MHD MICROPOLAR FLUIDS FLOW AND HEAT TRANSFER OVER A STRETCHING SHEET

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ABSTRACT: *This study intends to investigate MHD stagnation point flow of micropolar fluids over a stretching sheet with heat transfer. The highly non linear partial differential equations of motion have been transformed to ordinary differential form by using similarity functions. Numerical solution of the resulting equations is obtained using Runge Kutta fourth order method with shooting technique. The results have been computed for several values of the parameters namely magnetic parameter Ha, velocity ratio parameter λ, Prandtl number Pr, Eckert number Ec and the non dimensional micropolar parameters* C_1, C_2 *and* C_3. *The effects o—f these parameters have been observed on fluid velocity, microrotation and heat function*.

AMS Subject Classification: 76M20.
Key Words: Micropolar fluids, MHD stagnation point flow, Stretching sheet, Prandtl number, Richardson's extrapolation.

1. INTRODUCTION

The dynamics of micropolar fluids has been an expanding research field since it is introduced by Eringen [1]. Micropolar fluids exhibit the microrotational effects and micro rotational inertia, e.g., liquid crystals which are made up of dumb bell molecules. In fact, animal blood happens to fall into this category. Other polymeric fluids and fluids containing certain additives may be represented by the mathematical model underlying micropolar fluids. Excellent reviews about the applications of micropolar fluids have been presented by Airman et al. [2, 3]. Eringen [4] extended his theory of structure continua to account for the thermal effects. A comprehensive review of the subject and applications of micropolar fluid mechanics was given by Khonsari and Brewe [5], Chamkha et al. [6], Bachok et al. [7] and Kim and Lee [8]. Takhar et al. [9] studied the mixed convective flow of a steady, incompressible micropolar fluid over a stretching sheet. Bhargava et al. [10] studied coupled fluid flow, heat and mass transfer phenomena of micropolar fluids over a stretching sheet with non-linear velocity. Magyari et al. [11] have studied Stokes' first problem for micropolar fluid and solved the problem analytically by Laplace transforms and numerically by Valk´ o Abate procedure. Sastry and Rao [12] studied the effects of suction parameter on laminar micropolar fluid in a porous channel.

The importance of the stagnation point flow is due to its wide applications such as cooling of electronic devices by fans, cooling of nuclear reactors, and many hydrodynamic processes and hence attracted attention of many researchers. MHD stagnation point flows with thermal effects have applications in many manufacturing processes in industry and engineering. Rehman *et al.* [13] studied the effect of the magnetic field on the flow of a micro polar fluid past a continuously moving plate. MHD stagnation point flow of a micro polar fluids towards surface over a stretching sheet plays a vital role in the heat transfer. The MHD flows over a stretching sheet in different contexts were considered by Pavlov [14]. Kumari and Nath [15] studied the micropolar fluid flow at a stagnation point of two dimensional as well as axisymmetric problems. Hussain and Ahmad [16-17] studied MHD flow of micropolar fluids over a shrinking sheet with mass suction and heat transfer. Kumaran *et al.* [18] obtained an exact solution for a boundary layer flow of an electrically conducting fluid past a quadratically stretching and linearly permeable sheet.The steady two-dimensional stagnation point flow of an incompressible viscous fluid over a flat deformable sheet stretching in its own plane with a velocity proportional to the distance from the stagnation point was considered by Mahapatra and Gupta [19].

The boundary layer problem due to a stretching sheet has relevance to extrusion problems and has received considerable interest due to its practical application to boundary layer control and thermal protection in high energy flow. The surface stretching problem was first proposed and analyzed by Sakiadis [21] based on the boundary layer approximation. Crane [22] presented an exact solution of the two dimensional Navier Stokes equations for a stretching sheet problem with the closed analytical form, where the surface stretching velocity was proportional to the distance from the slot. Lok et al. [23] investigated nonorthogonal stagnation point flow of a micropolar fluid. Kamal and Sifat [24] studied 3-dimensional micropolar fluid motion caused by the stretching surface. Shafique and Rashid [25] obtained numerical solution of three dimensional micropolar fluid flows due to a stretching flat surface. Sajjad et al. [26] studied MHD stagnation point flow of micropolar fluids towards a stretching sheet is studied in the presence of variable surface temperature. Ali et al. [27] obtained numerical solution of a viscous, incompressible, electrically conducting fluid flow and heat transfer over porous stretching sheet with injection.

In this article, numerical solution has been obtained for MHD stagnation point flow and heat transfer over a stretching sheet for micropolar fluids. The results have been computed for sufficient values of the parameters which are involved in this study in order to elaborate their effects on fluid velocity, microrotation and heat function.

2. MATHEMATICAL ANALYSIS

The micropolar fluids flow is assumed to be steady, incompressible, and two dimensional. The fluid is electrically conducting. The flow is over a stretching sheet which coincides with the plane y=0and the flow is confined in the region$y$>0. A uniform magnetic field of strength B_0 is applied perpendicular to the fluid flow. The magnetic Reynolds number is taken to be small and therefore the induced magnetic field is neglected. The fluid flows with velocity $\underline{V}$ = V(u ,v).The microrotation vector is $\underline{W} = (0,0,\omega_3)$ All the fluid properties are assumed to be constant. The body couple is neglected.

Keeping in view, the governing equations of motion as given by Eringen [1] become:

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0 \qquad (1)$$

$$(\mu+\kappa)(\frac{\partial^2 u}{\partial y^2})+\kappa(\frac{\partial \omega_3}{\partial y})+\rho\sigma H_0^2(U-u) = \rho(u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}) \qquad (2)$$

$$\gamma(\frac{\partial^2 \omega_3}{\partial x^2}+\frac{\partial^2 \omega_3}{\partial^2 y})+\kappa(-\frac{\partial u}{\partial y})-2\kappa\omega_3 = \rho j(u\frac{\partial \omega_3}{\partial x}+v\frac{\partial \omega_3}{\partial y}) \qquad (3)$$

$$u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}=\frac{K}{\rho C_p}\frac{\partial^2 T}{\partial y^2}+\frac{\mu}{\rho C_p}(\frac{\partial u}{\partial y})^2+\frac{\sigma\mu^2 B_0^2}{\rho C_p}u^2 \qquad (4)$$

where κ and γ are material constants, ρ is the density, μ is the coefficient of viscosity, ν is kinematic viscosity coefficient, σ is the electrical conductivity of the fluid, T is the temperature, T_∞ is the free stream temperature, K is the thermal conductivity of the fluid, C_p is the specific heat at constant pressure.

The boundary conditions are

The boundary conditions are:

$$u=ax, \quad v=0, \quad T=T_w, \quad \omega_3 \to 0 \quad \text{at } y=0$$

$$u=bx, \quad T\to T_\infty, \quad w_3\to 0 \quad \text{as } y\to\infty \qquad (4)$$

Where a is stretching velocity constant, b proportionality constant of free steam velocity and T_∞ is the free stream temperature.

The stream function ψ (x, y) is such that

$$u=\frac{\partial\psi}{\partial y}, \quad v=-\frac{\partial\psi}{\partial x} \qquad (5)$$

Using similarity transformations:

$$\psi=\sqrt{a\upsilon}\,xf(\eta), \quad \eta=y\sqrt{\frac{a}{\upsilon}} \qquad (6)$$

$$u=axf'(\eta), \quad v=-\sqrt{a\upsilon}\,f(\eta),$$

$$\omega_3=a\sqrt{\frac{a}{\upsilon}}\,xh(\eta) \qquad (7)$$

Equation of continuity (1) is identically satisfied.

Substituting (7) in to equations (2) and (3), we have

$$(1+C_1)f'''+ff''-f'^2-H_a f' + C_1 h'+H_a^2(\lambda-f')+\lambda^2=0 \qquad (8)$$

$$C_3 h''-C_1C_2 f''-2C_1C_2 h=(f\hbar-f\hbar') \qquad (9)$$

$$\theta''+\Pr(f\theta'+Ecf'''+H_a^2 Ecf'^2=0 \qquad (10)$$

where C_1 and C_2 are dimensionless material constants, $H_a^2=\frac{\sigma B_0^2}{\rho c}$ is magnetic parameter, $\lambda=\frac{b}{a}$ is velocity ratio parameter, $\Pr=\frac{\mu c_p}{\kappa}$ is Prandtl number and $Ec=\frac{U_w}{C_p(T_w-T_\infty)}$ is Eckert number.

$$f(0)=0, \quad f'(0)=1, \quad h(0)=0, \theta(0)=1$$
$$f'(\infty)=\lambda, h(\infty)=0, \quad \theta(\infty)=0 \qquad (11)$$

3. RESULTS AND DISCUSION

By using FORTRAN 90 coding, Runge Kutta fourth order method with shooting technique has been employed to solve the governing ordinary differential equations (8) to (10) with boundary conditions (11). The results for non dimensional velocity f', micro rotation h and temperature function θ have been obtained for representative values of the physical parameter namely, H_a, λ, *Pr* and *Ec*. It is to be mentioned that when κ =0, (C_1=0) and w_3 are zero, the problem reduces to Newtonian fluid flow. Hence four different values of the material constant C_1 have been taken arbitrarily to elaborate the micropolar behavior of the fluid flow. The values C_2 =0.1 and C_3 = 0.5 have been fixed, because these two material parameters have little effect. The parameter λ is the ratio of stretching velocity of the sheet to the free stream velocity of fluid. The numerical results have been computed for two different values of λ ($\lambda > 1$ and $\lambda < 1$).

Table 1 and table 2 respectively contain the results for skin friction coefficient for λ= 3 and λ =0.1. It is noticed that magnitude of skin friction coefficient increases with increase in the values of Hartmann number and it is lesser for micropolar fluids than for Newtonian fluids.

Table 3 and table 4 respectively presents the results for local Nusselt number for Prandtl number and Eckert number. The magnitude of Nusselt number increases with increasing values of Prandtl number but it decreases with increasing values of Eckert number. Table 5 shows that magnitude of couple stress increases with increase in the values of C_1.

Fig.1 and Fig.2 respectively depict the effect of Ha on f' when λ = 3 and λ =0.1. The horizontal velocity decreases with increase in the values of Ha when λ= 3 and it decreases when λ =0.1.

Fig.3 shows that f' decreases with increase in the values of C_1 but increases with increasing values of C_1 as shown in Fig.4.

Fig.5 demonstrates that when λ= 3, the microrotation function *h* is higher near the sheet than the region away from the sheet. It is also noticed that *Ha* has same effect on h near the sheet but it Increases *h*, in the region away from the sheet. Fig.6 shows that *h* increases near the sheet but it decreases away from the sheet with increase in the values of *Ha* when λ= 0.1.

Fig.7 and Fig.8 show that magnitude of *h* increases with increase of the value of C_1, both for λ =3 and λ =0.1.

Fig.9 depicts that temperature function θ increases with increase in *Ha*. Fig.10 and Fig.11 demonstrate that the temperature distribution decreases with increase in the values of *Pr* both for λ= 3 and λ =0.1. The temperature function θ increases slowly when λ= 3 but it increases significantly when λ =0.1 with increase in the values of *Ec* as shown in Fig.12 and Fig.13.

Table1: Results of $f''(0)$ λ=0.3, H_a =0.5

Ha	Newtonian fluid	Micropolar fluids	
		$C_1 = 0.5$	$C_1 = 1.5$
0.0	4.72939	3.85978	1.97008
0.5	4.83236	3.94414	1.91317
1.0	5.13036	4.18741	2.03731
1.5	5.59269	4.56497	2.22960

Table 2: Results of $-f''(0)$ when □□□□□□ H_a =0.5

Ha	Newtonian fluid	Micropolar fluids	
		$C_1 = 0.5$	$C_1 = 1.5$
0.0	0.97077	0.79189	0.36418
0.5	1.06855	0.87087	0.39889
1.0	1.32119	1.07640	0.49405
1.5	1.51764	1.35339	0.62985

Table 3: Results of $-\theta'(0)$ when H_a =0.5.

Pr	λ=3.0	λ=0.1
0.05	0.26665	0.25599
0.1	0.28378	0.33973
0.5	0.43007	0.53180
1.0	0.60821	0.61419

Table 4: Results of $-\theta'(0)$ when H_a =0.5.

Ec	λ=3.0	λ=0.1
0.1	0.28378	0.33973
0.3	0.27504	0.20654
0.5	0.26629	0.07335
0.7	0.25754	0.05984
1.0	0.24442	0.002596

Table 5: The values of couple stress $g'(0)$

C_1	0.5	1.0	1.5	5.0
λ=3.0	0.09809	0.18184	0.25552	0.60731
λ=0.1	-0.04251	-0.07470	-0.10004	-0.18763

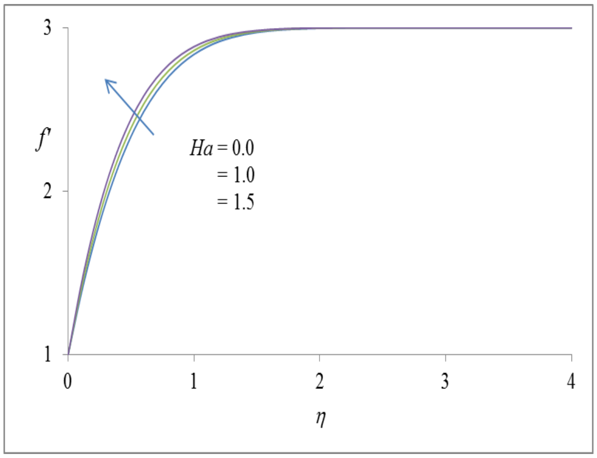

Fig.1: Graph of f' for different values of *Ha* when λ=3.0

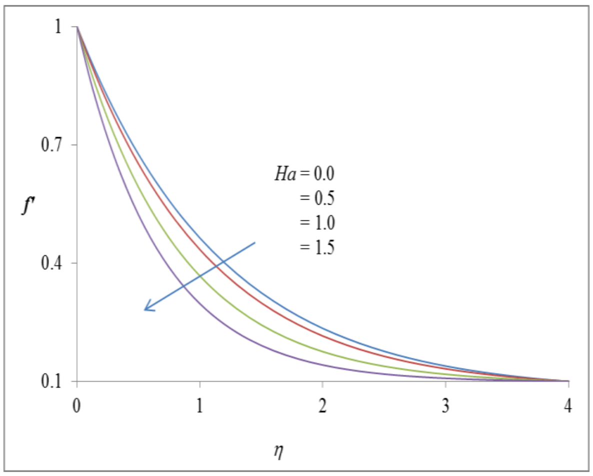

Fig.2: Graph of f' for different values of *Ha* when λ=0.1

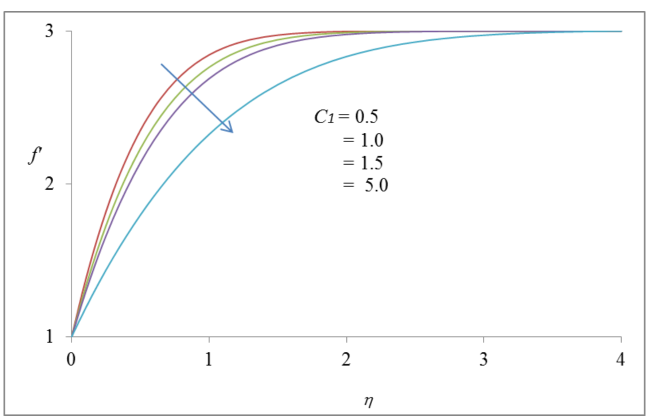

Fig.3: Graph of f' for different values of C_1 when λ=3.0

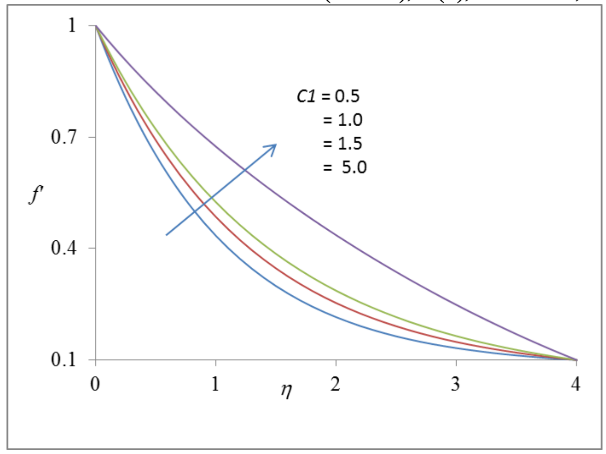

Fig.4: Graph of f' for different values of C_1 when λ=0.1

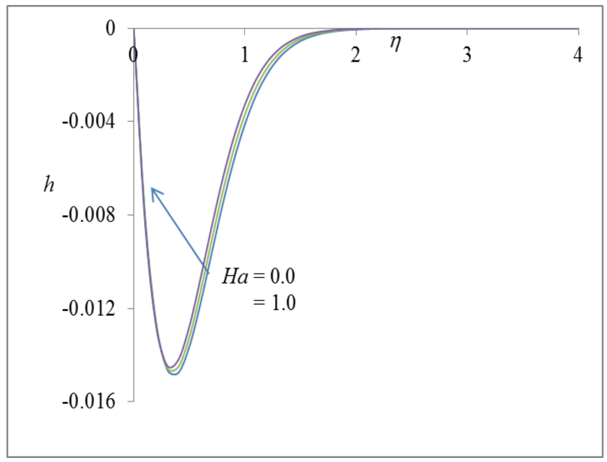

Fig.5: Graph of *h* for different values of *Ha* when λ=3.0

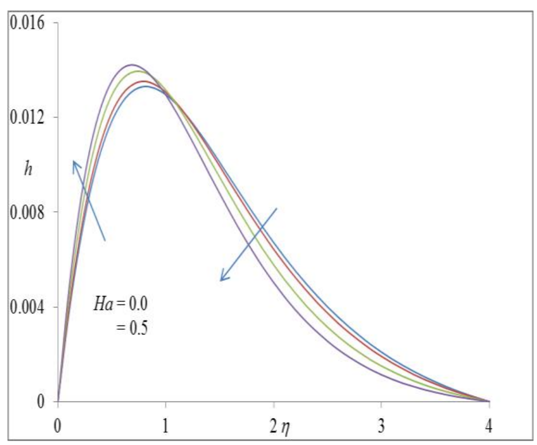

Fig.6: Graph of *h* for different values of *Ha* when λ=0.1

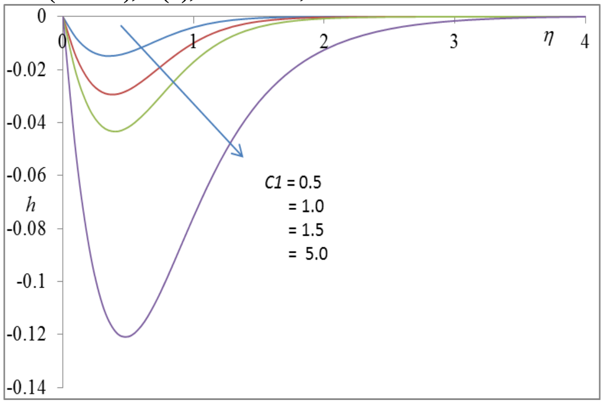

Fig.7: Graph of *h* for different values of C_1 when λ=3.0

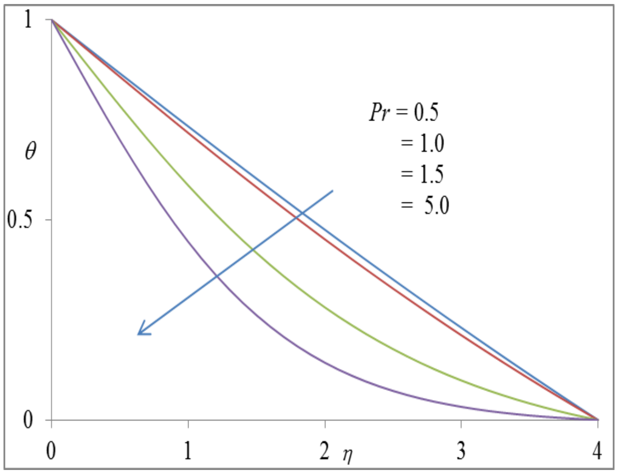

Fig.10: Graph of θ for different values of *Pr* when λ=3.0

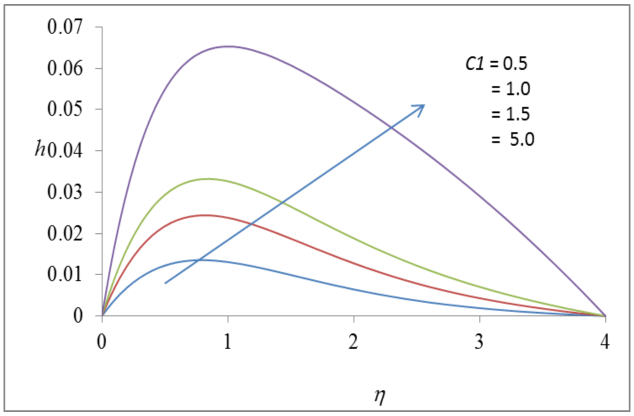

Fig.8: Graph of *h* for different values of C_1 when λ=0.1

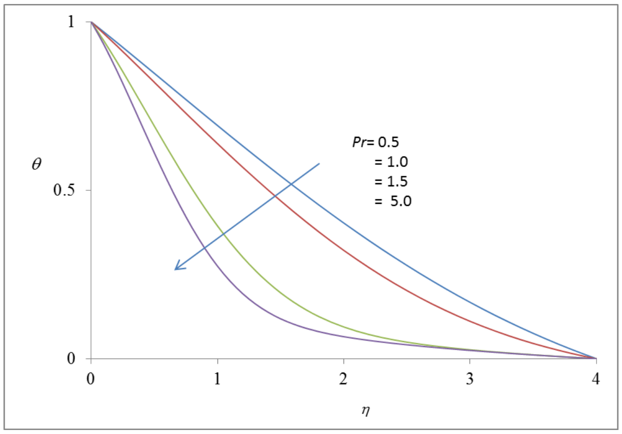

Fig.11: Graph of θ for different values of *Pr* when λ=0.1

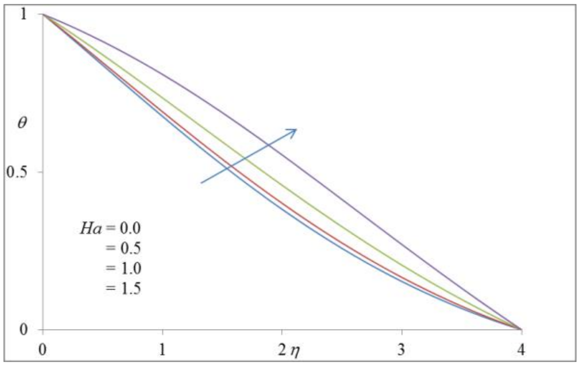

Fig.9: Graph of θ for different values of *Ha* when λ=3.0

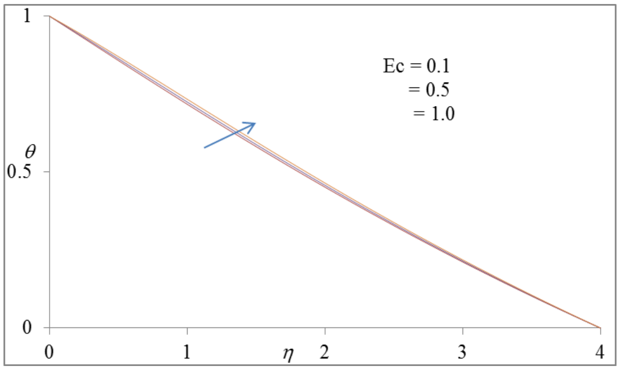

Fig.12: Graph of θ for different values of *Ec* when λ=3.0

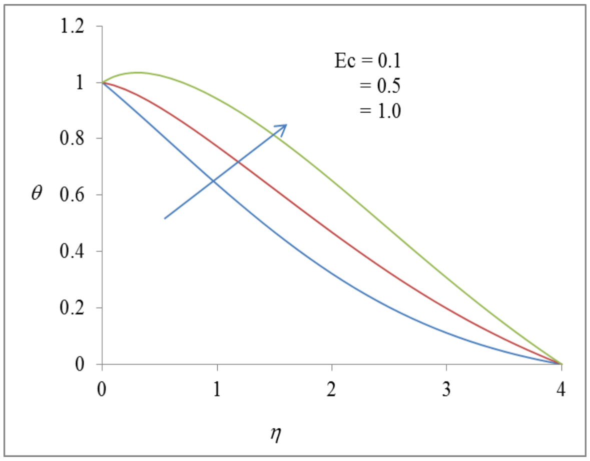

Fig.13: Graph of θ for different values of Ec when λ=0.1

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