NUMERICAL SOLUTION FOR MHD STAGNATION POINT FLOW TOWARDS A SHRINKING SHEET

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ABSTRACT: *Numerical solution is sought for steady, two dimensional stagnation point flow of an incompressible, electrically conducting fluid due to a shrinking sheet. The appropriate similarity functions reduce the fluid momentum and energy equation to ordinary differential form. The effects of parameters namely the magnetic parameter M, the velocity ratio parameter λ , Prandtl number Pr, Suction parameter f_w have been computed using coding in Mathematica. The results for velocity and temperature have been obtained and presented graphically.*

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1. INTRODUCTION

The flow and heat transfer over a stretching surface bears important research interest due to its various applications in industries such as hot rolling, wire drawing, glass fiber production, manufacturing plastic films and extrusion of a polymer in a melt spinning process. Sakiadis [1, 2] was the first to propose and analyze the surface stretching problem based on the boundary layer approximation. Crane [3] investigated the flow caused by the stretching of a sheet. Many researchers such as Gupta and Gupta [4], Dutta et al. [5], Chen and Char [6] extended the work of Crane [3] by including the effect of heat and mass transfer analysis under different physical situations. Many authors including Xu and Liao [7], Cortell [8, 9], Hayat et al. [10] and Hayat and Sajid [11] studied various aspects of the fluid flow due to moving boundaries. Ahmad et al. [12] obtained closed form solution for a viscous, incompressible, MHD flow over a porous stretching sheet.

The flow induced by a shrinking sheet is different from forward stretching flow, as first observed by Wang [13]. Goldstein [14] opinioned the shrinking flow is essentially a backward flow. After few years, Miklavcˇicˇ and Wang [15] established the existence and uniqueness of the similarity solution of the equation for the steady flow due to a shrinking sheet and they also reported that an adequate suction is necessary to maintain the steady flow. If the physical background of the flow is examined then it can be observed that the vorticity generated due to the shrinking of sheet is not confined within the boundary layer, and the steady flow exists only when adequate suction on the boundary is imposed. Yacob and Ishak [16] and Bhattacharyya et al. [17] discussed the micropolar fluid flow over a shrinking sheet with out and with thermal radiation, respectively. Sajjad et al. [18] considered MHD boundary layer flow of micropolar fluids over a permeable shrinking sheet. Sajjad et al. [19] studied MHD stagnation point flow of micropolar fluids towards a stretching sheet in the presence of variable surface temperature.

This work examines the numerical solution for MHD stagnation point flow towards a shrinking sheet to extend the work of Mahapatra and Gupta [20] who studied this problem for stretching surface. We obtained the results for velocity and temperature distribution for flow due to shrinking surface. The effects of physical parameters of interest have been observed and presented in graphical form.

2. MATHEMATICAL ANALYSIS

The fluid flow is steady, two-dimensional, incompressible and electrically conducting. There is stagnation point flow at the surface coincident with y=0. The surface shrinks, where the origin is fixed. A magnetic field of strength B_0 is applied normal to the surface. The magnetic Reynolds number is small and the induced magnetic field is negligible. $U_w(x)$, the stretching velocity and $U(x)$, the free stream velocity are considered proportional to x , x being the distance from stagnation point. It is assumed that the surface is at temperature $T_w(x)$, and T_∞ is ambient fluid temperature.

Under the above assumptions, the governing equations of motion become:

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0 \tag{1}$$

$$u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=U\frac{dU}{dx}+\upsilon(\frac{\partial^2 u}{\partial y^2})-\frac{\sigma B_0^2}{\rho}(U-u) \tag{2}$$

$$u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}=\alpha(\frac{\partial^2 T}{\partial y^2}) \tag{3}$$

where u,v are velocity components along x and y directions and ρ,υ,α and T are fluid density, kinametic viscosity, thermal diffusivity and fluid temperature respectively subject to the boundary conditions :

$$\begin{aligned} &y=0, u=-cx, v=-V, T=T_w(x) \\ &y\to\infty, u\to U(x)=ax, T\to T_\infty \end{aligned} \tag{4}$$

where a and c are constants such that $a>0$ and $c>0$.
Introducing the stream function $\psi(x,y)$, we have

$$u = \frac{\partial \psi}{\partial y} \text{ and } v = \frac{-\partial \psi}{\partial x} \qquad (5)$$

The continuity equation (1) is satisfied identically. By using the similarity transformation:

$$\psi = \sqrt{c\upsilon}\, x\, f(\eta), \quad \theta(\eta) = \frac{T - T_\infty}{T_w - T_\infty}, \ \eta = \sqrt{\frac{c}{\upsilon}} y, \qquad (6)$$

The equations (2) and (3) in dimensionless form lead to:

$$f''' + ff'' - f'^2 + \lambda^2 + M(\lambda - f') = 0 \qquad (7)$$

$$\theta'' + P_r f\theta' = 0 \qquad (8)$$

The corresponding boundary conditions are:

$$\left.\begin{array}{l} \eta = 0, f = f_w, f' = -1, \theta = 1 \\ \eta \to \infty, f' = \lambda, \theta = 0 \end{array}\right\} \qquad (9)$$

where prime denotes differentiation with respect to η , $\lambda = \frac{a}{c}$ is the velocity ratio parameter, $f_w = \frac{V}{\sqrt{c\upsilon}}$ is suction parameter, $P_r = \frac{\upsilon}{\alpha}$ is the Prandtl number, $M = \frac{\sigma B_0^{\ 2}}{\rho a}$ is the magnetic parameter.

3. RESULTS AND DISCUSION

Mathematica software version 6.0 has been utilized to solve the equations (7) and (8) together with boundary conditions (9).The numerical results for non dimensional velocity f' and temperature function have been computed for some values of parameters namely M, λ , Pr, f_w .The fixed values of these parameters have been chosen arbitrarily M=1.0, λ =-0.1 , f_w =0.1 , Pr=0.7. Fluid flow and heat transfer behavior is presented in the form of graphs plotted for velocity and temperature functions. Fig.1 and Fig.2 respectively show that the velocity increases in magnitude with increase in the values of λ and M. This pattern is opposite to the situation for flow due to stretching surface, as presented by Mahapatra and Gupta [20]. The velocity decreases in magnitude with increase in suction parameter f_w.

The temperature function increases with increasing values of Prandtl number Pr but it decreases with increase in the values of suction parameter f_w as depicted respectively in fig.4 and fig.5.

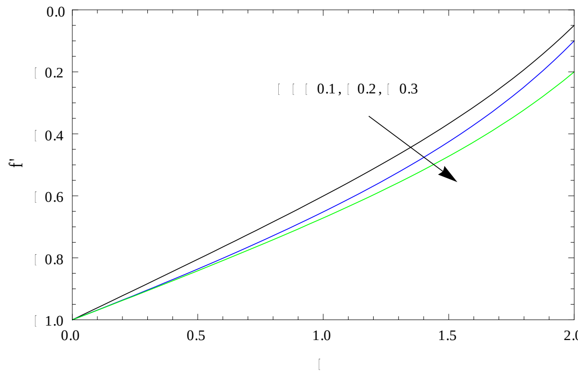

Fig.1: Graph of f' for different values of λ .

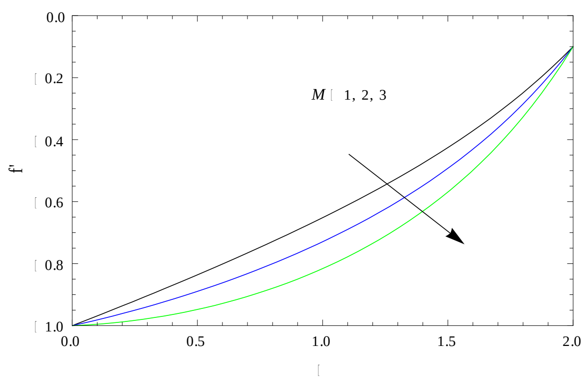

Fig.2: Graph of f' for different values of M

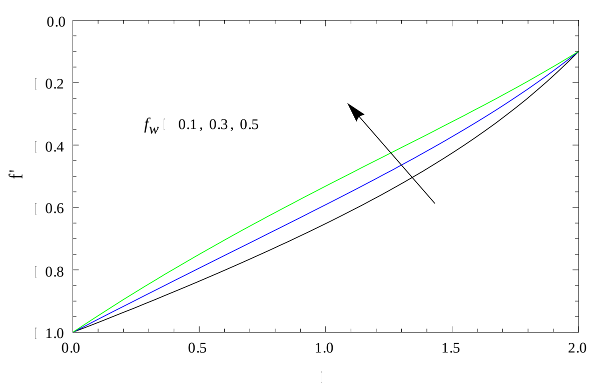

Fig.3: Graph of f' for different values of f_w .

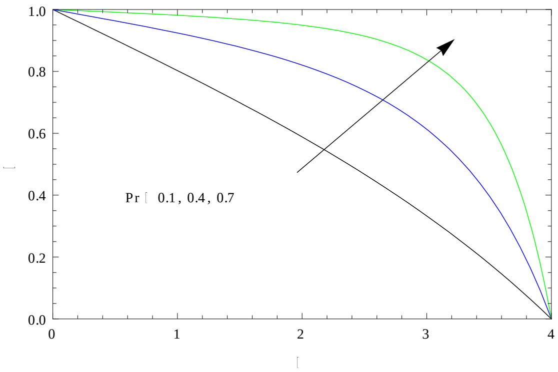

Fig.4: Graph of θ for different values of *Pr*.

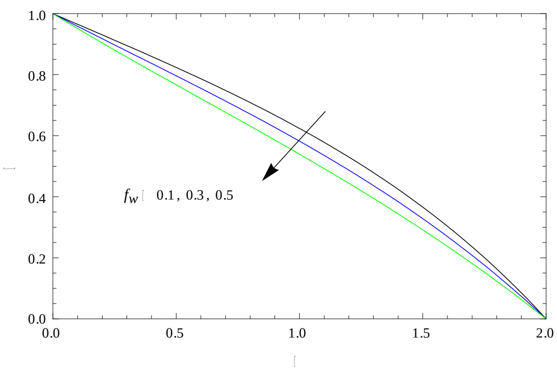

Fig.5: Graph of θ for different values of f_w.

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