TOTAL ANNUAL RAINFALL FREQUENCY ANALYSIS IN PAKISTAN USING METHODS OF L-MOMENTS AND TL-MOMENT

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ABSTRACT: *The extreme high rainfall amounts are among environmental events with the most harmful consequences for human society. In this study the data has been collected from 44 metrological observatories of Pakistan. Generalized Logistic distribution (GLO), Generalized Pareto Distribution (GPA) and Generalized Extreme Value distributions (GEV) are included in this study whose parameters are estimated by the method of L-moments and TL-moments. Mean Absolute Deviation Index ($MADI$), L-moments and TL-moments ratio diagrams are used for the goodness-of-fit measure to identify the most suitable distribution for each site separately . The result indicates that GPA distribution is the best fitted distribution to the data of annual total rainfall for maximum stations in Pakistan followed by GLO and GEV using both estimation methods . Quintiles are calculated for each station, for the period of 1, 2, 5, 10, 20, 50, 100, 500 and 1000 years only for that specific distribution which is the best according to the values of MADI for both L-moments and TL-moments.*

Keywords*: L-Moments, TL-Moments, Mean Absolute Deviation Index, Generalized Pareto Distribution*

1. INTRODUCTION

Rainfalls play an important role in hydrology and projects related to economic evaluation of water resources. The extreme rainfall magnitude and its frequency is needed for hydrological planning. This objective is gained through frequency analysis. Pakistan has been facing a lot of natural disasters in the form of floods, droughts and storms. These floods destruct many properties, crops and even lives of many people in different areas of Pakistan. The attempt has been made for this purpose to perform the statistical modeling of precipitation data using L-moments and TL-moments. Hosking [1] introduced the method of L-moments for the estimation of parameters that is similar to conventional method of moments. Elamir and Seheult [2] introduce TL-moments which is the extension of L-oments and decleared TL-moments are more robust then L-moments in case of outlires by assigning zero weight to all extreme values. A few studies have been attempted by many researchers on dealing with L-moments and TL-moments methods. A brief literature review of extreme value distributions is as follows.Chen et al [3] used L-moments method on low flow data and lognormal distribution is identified as the best distribution. Moniem and Selim [4] estimated parameters of generalized Pareto distribution by the methods of L-moments and TL-moments. Shabri et al [5] identified generalized logistic and generalized extreme value distribution by the method of TL-moments for annual maximum rainfall. Ahmad et al [6] performed L-moments and TL-moments method on annual maximum stream flow and found generalized logistic distribution as the best. Ahmad et al [7] investigated random behavior of monsoon rainfall in Pakistan using L-moments and quantiles are calculated for different return periods. Deka [8] identified generalized extreme value distribution as the best among all extreme value distributions by the method of TL-moments. Bilkova [9] used L-moments and TL-moments method for the construction of models of wage distribution. There is no any study in Pakistan using trimmed L.moments. The remaining of the paper is as follows: Section 2 is about methodology, section 3 about estimation methods, section 4 presesnts the goodness of fit tests, section 5 quanties estimationand section 6 concludes the paper.

2. METHODOLOGY

2.1 L-moments:

L-moments are defined as the linear combination of probability weighted moments (PWM's) defined by Hosking [1]. L stands for linear and they are used to calculate summary statistic. Measure of location, measure of dispersion, measure of (shape) skewness, and kurtosis are summary statistic. The r^{th} L-moment defined by Hosking [1] is given below

$$\lambda_{r+1} = \sum_{k=0}^{r} \beta_k (-1)^{r-k} \binom{r}{k} \binom{r+k}{k}$$

In terms of PWM's first four L-moments are as follows

$$\lambda_1 = \beta_0$$
$$\lambda_2 = 2\beta_1 - \beta_0$$
$$\lambda_3 = 6\beta_2 - 6\beta_1 + \beta_0$$
$$\lambda_4 = 20\beta_3 - 30\beta_2 + 12\beta_1 - \beta_0$$

The L-moment ratiosτ, τ_3 and τ_4 represents L-CV, L-skewness and L-kurtosis respectively which are written as follows

$$(\tau) = \lambda_2/\lambda_1, \qquad (\tau_3) = \lambda_3/\lambda_2,$$
$$(\tau_4) = \lambda_4/\lambda_2$$

From sample order statistic, the sample L-moments are estimated which are defined by Asquith [10].

$$l_r = \frac{1}{r}\sum_{i=1}^{n}\left[\frac{\sum_{j=0}^{r-1}(-1)^j \binom{r-1}{j}\binom{i-1}{r-1-j}\binom{n-i}{j}}{\binom{n}{r}}\right] X_{i:n}$$

Sample L-moments are denoted by l_1, l_2, l_3 and l_4 and t, t_3, t_4 denotes sample L-moments ratios defined by Hosking [1], where t denotes ($L\text{-}C_v$) L-coefficient of variation, t_3 denotes

(*L-C$_s$*) L-coefficient of skewness and t_4 denotes (*L-C$_k$*) L-coefficient of kurtosis.

2.2 TL-Moments:

Trimmed L-moments are the special case of L-moments which has been introduced by Elamir and Seheult [2]. Trimmed L-moments are based on trimming of the t_1- smallest and t_2- largest order statistic of a distribution or values from a sample. The theoretical TL-moments defined by Elamir and Seheult [2] is as follows:

$$\lambda_r^{(t_1,t_2)} = \frac{1}{r}\sum_{k=0}^{r-1}(-1)^k \binom{r-1}{k} E(Xr+t_1-k:r+t_1+t_2)$$

Where r represent order of the TL-moments, t represents the level of trimming of the t_1- smallest and t_2 represents the level of the trimming of the t_2- largest value. The TL-moments becomes L-moments when $t_1 = t_2 = 0$. In this research work we use unequal trimming that is $t_1 = 1$ and $t_2 = 0$ which means only smallest sample values will be trimmed. The first four TL-moments are given below:

$$\lambda_1^{(1,0)} = 6\beta_1 - 6\beta_2$$

$$\lambda_2^{(1,0)} = 6(-2\beta_3 + 3\beta_2 - \beta_1)$$

$$\lambda_3^{(1,0)} = \frac{20}{3}(-5\beta_4 + 10\beta_3 - 6\beta_2 + \beta_1)$$

$$\lambda_4^{(1,0)} = \frac{15}{2}(-14\beta_5 + 35\beta_4 - 30\beta_3 + 10\beta_2 - \beta_1)$$

The L-moment ratios $\tau^{(1,0)}$, $\tau_3{}^{(1,0)}$and $\tau_4{}^{(1,0)}$ represents TL-CV, TL-skewness and TL-kurtosis respectively which are written as follows:

$$\tau^{(1,0)} = \frac{\lambda_2{}^{(1,0)}}{\lambda_1{}^{(1,0)}}$$
$$\tau_3{}^{(1,0)} = \frac{\lambda_3{}^{(1,0)}}{\lambda_2{}^{(1,0)}}$$
$$\tau_4{}^{(1,0)} = \frac{\lambda_4{}^{(1,0)}}{\lambda_2{}^{(1,0)}}$$

2.3 Description of the Data and Study Area

The Annual total rainfaell data are collected for the proposed study, from Pakistan Meteorological centers Karachi and Islamabad, which waere measure in millimeter. Fig.1 shows geographical location of all 44 meteorological observatories. The data were collected from all over Pakistan included Azad Kashmir and Northrern Areas. The recorded length of this collected data varies from 29 to 51 years. Observations follow the basic assumptions of flood frequency analysis at various sites which are stationary, independent and identically distributed. We applied different plots and tests for this purpose which included time series plots, Mann-Whitney test, Kendall's tau test and Ljung-Box Q-Statistic.

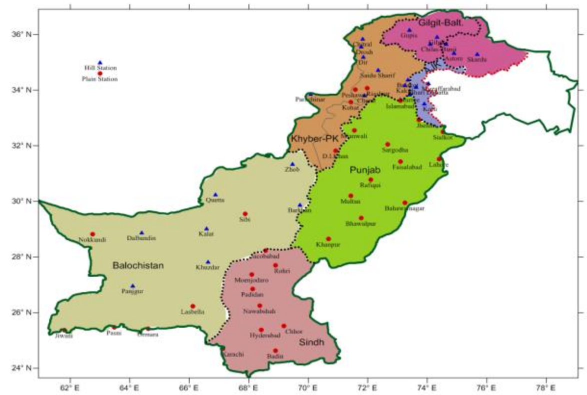

Fig.1: Geographical location of meteorological stations used in the study

3. Estimation of Parameters:

Three selected distributions were used for the proposed study of annual total rainfall that are generalized extreme value distribution (GEV), generalized logistic distribution (GLO) and generalized Pareto distribution (GPA). The basic purpose of this study is to find the best distribution among the selected distributions for each site. The parameters of these selected distributions were estimated by using L-moments and TL-moments approach. Distributions with their quantile functions of L-moments and TL-moments are given in Table 1 and Table 2.

Table 1: L-Moment Parameter Estimation for selected Distribution.

Distribution	**Quantile Function**	**Parameter Estimates**
GLO	$X(F) = \xi + \frac{\alpha}{K}\left[1 - \left(\frac{1-F}{F}\right)^K\right]$	$\hat{\alpha} = \frac{\ell_2}{\Gamma(1+K)\Gamma(1-K)}$ $\hat{\xi} = \ell_1 + \frac{(\ell_2 - \alpha)}{K}$ $\hat{K} = -t_3$
GEV	$X(F) = \xi + \frac{\alpha}{K}[1 - (-\ln F)^K]$	$\hat{\alpha} = \frac{\ell_2 K}{\Gamma(1+K)\Gamma(1-2^{-K})}$ $\hat{\xi} = \ell_1 + \frac{\alpha[\Gamma(1+K) - 1]}{K}$ $\hat{K} = 7.8590C + 2.9554C^2$ $C = \frac{2}{3+t_3} - \frac{\ln 2}{\ln 3}$
GPA	$X(F) = \xi + \frac{\alpha}{K}[1 - (1-F)^K]$	$\hat{\alpha} = \ell_2[(K+1)(K+2)]$ $\hat{\xi} = \ell_1 - \ell_2(K+2)$ $\hat{K} = \frac{4}{t_3+1} - 3$

Table 2: TL-Moment Parameter Estimation for selected Distribution.

Distribution	Quantile Function	Parameter Estimates
GLO	$X(F)=\xi+\frac{\alpha}{K}\left[1-\left(\frac{1-F}{F}\right)^{K}\right]$	$\hat{\alpha}=\frac{4K\ell_2}{-6\Gamma(1-K)\Gamma(K+1)-3\Gamma(1-K)\Gamma(K+1)(K-3)}$ $\hat{\xi}=\ell_1+\frac{\alpha[\Gamma(1-K)\Gamma(K+1)-1]}{K}-\alpha\Gamma(1-K)\Gamma(K+1)$ $\hat{K}=\frac{4-27t_3}{20}$
GEV	$X(F)=\xi+\frac{\alpha}{K}[1-(-\ln F)^{K}]$	$\hat{\alpha}=\frac{-2(6)^{K}\ell_2}{3\Gamma(1+K)(2^{K}-3^{K})}$ $\hat{\xi}=\ell_1+\frac{\alpha}{K}+\frac{\alpha\Gamma(K)}{2^{K}}$ $\hat{K}=0.49-2.08t_3+0.61(t_3)^2-0.60(t_3)^3+0.48(t_3)^4$
GPA	$X(F)=\xi+\frac{\alpha}{K}[1-(1-F)^{K}]$	$\hat{\alpha}=\frac{[\ell_2(K+1)(K+2)(K+3)]}{3}$ $\hat{\xi}=\ell_1-\alpha\left[\frac{K+3}{(K+1)(K+2)}\right]$ $\hat{K}=\frac{4-12t_3}{3t_3+4}$

4. Goodness of Fit Measure

4.1 L-moment and TL-moment Ratio Diagram:

L-moment and TL-moment ratio diagram is the better and simplest way to check which distribution is appropriate for the actual data for a specific site. The distribution which is nearest to the coordinates of the sample L-moment and TL-moment is chosen as the best fitted for the actual data. The L-moment and TL-moment ratio diagram for 44 stations are shown in Fig.2 and Fig.3 respectively. Fig.2 and Fig.3 indicate that for both L-moment ratio diagram and TL-moment ratio diagram GPA distribution is the best fitted distribution for most of the sites because mostly stations lie near GPA and its average also lies closest to GPA distribution. The average values of sample L-moment and TL-moment ratios which are denoted by t_3, t_4 which are given below: After the GPA the second and third best fit distributions are GLO and GEV.

For L-moments t_3= 0.1614843 and t_4=0.1394614

For TL-moments t_3^*= 0.224406 and t_4^*=0.1315963

4.2 Mean Absolute Deviation Index (MADI)

The Mean Absolute Deviation Index ($MADI$) has been used in this study for the comparison between the probability distribution for fitting the data. This goodness of fit method was used by Jain and Singh [11]. The goal of this method is to check whether a given distribution fits the data closely and to select from a number of candidate distributions, the one that gives the best fits to the data . The MADI is computed by using following formula:

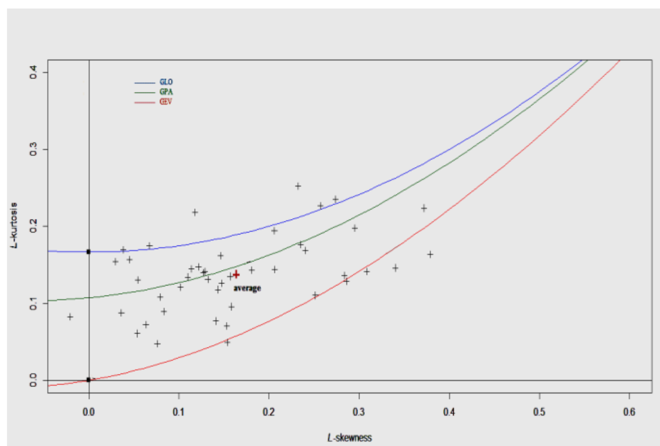

Fig 2: L-Moments ratio diagram

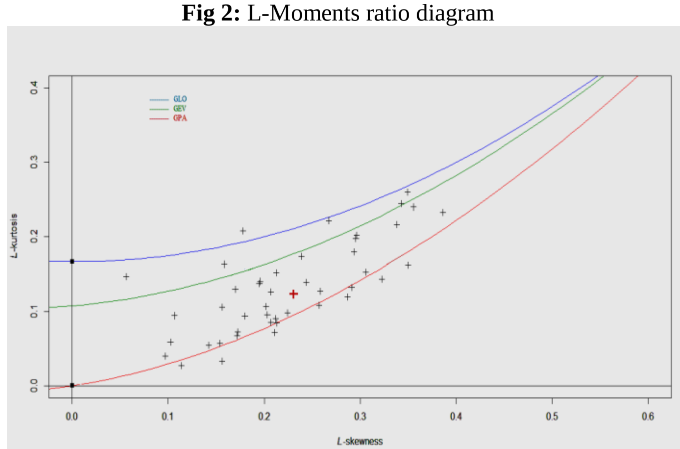

Fig 2: TL-Moments ratio diagram

$$MADI = \frac{1}{N}\sum_{i=1}^{N}\left|\frac{x_i - y_i}{x_i}\right|$$

Where N represents the number of observations, x_i represents the observed values and y_i represents the predicted value. The smallest value of ($MADI$) indicates that this distribution is more appropriate for the actual data. $MADI$ for 44 stations are given in the Table 3, both for L-moments and TL-moments. After this the values given in the table 3 are ranked for order 1 to 3 and results are given in the table 4. According to table 4 using L-moments GLO distribution receives rank 1 for 14 sites, GEV receives rank 1 for 8 sites and GPA receives rank 1 for 22 sites. Similarly using TL-moments GLO distribution receives rank 1 for 19 sites, GEV does not receive rank 1 for

Table 3: MADI for stations in Pakistan using L-Moments and TL-moments

Sr. No	Station Name	MADI for stations in Pakistan using L-Moments			MADI for stations in Pakistan using TL-Moments		
		GLO	GEV	GPA	GLO	GEV	GPA
1	Badin	4.180	1.822	1.556	0.608	0.993	0.708
2	Choor	2.817	2.856	2.949	2.649	556.2	9.983
3	Hyderabad	1.598	1.776	2.572	0.948	0.999	0.732
4	Jacobabad	1.730	71.63	1.690	2.988	0.992	0.742
5	Jiwani	3.019	2.934	13.32	2.015	0.996	0.727
6	Karachi	1.714	1.767	8.00	1.921	0.977	0.730
7	Khuzdar	0.586	0.560	0.555	0.524	0.998	0.568
8	Nawabshah	1.343	1.709	1.637	1.534	0.994	0.719
9	Pasni	2.005	9.486	1.920	1.823	1.814	0.711
10	Rohri	1.403	1.503	1.545	5.784	0.978	0.737
11	Cherat	0.335	0.333	0.337	0.258	0.982	0.488
12	Chitral	0.276	0.274	0.275	0.275	0.963	0.371
13	DI Khan	0.399	0.397	0.402	0.419	0.999	0.557
14	Faisalabad	0.421	0.423	0.437	11.26	0.992	0.556
15	Kakul	0.579	0.581	0.589	13.01	0.999	0.353
16	Lahore	0.386	0.392	0.407	0.416	0.999	0.532
17	Multan	0.508	0.500	0.495	1.035	0.985	0.670
18	Muzaffarabad	0.199	0.200	0.202	3.094	0.906	0.393
19	Peshawar	0.285	0.281	0.279	0.321	0.991	0.547
20	Sargodha	0.335	0.336	0.342	0.463	0.878	0.535
21	Kanpur	1.243	5.669	1.299	1.327	0.995	0.747
22	Kotli	0.299	0.319	0.322	20.60	0.868	0.465
23	Garhi Dupatta	3.058	0.211	0.217	0.236	0.682	0.379
24	Para Chinar	0.183	0.182	0.180	0.215	0.831	0.462
25	Gilgit	0.424	0.419	0.416	9.887	0.996	0.565
26	Dir	0.229	0.234	0.247	0.214	0.918	0.416
27	Barkhan	0.498	0.492	0.485	0.535	0.992	0.562
28	Lasbella	14.79	1.338	0.977	2.222	0.993	0.668
29	Moen-jo-doro	3.652	2.582	1.876	5.021	0.986	0.749
30	Padidan	3.497	2.858	2.500	49.22	0.947	0.758
31	Panjgar	1.116	1.070	1.024	1.087	0.968	0.649
32	Skardu	0.428	0.423	0.420	0.739	0.973	0.611
33	Zhob	0.458	0.454	0.449	0.471	0.947	0.518
34	Bahawalnagar	0.583	0.573	0.578	2.399	0.991	0.761
35	Chilas	0.522	0.521	0.526	0.516	0.939	0.565
36	Jhelum	0.321	0.320	0.318	0.362	0.858	0.476
37	Nokandi	25.44	3.898	5.714	3.247	0.976	0.744
38	Sibbi	0.808	0.654	0.612	0.828	0.993	0.639
39	Astor	0.319	0.320	0.323	0.338	0.978	0.497
40	Mian wali	0.390	0.384	0.373	0.443	0.969	0.526
41	Muree	2.257	0.259	0.217	0.277	0.887	0.397
42	Bahawalpur	0.953	0.912	0.900	0.952	0.959	0.655
43	Balakot	0.325	0.327	0.335	19.64	0.945	0.430
44	Bunji	0.517	0.498	0.485	0.594	0.990	0.573

Table 4: Ranking of the distribution for 44 stations by MADI using L-Moments and TL-moments approach (on a scale 1 to 3 with 1 being the best)

MADI using Method of L-Moments				**MADI using Method of TL-Moments**			
Number of times a distribution had the ranking				Number of times a distribution had the ranking			
Distribution	1	2	3	Distribution	1	2	3
GLO	15	04	23	GLO	19	05	20
GEV	08	32	04	GEV	0	20	24
GPA	21	06	17	GPA	25	19	0

Table 5: Estimated Quantiles (hundered mm) for different Return Periods based on L-moments

Station Name	**Dist**	0.100 1	0.500 2	0.800 5	0.900 10	0.950 20	0.980 50	0.990 100	0.998 500	0.990 1000
Badin	GPA	0.202	0.809	1.600	2.093	2.511	2.967	3.251	3.755	3.919
Choor	GLO	0.274	0.924	1.442	1.793	2.151	2.658	3.078	4.206	4.769
Hyderabad	GLO	0.140	0.820	1.488	2.002	2.577	3.481	4.306	6.860	8.318
Jacobabad	GPA	0.152	0.731	1.596	2.221	2.821	3.579	4.126	5.316	5.795
Jiwani	GEV	0.138	0.777	1.514	2.088	2.713	3.646	4.451	6.726	7.916
Karachi	GLO	0.160	0.897	1.507	1.930	2.368	3.001	3.535	5.004	5.755
Khuzdar	GPA	0.477	0.938	1.406	1.628	1.777	1.900	1.959	2.031	2.047
Nawabshah	GLO	0.165	0.834	1.478	1.967	2.511	3.356	4.121	6.458	7.774
Pasni	GPA	0.190	0.885	1.629	2.003	2.266	2.498	2.614	2.768	2.804
Rohri	GLO	0.146	0.786	1.465	2.011	2.644	3.676	4.651	7.835	9.742
Cherat	GEV	0.647	0.979	1.237	1.383	1.507	1.647	1.738	1.914	1.975
Chitral	GEV	0.636	0.987	1.245	1.384	1.497	1.621	1.698	1.838	1.885
DI Khan	GEV	0.636	0.987	1.245	1.384	1.497	1.621	1.698	1.838	1.885
Faisalabad	GLO	0.574	0.956	1.260	1.464	1.672	1.967	2.210	2.859	3.183
Kakul	GLO	0.638	0.939	1.212	1.412	1.629	1.956	2.244	3.088	3.547
Lahore	GLO	0.615	0.943	1.229	1.432	1.648	1.967	2.242	3.023	3.435
Multan	GPA	0.518	0.923	1.373	1.609	1.781	1.940	2.023	2.139	2.168
Muzaffarabad	GLO	0.785	0.986	1.132	1.226	1.317	1.440	1.538	1.786	1.902
Peshawar	GPA	0.600	0.940	1.310	1.498	1.633	1.754	1.815	1.899	1.919
Sargodha	GLO	0.654	0.979	1.214	1.363	1.509	1.707	1.863	2.255	2.439
Kanpur	GLO	0.153	0.797	1.466	1.998	2.609	3.596	4.520	7.495	9.253
Kotli	GLO	0.712	0.985	1.179	1.300	1.418	1.575	1.698	2.002	2.142
Garhi Dupatta	GEV	0.797	1.002	1.135	1.200	1.250	1.298	1.327	1.372	1.385
Para Chinar	GPA	0.723	0.985	1.212	1.302	1.354	1.390	1.405	1.419	1.421
Gilgit	GPA	0.569	0.932	1.333	1.543	1.696	1.836	1.910	2.012	2.037
Dir	GLO	0.753	0.989	1.154	1.256	1.354	1.484	1.585	1.833	1.946
Barkhan	GPA	0.539	0.944	1.358	1.556	1.690	1.802	1.856	1.923	1.938
Lasbella	GPA	0.281	0.849	1.549	1.958	2.286	2.623	2.821	3.142	3.237
Moen-jo-doro	GPA	0.142	0.699	1.578	2.251	2.932	3.843	4.540	6.189	6.912
Padidan	GPA	0.129	0.666	1.557	2.278	3.043	4.125	5.001	7.245	8.310
Panjgar	GPA	0.355	0.830	1.477	1.899	2.271	2.697	2.975	3.500	3.683
Skardu	GPA	0.510	0.924	1.380	1.616	1.787	1.942	2.022	2.132	2.160
Zhob	GPA	0.584	0.988	1.316	1.436	1.501	1.544	1.560	1.574	1.576
Bahawalnagar	GEV	0.421	0.927	1.382	1.672	1.941	2.278	2.523	3.061	3.281
Chilas	GEV	0.564	0.892	1.263	1.548	1.854	2.304	2.686	3.747	4.291
Jhelum	GPA	0.682	0.988	1.242	1.337	1.389	1.424	1.438	1.450	1.451
Nokandi	GEV	0.093	0.681	1.470	2.162	2.991	4.368	5.686	10.03	12.66
Sibbi	GPA	0.395	0.923	1.470	1.735	1.916	2.071	2.146	2.241	2.262
Astor	GLO	0.674	0.972	1.200	1.351	1.502	1.712	1.883	2.331	2.549
Mian wali	GPA	0.610	0.960	1.302	1.457	1.557	1.637	1.674	1.716	1.724
Muree	GPA	0.773	0.981	1.175	1.259	1.310	1.350	1.367	1.385	1.389
Bahawalpur	GPA	0.368	0.850	1.476	1.864	2.191	2.546	2.766	3.154	3.279
Balakot	GLO	0.719	0.990	1.175	1.289	1.398	1.541	1.651	1.919	2.040
Bunji	GPA	0.521	0.945	1.372	1.572	1.704	1.814	1.865	1.927	1.941

Table 6: Estimated Quantiles (hundered mm) for different Return Periods based on TL-moments

Station Name	**BestD ist**	0.100 1	0.500 2	0.800 5	0.900 10	0.950 20	0.980 50	0.990 100	0.998 500	0.990 1000
Badin	GLO	0.500	0.874	1.271	1.591	1.963	2.569	3.142	5.016	6.140
Choor	GLO	0.564	0.926	1.256	1.497	1.758	2.154	2.501	3.522	4.076
Hyderabad	GPA	0.444	0.854	1.411	1.774	2.095	2.461	2.701	3.152	3.310
Jacobabad	GPA	0.430	0.821	1.402	1.819	2.219	2.720	3.081	3.859	4.170
Jiwani	GPA	0.432	0.833	1.409	1.808	2.178	2.627	2.938	3.575	3.817
Karachi	GPA	0.473	0.917	1.408	1.665	1.851	2.022	2.111	2.234	2.265
Khuzdar	GLO	0.645	0.920	1.198	1.415	1.662	2.054	2.416	3.561	4.225
Nawabshah	GPA	0.656	0.896	1.246	1.491	1.721	2.003	2.201	2.614	2.774
Pasni	GPA	0.511	0.916	1.378	1.627	1.813	1.989	2.083	2.221	2.257
Rohri	GPA	0.425	0.799	1.388	1.837	2.290	2.894	3.355	4.439	4.911
Cherat	GLO	0.779	0.963	1.130	1.251	1.381	1.578	1.750	2.253	2.525
Chitral	GLO	0.779	0.968	1.131	1.247	1.369	1.550	1.705	2.146	2.377
DI Khan	GLO	0.713	0.943	1.164	1.331	1.517	1.805	2.066	2.860	3.307
Faisalabad	GPA	0.712	0.938	1.218	1.385	1.521	1.662	1.746	1.885	1.927
Kakul	GPA	0.732	0.942	1.150	1.312	1.494	1.781	2.044	2.867	3.340
Lahore	GLO	0.724	0.931	1.205	1.380	1.530	1.696	1.802	1.995	2.059
Multan	GPA	0.671	0.935	1.252	1.433	1.576	1.719	1.800	1.928	1.965
Muzaffarabad	GPA	0.848	0.981	1.091	1.167	1.246	1.360	1.457	1.722	1.858
Peshawar	GLO	0.729	0.953	1.158	1.309	1.473	1.722	1.942	2.591	2.946
Sargodha	GLO	0.753	0.974	1.192	1.290	1.354	1.405	1.428	1.455	1.460
Kanpur	GPA	0.444	0.834	1.398	1.793	2.161	2.611	2.926	3.579	3.830
Kotli	GPA	0.802	0.982	1.121	1.213	1.305	1.435	1.540	1.817	1.952
Garhi Dupatta	GLO	0.872	0.994	1.079	1.132	1.183	1.250	1.303	1.432	1.491
Para Chinar	GLO	0.812	0.981	1.145	1.219	1.266	1.302	1.319	1.338	1.342
Gilgit	GPA	0.710	0.946	1.168	1.333	1.514	1.792	2.041	2.785	3.197
Dir	GLO	0.833	0.979	1.100	1.184	1.271	1.397	1.504	1.799	1.950
Barkhan	GLO	0.705	0.936	1.224	1.395	1.535	1.680	1.766	1.910	1.953
Lasbella	GPA	0.558	0.866	1.315	1.632	1.930	2.297	2.556	3.099	3.310
Moen-jo-doro	GPA	0.681	0.894	1.220	1.462	1.700	2.009	2.237	2.754	2.970
Padidan	GPA	0.411	0.765	1.366	1.865	2.405	3.187	3.835	5.554	6.397
Panjgar	GPA	0.578	0.851	1.283	1.616	1.953	2.406	2.755	3.583	3.948
Skardu	GPA	0.668	0.954	1.198	1.370	1.552	1.819	2.047	2.690	3.026
Zhob	GLO	0.756	0.961	1.188	1.307	1.394	1.475	1.517	1.576	1.590
Bahawalnagar	GPA	0.629	0.938	1.218	1.422	1.642	1.974	2.265	3.115	3.575
Chilas	GLO	0.680	0.901	1.160	1.382	1.652	2.116	2.576	4.193	5.228
Jhelum	GLO	0.796	0.978	1.158	1.241	1.295	1.339	1.359	1.383	1.388
Nokandi	GPA	0.372	0.773	1.416	1.917	2.432	3.132	3.677	5.000	5.594
Sibbi	GPA	0.632	0.936	1.215	1.420	1.643	1.981	2.279	3.158	3.637
Astor	GLO	0.776	0.964	1.132	1.253	1.383	1.578	1.747	2.239	2.503
Mian wali	GLO	0.749	0.963	1.149	1.281	1.421	1.628	1.806	2.312	2.579
Muree	GLO	0.841	0.977	1.123	1.196	1.248	1.294	1.317	1.348	1.355
Bahawalpur	GPA	0.601	0.856	1.264	1.582	1.908	2.351	2.697	3.532	3.907
Balakot	GPA	0.814	0.967	1.143	1.239	1.310	1.378	1.415	1.468	1.483
Bunji	GPA	0.698	0.936	1.230	1.402	1.541	1.683	1.767	1.903	1.944

any of the site and GPA receives rank 1 for 25 sites. So it can be concluded that by using L-moments and TL-moments GPA distribution is the best fitted distribution followed by GEV and GLO distributions.

5 Estimation of Quantiles

The quantile estimates $\hat{Q}(F)$ with nonexceedance probability F for each site can be calculated by

$$\hat{Q}_i(\mathrm{F}) = \ell_1^{(i)}\hat{q}(\mathrm{F})$$

The quantiles estimates $\hat{q}(\mathrm{F})$, for various nonexceedance probabilities for L-moments and TL-moments are shown in Table 5 and 6. It is to be noted that the quantiles are calculated only for that specific distribution which is the best according to the values of MADI. According to MADI the value given in the Table 3 for site 1 Badin by using L-moments GPA distribution is the best and for TL-moments GLO is the best. So we estimate quantiles only these best fitted distributions. The return period which is also known as recurrence interval, is an estimation of the likelihood of an event, such as rainfall, flood, storms or a river discharge flow to occur. The return period (T) is expressed as $\frac{1}{P}$ where P represents the probability of exceedance. The probability of exceedance is explained as the chance of occurrence of some events over a given time period. It is to be noted that $P = \frac{1}{T}$exceedance probability. For example let for 10 years it becomes $\frac{1}{10}$ = 0.1. Now the non-exceedance probability is calculated by the relation $F = 1 - \frac{1}{T}$. For example let for 10 years the probability of non-exceedance become $1 - \frac{1}{10} = 0.90$. The values given in the table 5 for L-moments can be interpreted as; for example for site Badin$\hat{q}_{GPA}$(0.990) = 3.755 is that amount of rainfall which will happen once on the avearge in 100 These all values are interpreted in the same way. Similarly, the values given in the Table 6 for TL-moments can be interpreted as; for example for site Badin $\hat{q}_{GPA}$(0.990) = 3.142 is that amount of rainfall which will happen once in 100 years.

6. CONCLUSION

On the basis of MADI, L-moment and TL-moment ratio diagram, the best fitted distribution for annual total rainfall over metrological observatories in Pakistan was obtained. GLO, GEV and GPA distributions are considered in this study. Among these three distributions GPA is considered the best fit for both L-moment and TL-moment method for most of the sites followed by GLO and GEV. These results can be justified on the basis of L-moment and TL-moment ratio diagram where mostly sites lie closest to GPA distribution. However, GLO and GEV distributions received second and third rank respectively for MADI.

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