FACE ANTIMAGIC LABELING OF GENERALIZED KC_n SNAKE GRAPH

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ABSTRACT. *This paper deal with the labeling of type (1, 1, 1). If we assign labels from the set* $\{1,2,3,\cdots,|V(G)|+|E(G)|+|F(G)|\}$ *to the vertices, edges and faces of plane graph G in such a way that each vertex, edge and face receives exactly one label and each number is used exactly once as a label such a labeling is called magic labeling of type (1, 1, 1). In this paper super 1-antimagic labeling of type (1, 1, 1) on* KC_n *snake graph and subdivision of* KC_n *snake graph for string (1, 1, ... , 1) and string (2, 2, ... ,2) are discussed.*

Key Words: Super face antimagic labeling of type (1, 1, 1), snake graph for string (1, 1, … , 1) and string (2, 2,… , 2).

1. INTRODUCTION

Throughout this paper finite, connected, simple and plane graphs are considered. A graph G with vertex set $V(G)$, edge set $E(G)$ and face set $F(G)$ is said to have a labeling of type (α,β,γ) (where $(\alpha,\beta,\gamma)\in\{0,1\}$) if we assign labels from the set $\{1,2,3,\cdots,|V(G)|+|E(G)|+|F(G)|\}$ to the set vertices, edges and faces of plane graph G in such a way that each vertex, edge and face receives exactly one label and each number is used exactly once as a label. The weight of a face under a labeling of type (1, 1, 1) is the sum of the labels carried by that face and the edges and vertices surrounding it. A labeling is called face-magic, if for every integer $s\geq 3$, all S-sided faces have the same weight. We allow different weights for different S. The notion of face-magic labeling of type (α,β,γ) for plane graphs was defined by Lih [24], where are given face-magic labelings of type (1, 1, 0) for wheels, friendship graphs and prisms. The face-magic labeling for grid graphs, honeycomb, Mo¨bious ladders, m-prisms, m-antiprisms and for special classes of plane graphs are given in [1-12]. Miller et al. [21] provided the face-magic labeling of type (1, 1, 1) for wheels and subdivision of wheels.

A labeling of type (1, 1, 1) of plane graph G is called d-antimagic if for every positive integer S the set of weights of all S-sided faces is $W_s=\{a_s, a_s+d, a_s+2d,\cdots,a_s+(f_s-1)d\}$ for some integer a_s and $d\geq 0$, where f_s is number of S-sided faces. We allow different sets W_s for different S. For $d=0$, we have face-magic labeling. A *d*-antimagic labeling is called super if the smallest possible labels appear on the vertices. The super *d*-antimagic labeling of type (1, 1, 1) for disjoint union of prisms and for antiprisms are presented in [19] and [14]. The existence of *d*-antimagic labeling of type (1, 1, 1) for plane graphs containing a special Hamilton path for disconnected plane graphs are examined in [15]. Super *d*-antimagic labeling for Jahangir graph for certain differences *d* are determined by Siddiqui in [23]. A general survey of graph labeling is [20].

A KC_n snake is a connected graph with k blocks, each one isomorphic to cycle C_h, such that the block-cutpoint graph is a path. We call these graphs cyclic snakes. Let G be a KC_n snake with $K\geq 2$. Suppose that $v_1,v_2,v_3,\cdots,v_{k-1}$ are the consecutive cut-vertices of G. The string $(d_1,d_2,d_3,\cdots,d_{k-2})$ of integers d_i is the distance in G between v_i and v_{i+1}, $1\leq i\leq k-2$, characterizes the graph G in the class KC_n of n cyclic snakes. The definition of KC_n snake graphs introduced Barrientos in [18] as a natural extension of triangular snake graphs defined by Rosa [22]. The order and size of KC_n snake graph is defined as
$V=\{v_{i,j}\,;1\leq i\leq k, 1\leq j\leq n-1\}$ and
$E=\{v_{i,j}\,v_{i,j+1}\,;1\leq i\leq k, 1\leq j\leq n-1\}$

2. MAIN RESULTS 1

In this paper we investigate the existence of the super *d*−antimagic labeling of type (1, 1, 1) for KC_n snake graphs as well as subdivision of KC_n snake graphs with string (1, 1, ...1) and string (2, 2, ..., 2).

Theorem 1 For all $k\geq 2$ and n even, $H\cong KC_n$ snake graph of string (1, 1, ...1) and string (2, 2, ..., 2) admits super 1-antimagic labeling of type (1, 1, 1).

Proof.

Let $s=|V(H)|, e=|E(H)|$ and $f=|F(H)|$. Then $s=(n-1)k+1$, $e=nk$ and $f=k$. Now, we define the labeling
$\lambda: |V(H)|\cup|E(H)|\cup|F(H)|\rightarrow\{1,2,3,\cdots,s+e+f\}$ as follows

$\lambda(v_{i,1})=\{i, \quad 1\le i\le k+1\}$

$\lambda(v_{i,2})=\{2k+2-i, \quad 1\le i\le k\}$

$\lambda(v_{i,3})=\{4k+2-i, \quad 1\le i\le k\}$

$\vdots \quad \vdots$

$\lambda(v_{i,\frac{n}{2}})=\{(n-2)k+2-i, \quad 1\le i\le k\}$

$\lambda(v_{i,\frac{n+4}{2}})=\{2k+1+i, \quad 1\le i\le k\}$

$\lambda(v_{i,\frac{n+6}{2}})=\{4k+1+i, \quad 1\le i\le k\}$

$\vdots \quad \vdots$

$\lambda(v_{i,n-1})=\{(n-2)k+1+i, \quad 1\le i\le k\}$

We label the edges of H as follows

$\lambda(v_{i,1}\ v_{i,2})=\{s+i, \quad 1\le i\le k\}$

$\lambda(v_{i,2}\, v_{i,3})=\{s+2k+i, \quad 1\le i\le k\}$

$\vdots \quad \vdots$

$\lambda(v_{i,\frac{n}{2}}\ v_{i+1,1})=\{s+(n-1)k+i, \quad 1\le i\le k\}$

$\lambda(v_{i,1}v_{i,\frac{n+4}{2}})=\{s+2k+1-i, \quad 1\le i\le k\}$

$\lambda(v_{i,\frac{n+4}{2}}\, v_{i,\frac{n+6}{2}})=\{s+4k+1-i, \quad 1\le i\le k\}$

$\vdots \quad \vdots$

$\lambda(v_{i,n-1}\, v_{i+1,2})=\{s+nk+1-i, \quad 1\le i\le k\}$

We label the faces of H as follows

$\lambda(f_i)=\{s+e+f+1-i, \quad 1\le i\le k\}$

In this way the KC_n snake graph of string (1, 1, ..., 1) and string (2, 2, ..., 2) can be labeled to show super 1-antimagic labeling of type (1, 1, 1).

Theorem 2 For all $k\ge 2$ and n odd, $H\cong KC_n$ snake graph of string (1, 1, ...1) and string (2, 2, ..., 2) admits super 1-antimagic labeling of type (1, 1, 1).

Proof.

Let $s=|V(H)|, e=|E(H)|$ and $f=|F(H)|$. Then $s=(n-1)k+1$, $e=nk$ and $f=k$. Now, we define the labeling $\lambda: |V(H)|\cup|E(H)|\cup|F(H)|\to\{1, 2, 3, \cdots, s+e+f\}$ as follows

$\lambda(v_{i,1})=\{i, \quad 1\le i\le k+1\}$

$\lambda(v_{i,2})=\{2k+1+i, \quad 1\le i\le k\}$

$\lambda(v_{i,3})=\{4k+1+i, \quad 1\le i\le k\}$

$\vdots \quad \vdots$

$\lambda(v_{i,\frac{n-1}{2}})=\{(n-2)k+1+i, \quad 1\le i\le k\}$

$\lambda(v_{i,\frac{n+3}{2}})=\{2k+2-i, \quad 1\le i\le k\}$

$\lambda(v_{i,\frac{n+5}{2}})=\{4k+2-i, \quad 1\le i\le k\}$

$\vdots \quad \vdots$

$\lambda(v_{i,n})=\{(n-2)k+2-i, \quad 1\le i\le k\}$

We label the edges of H as follows

$\lambda(v_{i,1}\ v_{i,2})=\{s+i, \quad 1\le i\le k\}$

$\lambda(v_{i,2}\, v_{i,3})=\{s+2k+1-i, \quad 1\le i\le k\}$

$\lambda(v_{i,3}\, v_{i,4})=\{s+2k+i, \quad 1\le i\le k\}$

$\vdots \quad \vdots$

$\lambda(v_{i,n}\, v_{i,1})=\{s+(n-1)k+i, \quad 1\le i\le k\}$

We label the faces of H as follows

$\lambda(f_i)=\{s+e+f+1-i, \quad 1\le i\le k\}$

In this way the KC_n snake graph of string (1, 1, ..., 1) and string (2, 2, ..., 2) can be labeled to show super 1-antimagic labeling of type (1, 1, 1).

3. MAIN RESULTS 2

In this section we formulate super antimagic labeling of KC_n snake graph of string of string (1, 1, ..., 1) and string (2, 2, ..., 2) with 1 subdivision.

Theorem 3 For all $k\ge 2$ and n even, $H\cong KC_n$ snake graph of string (1, 1, ...1) and string (2, 2, ..., 2) with 1 subdivision, admits super 1-antimagic labeling of type (1, 1, 1).

Proof.

Let $s=|V(H)|, e=|E(H)|$ and $f=|F(H)|$. Then $s=2nk-k+1$, $e=2nk$ and $f=k$.

Now, we define the labeling $\lambda: |V(H)|\cup|E(H)|\cup|F(H)|\to\{1, 2, 3, \cdots, s+e+f\}$ as follows

$\lambda(v_{i,1})=\{i, \quad 1\le i\le k+1\}$

$\lambda(v_{i,2})=\{4k+2-i, \quad 1\le i\le k\}$

$\lambda(v_{i,3})=\{6k+2-i, \quad 1\le i\le k\}$

$\vdots \quad \vdots$

We label the partitions of H as follows

$\lambda(a_{i,1})=\{2k+2-i, \quad 1\le i\le k+1\}$

$\lambda(a_{i,2})=\{2k+1+i, \quad 1\le i\le k\}$

$\lambda(a_{i,3})=\{4k+1+i, \quad 1\le i\le k\}$

$\vdots \quad \vdots$

We label the edges as follows

$\lambda(v_{i,1}\ a_{i,1})=\{s+i, \quad 1\le i\le k\}$

$\lambda(a_{i,1}v_{i,2})=\{s+2k+1-i, \quad 1\le i\le k\}$

$\lambda(v_{i,21}\ a_{i,2})=\{s+2k+i, \quad 1\le i\le k\}$

$\lambda(a_{i,2}v_{i,3})=\{s+4k+1-i, \quad 1\le i\le k\}$

$\vdots \quad \vdots$

We label the faces of H as follows

$\lambda(f_i)=\{s+e+f+1-i, \quad 1\le i\le k\}$

In this way the KC_n snake graph of string (1, 1, ..., 1) and string (2, 2, ..., 2) with 1 subdivision can be labeled in the best way to show super 1-antimagic labeling of type (1, 1, 1).

Theorem 4 For all $k\ge 2$ and n odd, $H\cong KC_n$ snake graph of string (1, 1, ...1) and string (2, 2, ..., 2) with 1 subdivision, admits super 1-antimagic labeling of type (1, 1, 1).

Proof.

Let $s=|V(H)|, e=|E(H)|$ and $f=|F(H)|$. Then $s=2nk-k+1$, $e=2nk$ and $f=k$.

Now, we define the labeling $\lambda:|V(H)|\cup|E(H)|\cup|F(H)|\to\{1,2,3,\cdots,s+e+f\}$ as follows

$\lambda(v_{i,1})=\{i, \quad 1\le i\le k+1\}$

$\lambda(v_{i,2})=\{2k+2-i, \quad 1\le i\le k\}$

$\lambda(v_{i,3})=\{2k+1+i, \quad 1\le i\le k\}$

$\lambda(v_{i,4})=\{4k+2-i, \quad 1\le i\le k\}$

$\lambda(v_{i,5})=\{4k+1+i, \quad 1\le i\le k\}$

$\vdots \quad \vdots$

We label the partitions of H as follows, here $q=(n-1)k+1$

$\lambda(a_{i,1})=\{q+k+1-i, \quad 1\le i\le k\}$

$\lambda(a_{i,2})=\{q+k+i, \quad 1\le i\le k\}$

$\lambda(a_{i,3})=\{q+3k+1-i, \quad 1\le i\le k\}$

$\vdots \quad \vdots$

We label the edges as follows

$\lambda(v_{i,1}\ a_{i,1})=\{s+k+1-i, \quad 1\le i\le k\}$

$\lambda(a_{i,1}v_{i,2})=\{s+k+i, \quad 1\le i\le k\}$

$\lambda(v_{i,21}\ a_{i,2})=\{s+3k+1-i, \quad 1\le i\le k\}$

$\lambda(a_{i,2}v_{i,3})=\{s+3k+i, \quad 1\le i\le k\}$

$\vdots \quad \vdots$

We label the faces of H as follows

$\lambda(f_i)=\{s+e+f+1-i, \quad 1\le i\le k\}$

In this way the generalized KC_n snake graph of string (1, 1, ..., 1) and string (2, 2, ..., 2) with 1 subdivision can be labeled in the best way to show super 1-antimagic labeling of type (1, 1, 1).

4. OPEN PROBLEMS

Open Problem 1 For all $k\ge 2$, $H\cong mKC_n$ snake graph of string (2, 2, ..., 2) admits super 1-antimagic labeling of type (1, 1, 1).

Open Problem 2 For all $k\ge 2$, $H\cong mKC_n$ snake graph of string (1, 1, ...1) admits super 1-antimagic labeling of type (1, 1, 1).

Open Problem 3 For all $k\ge 2$, $H\cong KC_n$ snake graph of string (2, 2, ..., 2) with partition admits super 1-antimagic labeling of type (1, 1, 1).

Open Problem 4 For all $k\ge 2$, $H\cong mKC_n$ snake graph of string (1, 1, ...1) admits super 1-antimagic labeling of type (1, 1, 1).

REFERENCES

1. M. Bacˇa, On magic and consecutive labelings, for the special classes of plane graphs, Utilitas Math, 32(1987), 59 − 65.
2. M. Bacˇa, On magic labelings of grid graphs, Ars Combin., 33 (1992) , 295-299.
3. M. Bacˇa, On magic labelings of honeycomb Discrete Math., 105(1992), 305-311.
4. M. Bacˇa, On magic labelings of Mo¨bius ladders, J. Franklin Inst, 326(1989), 885–888.
5. M. Bacˇa, On magic labelings of type (1, 1, 1) for three classes of plane graphs, Math. Slovaca, 39 (1989), 233-239.

6. M. Bacˇa, Labelings of m-antiprisms, Ars Combin., 28, (1989), 242-245.
7. M. Bacˇa, Labelings of two classes of Convex polytopes, Utilitas Math., 34(1988) 24-31.
8. M. Bacˇa, On magic labelings of m-prisms, Math. Slovaca. 40 (1990), 11-14.
9. M. Bacˇa, Labelings of two classes of plane graphs, Acta. Math. Appl. Sinica, 9 (1993), 82 − 87.
10. M. Bacˇa, On magic labeling of type (1, 1, 1) for the special class of plane graphs, J. Franklin Inst., 329 (1992), 549 − 553.
11. M. Bacˇa, On magic labelings of convex polytopes, Ann. Disc. Math., 51(1992), 13-16 .
12. M. Bacˇa and I. Holl´ander, Labelings of a certain class of convex polytopes, J. Franklin Inst, 329 (1992), 539 − 547.
13. M. Bacˇa and M. Miller, On d−antimagic labeling of type (1, 1, 1) for prisms, J. Combin. Math. Combin. Comput. 44 (2003), 199-207.
14. M. Bacˇa, M Numan and M.K.Siddiqui, Super face antimagic labelings of union of antiprism, Mathematics in Computer Science., 7, No 2 (2013), 245-253.
15. M. Bacˇa, L. Brankovic and A.Semanicˇova´-Fenˇovcˇ´ikova´, Labelings of plane graphs containing Hamilton path, Acta Math. Sin. (Engl. Ser.), 26 (12) (2010), 2283-2294.
16. M. Bacˇa, F. Bashir and A.Semanicˇova´-Fenˇovcˇ´ikova´, On face antimagic la- belings of disjoint union of prisms, Utilitas Math., 85 (2011), 97-112.
17. M. Bacˇa, M. Miller, O. Phanalasy and A.Semanicˇova´-Fenˇovcˇ´ikova´, Super d-antimagic labelings of disconnected plane graphs, Acta Math. Sin. (Engl. Ser.) 26 (12) (2010), 2283-2294.
18. Barientos, Difference vertex labelings, Ph.D. Thesis, Universitat Po- litec- nica De Catalunya, Spain, 2004.
19. G. Ali, M. Bacˇa, F. Bashir andA.Semanicˇova´-Fenˇovcˇ´ikova´, On face antimagic labelings of disjoint union of prisms, Utilitas Math, 85 (2011) 97-112.
20. J.A. Gallian, A dynamic survey of graph labeling, The Electronic Journal of Combinatorics (2012).
21. M. Hussain, K. Ali, A. Ahmed, M. Miller, Magic labelings of type (a, b, c) of families of wheels, Math.Comput.Sci. (2013), 7:315-319.
22. A. Rosa, Cyclic steiner triple systems and labelings of triangular cacti, Sci. Ser. A Math Sci. (N. S.), 1 (1998), 87 - 95.
23. M.K. Siddiqui, M. Numan, M.A. Umar. Face Antimagic Labeling of Jahangir Graph. Mathematics in Computer Science. .2013:1-7.
24. Ko-Wei Lih, On magic and consecutive labelings of plane graphs, Utilitas Math, 24 (1983), 165 – 197.
25. W.D. Wallis, Magic Graphs, Birkh¨auser, Boston – Basel - Berlin, 2001.