \in$ IFS(Y) are defined by ,

$$\lambda_1 = \langle x, \left(\frac{a}{0.1}, \frac{b}{0.2}\right), \left(\frac{a}{0.2}, \frac{b}{0.2}\right) \rangle$$

$$\lambda_2 = \langle x, \left(\frac{a}{0.5}, \frac{b}{0.5}\right), \left(\frac{a}{0.4}, \frac{b}{0.5}\right) \rangle$$

$$\mu = \langle x, \left(\frac{a}{0.6}, \frac{b}{0.6}\right), \left(\frac{a}{0.4}, \frac{b}{0.4}\right) \rangle$$

$f(a) = y$, $f(b) = x$ Then , f is IF$_{\mu}$- pre continuous but not IF$_{\mu}$- semicontinuous mapping

Example 3.6 Let $X = Y = [0, 1]$, Let (X, δ) and (Y, η) be two IF-ts's where , $\delta = \left\{0, 1, C_{0.7,0.3}, C_{\alpha,\beta} : 0 \le \alpha \le \frac{1}{2}, 0 \le \beta \le \frac{1}{2}\right\}$ and $\eta = \{\underline{0}, \underline{1}, C_{0.0,0.3}, C_{0.4,0.2}\}$, $\mu = C_{0.2,0.2} f(x) = x$. Then $f : (X, \delta) \longrightarrow (Y, \eta)$ is an IF$_{\mu}$-semicontinuous but not IF$_{\mu}$-Pre continuous mapping .

4. IF$_{\mu}$-retract and IF$_{\mu}$- neighbourhood retract

Definition 4.1 Let (X, δ) be IF-ts , and $A \subset X$, Then , the IF- subspace (A, δ_A) is called a IF$_{\mu}$-retract (IF$_{\mu}$ − R) of (X, δ) if there exists a IF$_{\mu}$-continuous mapping $r : (X, \delta) \longrightarrow (A, \delta_A)$ such that $r(a) = a \ \forall \ a \in A$. In this case r is called a IF$_{\mu}$-retraction

Remark 4.1 Every IF – retract is a IF$_{\mu}$- R , but the converse is not true

Example 4.1 Let λ and μ be IF-sets on $X = \{a,$

$b, c\}$, defined by

$\lambda = \langle x, \left(\frac{a}{0.5}, \frac{b}{0.6}, , \frac{c}{0.4}\right), \left(\frac{a}{0.3}, \frac{b}{0.3}, , \frac{c}{0.3}\right)\rangle$

$\mu = \langle x, \left(\frac{a}{0.6}, \frac{b}{0.7}, , \frac{c}{0.3}\right), \left(\frac{a}{0.2}, \frac{b}{0.2}, , \frac{c}{0.2}\right)\rangle$

, $\delta = \{\underline{0}, \underline{1}, \lambda\}$, and $A = \{a, b\} \subset X$ Then, (A, δ_A) is a IF_μ-R of (X, δ), but not a IF retract

Remark 4.2 Let (X, δ) be a IF-ts. Since the identity map $\mathrm{id}_X : X \longrightarrow X$ is IF_μ-continuous, then X is a IF_μ- R or itself.

Proposition 3.1. Let $Z \subset Y \subset X$, $\mu \in \mathrm{IFS}(X)$, $r_1 : (X, \delta) \longrightarrow (Y, \delta_Y)$ be IF_μ-retraction, $r_2 : (Y, \delta_Y) \longrightarrow (Z, (\delta_Y)_Z)$ be $\mathrm{IF}_{r_{1\to(\mu)}}$-retraction. Then $r_2 r_1 : (X, \delta) \longrightarrow (Z, (\delta_Y)_Z)$ is a IF_μ-retraction

Proof It follows from theorem 3.2

Proposition 4.2 Let (X, δ) be a IF-ts, $A \subset X$ and $\mu \in IFS(X)$. Then the function $r : (X, \delta) \longrightarrow (A, \delta_A)$ is IF_μ-retraction iff for any IF-ts (Y, γ), every $\mathrm{IF}_{r\to(\mu)}$- continuous function $g : (A, \delta_A) \longrightarrow (Y, \gamma)$ has a IF_μ-continuous function $\bar{g} : (X, \delta) \longrightarrow (Y, \gamma)$ such that $\bar{g}| A = g$

Proof Let $r : (X, \delta) \longrightarrow (A, \delta_A)$ be IF_μ-retraction $g : (A, \delta_A) \longrightarrow (Y, \gamma)$ be $\mathrm{F}_{r\to(\mu)}$ continuous function, By Theorem 2.2.2. $\bar{g} = gr : (X, \delta) \longrightarrow (Y, \gamma)$ is IF_μ-continuous and $\forall$ a $\in A$, $\bar{g}(a) = gr(a) = g(a)$

Conversely, let $(Y, \gamma) = (A, \delta_A)$, then $\mathrm{g} = \mathrm{id}_A$. Since g is $\mathrm{IF}_{r\to(\mu)}$- continuous, then g has a fuzzy μ-continuous $\bar{g} : (X, \delta) \longrightarrow (A, \delta_A)$ and $\bar{g}| A = \mathrm{id}_A$.

Theorem 4.1 Let (X, δ) be a IF- ts, $A \subset X$ and $r : (X, \delta) \longrightarrow (A, \delta_A)$ be a mapping such that $r(a) = a \ \forall\ a \in A$. Then the graph $g : (X, \delta) \longrightarrow (X \times A, \theta)$ of r is IF_μ-continuous iff r is a IF_μ- retraction, where θ is the product topology generated by δ and δ_A

Proof. It follows directly from Theorem 3.3.

Definition 4.2 Let (X, δ) be a IF-ts, $\mu \in \mathrm{IFS}(X)$. Then (A, δ_A) is said to be a IF_μ-neighborhood retract (IF_μ- nbd R) of (X, δ) if (A, δ_A) is a $\mathrm{IF}_{\mu|Y}$ – R of (Y, δ_Y), such that $A \subset Y \subset X$, $1_Y \in \delta$.

Remark 4.3 Every IF_μ-R is a IF_μ-nbd R, but the converse is not true.

Example 4.2 Let $X = \{a, b, c\}$, $A = \{a\} \subset X$, λ_1, λ_2 and μ be IF-sets on X, defined by

$\lambda_1 = \langle x, \left(\frac{a}{0.2}, \frac{b}{0.2}, , \frac{c}{0.4}\right), \left(\frac{a}{0.3}, \frac{b}{0.3}, , \frac{c}{0.3}\right)\rangle$

$\lambda_2 = \langle x, \left(\frac{a}{1.0}, \frac{b}{1.0}, , \frac{c}{0.0}\right), \left(\frac{a}{0.0}, \frac{b}{0.0}, , \frac{c}{1.0}\right)\rangle$

$\mu = \langle x, \left(\frac{a}{0.8}, \frac{b}{0.8}, , \frac{c}{0.5}\right), \left(\frac{a}{0.3}, \frac{b}{0.3}, , \frac{c}{0.3}\right)\rangle$

Consider $\delta = \{\underline{0}, \underline{1}, \lambda_1, \lambda_2, \lambda_1 \vee \lambda_2, \lambda_1 \wedge \lambda_2\}$. Then (A, δ_A) is a IF_μ-nbd R of (X, δ), but not a IF_μ-R of (X, δ).

Proposition 4.3 Let (X, δ) and (Y, γ) be IF-ts's, $A \subset X$, $B \subset Y$, $\lambda \in \mathrm{IFS}(X)$ and $\in \mathrm{IFS}(Y)$. If (A, δ_A) is a IF_λ-nbd R of (X, δ) and (B, γ_B) is a IF_μ-nbd R of (Y, γ), then $(A \times B, (\delta \times \gamma)_{A\times B})$ is a $\mathrm{IF}_{\lambda\times\mu}$-nbd R of $(X \times Y, \delta \times \gamma)$.

Proof Since (A, δ_A) is a F_λ-nbd R of (X, δ), then (A, δ_A) is a $\mathrm{IF}_{\lambda|U}$- R of (U, δ_U) such that $A \subset U \subset X$, $1_U \in \delta$, this implies that, there exists a $\mathrm{IF}_{\lambda|U}$-continuous mapping $r_1 : (U, \delta_U) \longrightarrow (A, (\delta_U)_A)$ such that $r_1(a) = a\ \forall a \in A$. Also since (B, γ_B) is a IF_μ-nbd R of (Y, γ), then (B, γ_B) is a $\mathrm{IF}_{\mu|V}$ –R of (V, γ_V) such that $\mathrm{B} \subset \mathrm{V} \subset Y$, $1_V \in \gamma$, this implies that, there exists a $\mathrm{IF}_{\mu|V}$- continuous mapping $r_2 : (V, \gamma_V) \longrightarrow (B, (\gamma_V)_B)$, such that $r_2(b) = b\ \forall\ b \in B$, by using Theorem 3.3 and Lemma 3.1 we have $(r_1 \times r_2) : (U \times V, (\delta \times \gamma)_{U\times V}) \longrightarrow (A \times B, ((\delta \times \gamma)_{U\times V})_{A\times B}$ is a $\mathrm{IF}_{\lambda\times\mu}$- continuous mapping, $1_U \times 1_V \in \delta \times \gamma$, and $(r_1 \times r_2)(a, b) = (r_1(a), r_2(b)) = (a, b)\ \forall\ (a, b) \in A \times B$. Hence, $A \times B$ is a $\mathrm{IF}_{\lambda\times\mu}$-nbd R of $X \times Y$.

Proposition 3.4. Let (X, δ) and (Y, γ) be IF-ts's $\lambda \in \mathrm{IFS}(X)$, $\mu \in \mathrm{IFS}(Y)$, $A \subset X$, $B \subset Y$. If (A, δ_A) is a F_λ- R of (X, δ) and (B, γ_B) is a IF_μ – R of (Y, γ), then $(A \times B, (\delta \times \gamma)_{A\times B})$ is a $\mathrm{F}_{\lambda\times\mu}$-R of $(X \times Y, \delta \times \gamma)$

Proof The proof is much simpler than that of Proposition 4.3. and hence omitted

5 -IF_μ – semiretract, IF_μ – preretract, IF_μ – strongly semiretract and IF_μ – semi preretract

Definition 5.1 Let (X, δ) be a IF-ts, $\mu \in \mathrm{IFS}(X)$ and $A \subset X$. Then the F – subspace (A, δ_A) is called a IF_μ – semi retract ($\mathrm{IF}_\mu - \mathrm{SR}$) (resp . IF_μ- pre retract, IF_μ- strongly semiretract and IF_μ- semi preretract) (IF_μ -PR, IF_μ - SSR, IF_μ - SPR) of (X, δ) if there exists a IF_μ- semi continuous (resp . IF_μ- precontinuous, IF_μ-strongly semi continuous, IF_μ- semi pre continuous) mapping $r : (X, \delta) \longrightarrow (A, \delta_A)$ such that $r(a) = \forall\ a \in A$. In this case, f is called a IF_μ-semi retraction (resp . - IF_μ- perpetration, IF_μ- strongly semiretraction, IF_μ- semi preretraction)

$\Longrightarrow$ IF_μ-PR $\Longrightarrow$

IF -R $\Longrightarrow$ IF_μ-R $\Longrightarrow$ IF_μ -SSR

IF_μ- SPR

$\Longrightarrow$ IF_μ-SR $\Longrightarrow$

But, the converse is not true in general, as we indicate the following examples.

Remark 5.1 Example 3.1 shows that a IF_μ- R need not to be a IF- retract.

Example 5.1 Let $X = \{a, b\}$, $A = \{a\} \subset X$ and λ_1, λ_2 and μ be IF- sets on X defined by

$\lambda_1 = \langle x, \left(\frac{a}{0.2}, \frac{b}{0.3}, , \frac{c}{0.4}\right), \left(\frac{a}{0.5}, \frac{b}{0.6}, , \frac{c}{0.6}\right)\rangle$

$\lambda_2 = \langle x, \left(\frac{a}{0.1}, \frac{b}{0.2}, , \frac{c}{0.3}\right), \left(\frac{a}{0.6}, \frac{b}{0.7}, , \frac{c}{0.7}\right)\rangle$

$\mu = \langle x, \left(\frac{a}{0.3}, \frac{b}{04}, , \frac{c}{0.2}\right), \left(\frac{a}{0.2}, \frac{b}{0.2}, , \frac{c}{0.3}\right)\rangle$

Consider $\delta = \{\underline{0}, \underline{1}, \lambda_1, \lambda_2\}$. Then (A, δ_A) is a IF_μ- SPR, but not a IF_μ-PR, also, IF_μ- SR, but not a IF_μ- SSR.

Example 5.2. Let $X = \{a,b\}$, $A = \{a\} \subset X$ and λ , μ be IF- sets on X defined by

$\lambda = \langle x , \left(\frac{a}{0.5}, \frac{b}{0.6}\right) , \left(\frac{a}{0.1}, \frac{b}{0.3}\right) \rangle$

$\mu = \langle x , \left(\frac{a}{0..3}, \frac{b}{0.8}\right) , \left(\frac{a}{0.2}, \frac{b}{0.2}\right) \rangle$

Consider $\delta = \{\underline{0} , \underline{1} , \lambda \}$. Then (A , δ_A) is a IF$_\mu$- PR , but not a IF$_\mu$-SSR .

Example 5.3. Let $X = \{a,b\}$, $A = \{a\} \subset X$ and λ_1, $\lambda_2$and μ be IF- sets on X defined by

$\lambda_1 = \langle x , \left(\frac{a}{0.5}, \frac{b}{0.3}\right) , \left(\frac{a}{0.2}, \frac{b}{0.2}\right) \rangle$

$\lambda_1 = \langle x , \left(\frac{a}{0.6}, \frac{b}{0.4}\right) , \left(\frac{a}{0.2}, \frac{b}{0.2}\right) \rangle$

$\mu = \langle x , \left(\frac{a}{05}, \frac{b}{0.9}\right) , \left(\frac{a}{0.2}, \frac{b}{0.1}\right) \rangle$

Consider $\delta = \{\underline{0} , \underline{1} , \lambda_1 , \lambda_2 \}$. Then (A , δ_A) is a IF$_\mu$-SSR , but not a IF$_\mu$-R

Example 5.4 Let $X = \{a,b\}$, $A = \{a\} \subset X$ and λ_1, λ_2 and μ be IF- sets on X defined by

$\lambda = \langle x , \left(\frac{a}{0.2}, \frac{b}{0.3}, , \frac{c}{0.3}\right) , \left(\frac{a}{0.5}, \frac{b}{0.6}, , \frac{c}{0.7}\right) \rangle$

$\mu = \langle x , \left(\frac{a}{0.1}, \frac{b}{0.2}, , \frac{c}{0.4}\right) , \left(\frac{a}{0.5}, \frac{b}{0.7}, , \frac{c}{0.3}\right) \rangle$

Consider $\delta = \{\underline{0} , \underline{1} , \lambda \}$. Then (A , δ_A) is a IF$_\mu$- SPR , but not a IF$_\mu$-SR

Proposition 5.1 Let $\mu \in \mathrm{IFS}(X)$, (X , δ) be a IF-ts, $A \subset X$ and $r : (X , \delta) \longrightarrow (A , \delta_A)$ be a mapping such that $r(a) = a \; \forall \; a \in A$. If r is a IF$_\mu$- precontinuous and IF$_\mu$-semicontinuous , then (A , δ_A) is a IF$_\mu$-SSR of (X , δ)

Proof . It follows directly from Theorem 3.5

Remark 5.2 A IF$_\mu$- semiretract and IF$_\mu$- pre retract are independent concepts

Example 5.6 Let $X = \{a,b\}$, $A = \{a\} \subset X$ and λ and μ be IF- sets on X defined by

$\lambda = \langle x , \left(\frac{a}{0.3}, \frac{b}{0.1}\right) , \left(\frac{a}{0.6}, \frac{b}{0.6}\right) \rangle$

$\mu = \langle x , \left(\frac{a}{0.2}, \frac{b}{0.3}\right) , \left(\frac{a}{0.4}, \frac{b}{0.5}\right) \rangle$

onsider $\delta = \{\underline{0} , \underline{1} , \lambda \}$ .Then (A , δ_A) is a IF$_\mu$- SR of (X , δ) , but not a IF$_\mu$-PR

Example 5.7 Let $X = \{a,b\}$, $A = \{a\} \subset X$ and λ and μ be IF- sets on X defined by

$\lambda = \langle x , \left(\frac{a}{0.5}, \frac{b}{0.3}\right) , \left(\frac{a}{0.1}, \frac{b}{0.7}\right) \rangle$

$\mu = \langle x , \left(\frac{a}{0.3}, \frac{b}{0.8}\right) , \left(\frac{a}{0.2}, \frac{b}{0.2}\right) \rangle$

Consider $\delta = \{\underline{0} , \underline{1} , \lambda \}$ .Then (A , δ_A) is a IF$_\mu$- PR of (X , δ) , but not a IF$_\mu$-SR

CONCLUSION

The purpose of this paper is to define (IF$_\mu$- open sets) and many results of IF$_\mu$- strongly semi continuous Also, we define IF$_\mu$-retract and IF$_\mu$- neighborhood retract and IF$_\mu$-strongly semi retract as applications of IF$_\mu$- strongly semi continuous.

REFERENCES

1. K. Atanassov, Intuitionistic fuzzy sets, Fuzzy Sets and Systems 20(1986),87-96
2. D. Coker, An introduction to intuitionistic fuzzy Topological Spaces, Fuzzy Sets and Systems 88(1997)81-89
3. D. Coker, An introduction to fuzzy subspace in intuitionistic fuzzy topological spaces, J. Fuzzy Math., 4 (4)(1996) 749-764
4. H. Gurcay, D. Coker and A. Hayder. On fuzzy continuity in intuitionistic fuzzy topological spaces, J. Fuzzy Math. 5(2) (1997), 365-378
5. I. M. Hanafy, Completely continuous functions in intuitionistic fuzzy topological spaces, Czechoslovak Mathematical Journal, 53(4) (2003), 793-803
6. I. M. Hanafy, F.S. Mahmoud and Mohammed .M. Khalaf intuitionistic fuzzy retracts, International journal of logic and intelligent system, 5(1) (2005), 40-45
7.S.K.Samanta and T.K. Mondal, Topology on interval valued intuitionistic fuzzy sets, Fuzzy Sets and Systems 119 (2001) 483-494
8.L.A Zadeh, Fuzzy sets, Inform. and Control 8(1965)338-353