POSITION-FORCE CONTROL OF A 3-DOF DUAL COOPERATIVE ARM USING THE SLIDING MODE APPROACH

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ABSTRACT: Multi-arm cooperative robotic systems are prominently utilized to manipulate heavy and fragile objects. In this case, controlling the induced forces between the end-effectors and the object and also the object’s position are important. As the object may be fragile, ineffective control of the forces can lead to some damage. The present work studies the displacement of the rigid bodies by a 3-DOF (degree of freedom) dual arm in a predefined path and under predetermined forces. For this reason, the inverse kinematic problem is firstly solved. Then, considering the robot arms’ features and the desired goal, the sliding mode control is proposed to perform the simultaneous position-force control. The simulation results of a 3-DOF spatial robot prove the success of this approach compared to conventional PD controller. Also, the performance of the proposed approach is investigated under parametric uncertainties, external disturbances, sudden variations of the payload, and the measurement noises.

Keywords: cooperative robots, inverse kinematics, sliding mode control, position-force control

1. INTRODUCTION

Like the humans, robots can cooperate to manipulate heavy objects. This cooperation may also occur between a human and a robot[1]. Human-robot cooperation at home or at work seems useful for the tasks that require high accuracy (figure 1)[2]. Due to widespread application of the cooperative robots, modelling and designing these robots have received much attention.

Indeed, the purpose of implementing a control scheme is to provide a way so that a desired predetermined path could be travelled by a robot. This is called position control. In the position control, the center of mass travels on a pre-specified trajectory. Force control is also vital since its inefficiency can cause large contact forces which are troublesome in the case of fragile payloads[3].

Various force-position control schemes of the cooperative arms and the corresponding literature are reviewed in this section.

Figure 1: human-robot cooperation.

Classical control schemes are often designed and implemented in the time-domain. As an example, in [4] employed a PID position controller in a dual cooperative arm consisting of two arms which move an object on a specified path without considering the internal forces and the arms’ interactions. They assigned a spiral model to the object and adopted the PID scheme to diminish the position errors and to face the object’s oscillations. Despite the simplicity of this approach, some drawbacks like the system’s tardiness and instability are encountered under specific conditions. Other physical-based schemes have also been tried. In [5] studied the simultaneous position-force control of a dual cooperative arm. They controlled the cooperative robots by P and PI controllers using force sensors in the feedback.

Robust control has received special attention in this field. Using the robust scheme in the processes with either modeling or environmental uncertainty/indeterminacy, a reliable control method has been resulted. For example, in [6] proposed a robust position-force control scheme for the planar cooperative arms so that the consumed energy is minimized and the control target is also achieved.

Sliding Mode Control (SMC) is one of the most useful schemes in the robust control theory. SMC is nonlinear and robust. It is simple to design and facilitates the error reduction. As a result, it has been used to make the robot arms accurately track a predefined trajectory [7]. Another example is the simultaneous position-force control of a dual cooperative arm proposed by Yagiz, Hacioglu and Arsalan [8]. They adopted a smooth (non-chattering) sliding mode approach which was robust and accurate. Comparisons between their approach and the available PID controllers lead to acceptable results. Each arm, in their work, had 2 degrees of freedom and moved an object on a flat surface.

When the parametric or modeling variations occur, robust controllers are a proper choice. The major difference of the adaptive controllers and the robust ones is their independency to the prior knowledge of the uncertainties. The robust scheme guarantees to control the system in spite of the limited variations and uncertainties without any need to change the control law, while in the adaptive scheme, the control law changes with respect to the conditions.

The researchers have also focused on the adaptive schemes. Chen and Guo have also suggested the adaptive controllers for the cooperative robots[9]. They considered a planar dual robot arm and applied embedding the control problem to the joint space (i.e. the inverse kinematics) and employed an adaptive controller in this space. The advantages of their approach are its capability in the presence of uncertainties, the Liapanouv stability proof, and its applicability to a more number of arms in [10] and [11], offering a fault tolerance framework for cooperative robotic manipulators.

In Section 2, the mathematical equation the kinematics and dynamics of robots cooperating with three degrees of freedom, is provided. In Section 3, SMC for simultaneous control of force and position the system is designed. Also in this section, the issue of stability of the controller is investigated. Simulation results of the proposed method, is presented in Section 4.
2. ROBOT'S EQUATION

The adopted cooperative robot models are as in the figure 2. They have 3 degrees of freedom and manipulate a rigid body in the 3-dimensional space. Since the user's coordinate is Cartesian and the control scheme works in the joint space, a transformation is required to embed the joint space coordinates onto the Cartesian space. This transformation is called the forward kinematics. In general, the forward kinematics of the robot arms is as follows:

\[
\begin{align*}
\begin{bmatrix}
x \\
y \\
z
\end{bmatrix} &= \begin{bmatrix}
\cos(\theta_1) (l_3 \cos(\theta_2 + \theta_3) + l_2 \cos(\theta_2)) \\
\sin(\theta_1) (l_3 \cos(\theta_2 + \theta_3) + l_2 \cos(\theta_2)) \\
l_3 \sin(\theta_2 + \theta_3) + l_2 \sin(\theta_2)
\end{bmatrix} - \frac{d_2}{2}
\end{align*}
\]

(1)

The mid-point of the robot arm basis is assumed to be the common reference frame. The trajectory that each arm has to travel is slightly different on the x-axis due to the object's length. Considering this length, the kinematic equations of each arm will be:

\[
\begin{align*}
\begin{bmatrix}
x_1 \\
y_1 \\
z_1
\end{bmatrix} &= \begin{bmatrix}
\cos(\theta_1) (l_3 \cos(\theta_2 + \theta_3) + l_2 \cos(\theta_2)) \\
\sin(\theta_1) (l_3 \cos(\theta_2 + \theta_3) + l_2 \cos(\theta_2)) \\
l_3 \sin(\theta_2 + \theta_3) + l_2 \sin(\theta_2)
\end{bmatrix} - \frac{d_2}{2}
\end{align*}
\]

\[
\begin{align*}
\begin{bmatrix}
x_2 \\
y_2 \\
z_2
\end{bmatrix} &= \begin{bmatrix}
\cos(\theta_4) (l_3 \cos(\theta_5 + \theta_6) + l_2 \cos(\theta_5)) \\
\sin(\theta_4) (l_3 \cos(\theta_5 + \theta_6) + l_2 \cos(\theta_5)) \\
l_3 \sin(\theta_5 + \theta_6) + l_2 \sin(\theta_5)
\end{bmatrix} + \frac{d_2}{2}
\end{align*}
\]

(2)

Given the positions in the joint space and using the above equations, the corresponding Cartesian coordinates can be obtained. The inverse of this transformation (Cartesian space to the joint space) is called the inverse kinematics which is:

\[
\begin{align*}
\theta_i &= \tan^{-1}(y/x) \\
\cos(\theta_3) &= \frac{k}{\sqrt{2l_1l_3 \cos(\theta_i)^2}}
\end{align*}
\]

\[
k = x^2 + (\cos(\theta_i)z)^2 - (\cos(\theta_i)l_1)^2 - (\cos(\theta_i)l_3)^2
\]

Another transformation, called the Jacobian matrix, converts the joint space velocities into their corresponding Cartesian coordinates. The Jacobian matrix has a size of 3x3. The dynamic equation of the robot arms will be:

\[
\begin{align*}
M(\theta) + C(\theta, \dot{\theta}) + G(\theta) &= \tau + J^T F
\end{align*}
\]

(11)

where \(M(\theta) \in \mathbb{R}^{3 \times 3}\) is the mass matrix, \(C(\theta, \dot{\theta}) \in \mathbb{R}^{3 \times 1}\) denotes the Coriolis and centrifugal vectors, \(G(\theta) \in \mathbb{R}^{3 \times 1}\) is the gravitational vector, \(\tau \in \mathbb{R}^{3 \times 1}\) is the torque exerted to the robot joints, \(J \in \mathbb{R}^{3 \times 3}\) designates the Jacobian matrix, and \(F \in \mathbb{R}^{3 \times 1}\) represents the contact force between the end-effectors and the environment. The values used for the simulation of the robot arm, as shown in Table 1. In this table, \(m, L\) represents mass and length robot arm, and \(K_e\) is object stiffness factor.

Table 1: The parameters used for the simulation of the robot

<table>
<thead>
<tr>
<th>L_{1,4}</th>
<th>L_{2,5}</th>
<th>L_{3,6}</th>
<th>m_{1,4}</th>
<th>m_{2,5}</th>
<th>m_{3,6}</th>
<th>m</th>
<th>d_{1,2}</th>
<th>k_e</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.2</td>
<td>0.4</td>
<td>0.5</td>
<td>0.2k</td>
<td>0.5k</td>
<td>0.5k</td>
<td>1k</td>
<td>0.1</td>
<td>50</td>
</tr>
</tbody>
</table>

3. SLIDINGMODE CONTROL

From all approaches mentioned in section 1 to control the robotic systems, an approach should be adopted that is appropriate to handle the special features of the cooperative arms model. Cooperative robotic systems have a dynamic and nonlinear behavior. They need a simultaneous and fast control to correctly manipulate the payload. As a result, the SMC seems an appropriate choice since it is fast, robust and provides an acceptable performance in the presence of uncertainties. If the control problem is defined as a tracking problem, the SMC with guaranteed stability and fast convergence in annihilating the tracking error could be the first choice.

Assuming the joint angles as the state variables, the tracking error is defined as:

\[
e = \theta_d - \theta
\]

(13)

where \(\theta_d\) is the desired angle derived from the inverse kinematics and \(\theta\) is the robot's current angle. Once the error is defined, the sliding surface will be:

\[
S = k_e + \dot{e}
\]

(14)
\[ \dot{\theta} = f(\theta) + gu \] (15)

where \( f(\theta) \) and \( g \) are nonlinear functions and \( u \) represents the torque input. Hence, using the SMC theory, the control signal to put the system’s states on the sliding surface in the steady state will be:

\[ u_{eq} = \left( \frac{d\theta_d}{dt} - f(\theta) \right) / g \] (16)

Another component is added to \( u_{eq} \) so that the state variables reach the sliding surface in the transient state. The final relation for the control signal is as follows:

\[ u = u_{eq} + \lambda \text{sign}(S) / kg \] (17)

where \( \lambda \) is a positive number. As the control signals are applied, the system’s states are expected to move towards the sliding surface and remain there. So, the error is reduced to zero or is confined in an acceptable range.

The proposed method is stable and this can be shown using the Liapanouv control theory. In order to do so, a positive definite function is presented as the Liapanouv function:

\[ V = \frac{1}{2} SS^T \] (18)

Using the previous relations and differentiation with respect to time yields:

\[ \frac{dV}{dt} = -\lambda \| S \| \leq 0 \] (19)

Therefore, according to the Liapanouv theorem, the system is asymptotically stable because the variations of the positive function \( V \) are descending.

4. SIMULATION RESULTS

In this section, the proposed method is applied to a 3-DOF dual cooperative arm to manipulate an object on a circular and/or square path and the simulation results are presented. Then, comparisons were made between this method and a PD controller. The circular paths are as in the following figure. The paths are one object’s length far from each other.

After the SMC implementation, the path tracking is evaluated the tracking error as follows:

![Figure 4: the tracking error of the first robot arm for the circular path.](image)

![Figure 5: the tracking error of the second robot arm for the circular path.](image)

As shown in the figure 3 and 4, the tracking error is smaller than 1mm which is quite acceptable. The proposed method is evaluated by the square paths with curved corners. These paths are shown in the figure 6.

![Figure 6: the desired square paths for the robot arms.](image)

As the square paths are intrinsically different form their circular counterparts, the tracking errors will also differ. This is shown in the following figure.

![Figure 7: the tracking error of the first robot arm for the square path.](image)
As it is seen, the tracking procedure faces some deviations. At deviation points, the arms’ configuration changes and the error is resulted. The following figure depicts this error.

Figure 9: Tracking of the square path for the first robot arm.

As mentioned earlier, in addition to the position control, the force control is also vital in cooperative arms. Figure 9, the controlling force in moving the object weight of 0.5 kg, shows the circular path. In this figure, F1 force applied to the object by the first robot arm, and F2 force applied to the object by the second robot arm. As you can see the outcome of the force applied by the robot arm is always constant (5 N).

Figure 10: force control on the circular paths for the robot arms.

Figure 11, the force to control the movement of the object, when the object is added to the mass disturbance, is shown. This disturbance of mass 1 N, around the time of 2.3 seconds is applied to the object. As you can see, the controller controls the force applied to ease disturbance and exchange a small error in the desired direction continues.

Figure 12. Control the position of the first robot arm, in the presence of noise

Figure 12 and 13, performance of the SMC, in the presence of noise is exhibited. In these figures, the Gaussian noise with σ = 0.005, is applied to the robot arm angles. Clearly, in conditions of noise, SMC could well follow the desired path. In Table 2, the position error controller PD and SMC, in the presence and absence of disturbance and noise are compared. As can be seen, in all conditions of the above, SMC, better performance than PD controller is.

Table 2: Comparison of position error controller PD and SMC

<table>
<thead>
<tr>
<th>Condition</th>
<th>PD Controller</th>
<th>SMC Controller</th>
</tr>
</thead>
<tbody>
<tr>
<td>No Disturbance</td>
<td>0.02</td>
<td>0.01</td>
</tr>
<tr>
<td>With Disturbance</td>
<td>0.04</td>
<td>0.03</td>
</tr>
</tbody>
</table>
Table 2: compare performance SMC and PD in ISE

<table>
<thead>
<tr>
<th></th>
<th>$ISE\theta_{11} \times 10^{-3}$</th>
<th>$ISE\theta_{12} \times 10^{-3}$</th>
<th>$ISE\theta_{13} \times 10^{-3}$</th>
<th>$ISE\theta_{21} \times 10^{-3}$</th>
<th>$ISE\theta_{22} \times 10^{-3}$</th>
<th>$ISE\theta_{23} \times 10^{-3}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Circular motion</td>
<td>SMC 1.6</td>
<td>0.053</td>
<td>0.782</td>
<td>0.012</td>
<td>1.2</td>
<td>1.1</td>
</tr>
<tr>
<td>Square motion</td>
<td>PD 21.18</td>
<td>0.0005</td>
<td>0.001</td>
<td>0.188</td>
<td>10.3</td>
<td>8.8</td>
</tr>
<tr>
<td>Mass Disturbance</td>
<td>SMC 850.4</td>
<td>3.2</td>
<td>1</td>
<td>785.5</td>
<td>2</td>
<td>8.9</td>
</tr>
<tr>
<td>Increase Noisi Sigma=0.1</td>
<td>PD 322.2</td>
<td>88</td>
<td>32</td>
<td>114</td>
<td>112</td>
<td></td>
</tr>
<tr>
<td>Noisi Sigma=0.1</td>
<td>SMC 0.0656</td>
<td>0.862</td>
<td>1.1</td>
<td>0.079</td>
<td>1.2</td>
<td>1.1</td>
</tr>
<tr>
<td></td>
<td>PD 88</td>
<td>32</td>
<td>211</td>
<td>114</td>
<td>111.3</td>
<td>212</td>
</tr>
</tbody>
</table>

Figure 13. Control the position of the second robot arm, in the presence of noise.

5. CONCLUSION

In the present work, a sliding mode controller is modeled and designed to simultaneously control the position-force of 3-DOF dual cooperative arm that manipulates a rigid object. Simplicity of the design, robustness against the uncertainties and indeterminacy, and fast convergence of the tracking error to zero are some benefits of this approach. In order to compare the outputs of the proposed method with a conventional controller, a PD controller was applied to the robotic system and the results proved the effectiveness of the proposed method compared to the PD controller. To evaluate the SMC, the model’s equations were derived firstly. Then, the inverse kinematics was applied to the circular and square paths. As last, the position-force sliding mode controller could result the minimum errors, but some errors were accompanied by the square path. These errors are shown in the position and force control curves.

6. REFERENCES


