F_μ-PERFECTELYRETRACTS, F_μ-SEMI INTERIOR AND F_μ-IRRESOLUTING MAPPING

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ABSTRACT: *Throughout the literature, if (X, δ) is an F-ts, and $Y \subset X$, the induced F-topological Vicente [Fuzzy Sets and Systems 58 (1993) 365] introduced a new concept of F-topological subspaces, which coincides with the usual definition in the case that $\mu = \chi_Y$. Also, they introduced the concepts of F_μ-open sets and F_μ -continuity. In this paper, using the previous concepts, we introduce weaker forms of F_μ-continuity. The notion of an F-retract was introduced by Rodabough [J. Math. Anal. Appl. 79 (1981) 273]. Here, we introduce the weaker forms of it. The notions of F_μ-semi closure, F_μ-semi interior and F_μ-irresolute mapping are given. Many results have been obtained.*

Keyword F_μ-perfectelyretract, F_μ-semi closure, F_μ-semi interior , F_μ-irresolute mapping

INTRODUCTION AND PRELIMINARIES

Weaker forms of F-continuity between fuzzy topological spaces have been Considered by many authors [1,4,5,22] using the concepts of F-semi open sets [1], F-preopen sets [20], F-strongly semi open sets [2], F-semi preopen sets [7], F-Regular open sets [15]. Macho Stadler and de Prada Vicente [12] introduced and investigated F-topological subspaces and F_μ-continuity. We introduce and study in Section 1 a new F-topological notions called F_μ-perfectly continuous , F_μ- completely continuous and F_μ- R- Continuous, F_μ-perfectely retract, Using these notions in the same section we define and study F_μ-completely retract, F_μ-R-retract , F_μ-neighbourhood perfectly retract, F_μ-neighborhood completely retract and F_μ-neighbourhood R-retract. .In Section 2, the notions of F_μ-semi closure, F_μ-semi interior and F_μ-irresolute mapping are introduced. Some of the fundamental properties of these concepts are investigated.
For definitions and results not explained in this paper, we refer to the papers [3,8,11,21,24], assuming them to be well known. For further reading see [6,10,13,14,16–20]. Let X be a non-empty set. A fuzzy set in X is a function with domain X and values in I [23]. The words fuzzy set and fuzzy topological space will be abbreviated as F-set and F-ts, respectively [9]. Also by $\mathrm{Int}_\mu(\upsilon), \mathrm{InCl}_\mu(\upsilon)$ and $\mu - \upsilon$ we will denote, respectively, the interior, closure, and complement of the F-set υ of F-topological subspace.
We mention the following definitions and results Let $(X; \delta)$ be an F-ts and $\mu \in I^X$. We call $\mathcal{A}_\mu = \{\upsilon \in I^X : \upsilon \le \mu\}$

Definition [12]. The family $\delta_\mu = \{\upsilon \wedge \mu : \upsilon \in \delta\}$ is the F_μ-topology induced over μ by δ . The elements of δ_μ are called F_μ-open sets

Proposition [12]. δ_μ *verifies the following properties*:
(i) $if\ \upsilon \in \delta_\mu$, then $\upsilon \in \mathcal{A}$;
(ii) $C0$, ; $\mu \in \delta_\mu$
(iii) $if\ \mu_1, \mu_2 \in \delta_\mu$, then $\mu_1 \wedge \mu_2 \in \delta_\mu$;
(iv) $if\ \{\upsilon_j : j \in J\} \subset \delta_\mu , then\ \vee_{j\in J} \upsilon_j \in \delta_\mu$

Definition [12]. $\upsilon \in \mathcal{A}_\mu$ is a F_μ-closed set if $\mu - \upsilon \in \delta_\mu$ we note δ_μ^c the family of all F_μ-closed sets.

1- ON F_μ- RETRACTS

Definition 1.1 Let $f : (X, \delta) \to (Y, \gamma)$ be a mapping from a F- ts (X, δ) to another F- ts (Y, γ) , $\mu \in I^X$. Then f is called :
(i) a F- perfectlycontinuous (briefly, F_μPC) mapping F_μ for each $\upsilon \in \gamma_{f(\mu)}$, we have $\mu \wedge f^{-1}(\upsilon)$ is both F_μ-open and F_μ-closed set of X.
(ii) a F_μ- completelycontinuous (briefly, F_μCC) mapping F_μ for each $\upsilon \in \gamma_{f(\mu)}$ we have $\mu \wedge f^{-1}(\upsilon)$ is regularopen set of X.
(iii) a F_μ-R-continuous (briefly, F_μRC) mapping F_μ for each F_μ-regular open $\in \gamma_{f(\mu)}$, We have $\mu \wedge f^{-1}(\upsilon)$ is F_μ- regular open of X.

Remark 1.1 The implications between these different concepts are given by the following diagram:

$$F_\mu PC \Longrightarrow F_\mu CC \Longrightarrow F_\mu RC$$

The converse of the above implication need not be true in general, as shown by the following examples.

Example 1.1 Let $X = \{a, b\}$, $Y = \{y\}$, $\delta = \{\underline{0}, \underline{1}, \lambda_1, \lambda_2, \lambda_3\}$. and

$\gamma = \{\underline{0}, \underline{1}, \theta_1, \theta_2, \theta_3\}$. $\lambda_1, \lambda_2, \lambda_3$ and $\mu \in I^X$, $\theta_1, \theta_2, \theta_3$ I^Y, defined by

$\lambda_1 = a_{0.4} \vee b_{0.3}$
$\lambda_2 = a_{0.3} \vee b_{0.2}$
$\lambda_3 = a_{0.2} \vee b_{0.1}$
$\mu = a_{0.6} \vee b_{0.7}$
$\theta_1 = y_{0.4}$
$\theta_2 = y_{0.5}$
$\theta_3 = y_{0.6}$

Then, the constant function f is F_μ- Rcontinuous, but not F_μ-C continuous.

Example 1.2 Let $X =$ Y={ a, b }, $\delta = \{\underline{0}, \underline{1}, \lambda_1, \lambda_2\}$. and $\gamma = \{\underline{0}, \underline{1}, \theta_1, \theta_2\}$. λ_1, λ_2 and $\mu \in I^X$, θ_1, $\theta_2 \in I^Y$, defined by

$\lambda_1 = a_{0.1} \vee b_{0.1}$
$\lambda_2 = a_{0.2} \vee b_{0.3}$
$\mu = a_{0.5} \vee b_{0.4}$
$\theta_1 = a_{0.3} \vee a_{0.2}$
$\theta_2 = a_{0.1} \vee a_{0.1}$

$f(\text{a}) = \text{b}$, $f(\text{b}) = \text{a}$. Then f is F_μ-C – continuous, but not F_μ-P continuous.

Definition 1.*2* $\mu \in I^X$, A F- ts (X, δ) is called a F_μ-extremally disconnected space (abbreviated as F_μED-space), μ-closure of every F_μ-open set of X is F_μ-open

Lemma 1.1 Let (X, δ) be an F_μED- space, $\mu \in I^X$. Then, if λ is F_μ-regular open set of X, it is both F_μ –open and F_μ -closed

Theorem 1.1 Let (X, δ) be an F_μED- space, $\mu \in I^X$, and f :(X, δ) $\to (Y, \gamma)$ be a mapping . Then the following are equivalent.
(i) f is F_μ -PC
(ii) f is F_μ -CC
Proof It follows from lemma 1.1

Theorem 1.2 Let $f:(X, \delta) \longrightarrow (Y, \gamma)$ be a mapping , $\mu \in I^X$. Then, f isF_μ-perfectelycontinuous (resp., F_μ-completely continuous) iff the inverse image of every F_μ-closed set of Y is F_μ-open an F_μ-closed (resp., F_μ-regular open set of X)

Proof obvious.

Theorem 1.3. Let (X, δ), (Y, γ) be F-ts's. and $f : (X, \delta) \longrightarrow (Y, \gamma)$ be a mapping .if the graph $g : (X, \delta) \longrightarrow (X \times Y, \theta)$ of f is F_μ-perfectelycontinuous (resp., F_μ-completely continuous) so is f, where θ is the F- product topology generated by δ and γ

Proof . Suppose the graph $g : (X, \delta) \longrightarrow (X \times Y, \theta)$ is F_μ-Perfectelycontinuous
Let $v \in \gamma_{f\rightarrow(\mu)}$, i.e. ,$v = f^{\rightarrow}(\mu) \wedge \eta$ where $\eta \in \gamma$, we want to show that , $\mu \wedge f^{\leftarrow}(f^{\rightarrow}(\mu) \wedge \eta) \in \delta_\mu$. since $\underline{1} \times \eta \in \theta$, $g^{\rightarrow}(\mu) \wedge (\underline{1} \times \eta) \in \theta_{g\rightarrow(\mu)}$,

then $\mu \wedge g^{\leftarrow}(g^{\rightarrow}(\mu) \wedge (\underline{1} \times \eta)) = \mu \wedge g^{\leftarrow}(\underline{1} \times \eta) = \mu \wedge (\underline{1} \wedge f^{\leftarrow}(\eta)) = \mu \wedge f^{\leftarrow}(\eta) = \mu \wedge f^{\leftarrow}(f^{\rightarrow}(\mu) \wedge \eta)$ is an F_μ-open and an F_μ-closed set of δ_μ so f is F_μ-perfectecontinuous. The proof of F_μ-completelycontinuous by the same fashion.

Definition 1.3 [13] Let (X, δ) be a F-ts, and $\text{A} \subset \text{X}$, Then, the F- subspace (A, δ_A) is called a F_μ-retract of (X, δ) F_μ there exists a F_μ-continuous mapping $r : (X, \delta) \longrightarrow (A, \delta_A)$ such that r (a)= a for all a $\in A$. In this case r is called a F_μ-retraction.

Definition 1.4 Let (X, δ) be a F-ts, and $\text{A} \subset \text{X}$, Then, the F- subspace (A, δ_A) is called a F_μ-perfectely retract (F_μ-completely retract, F_μ-R-retract) of (X, δ) F_μ there exists a F_μ - F_μ-perfectly continuous (F_μ- completely continuous, F_μ- R- continuous) mapping $r : (X, \delta) \longrightarrow (A, \delta_A)$ such that r (a) = a for all a $\in A$. In this case r is called a F_μ-perfectlyretraction (F_μ-completelyretraction, F_μ-R-retractretraction)

Remark 1.2 The implications between these different concepts are given by the following diagram:

F_μP retract ⇒ F_μC retract ⇒ F_μR- retract

The converse of the above implication need not be true in general, as shown by the following examples.

Example 1.3 .Let λ and μ be F- sets on $\text{X} = \{\text{a}, \text{b}\}$, defined by

$\lambda = a_{0.2} \vee b_{0.3}$
$\mu = a_{0.4} \vee b_{0.7}$

,$\delta = \{\underline{0}, \underline{1}, \lambda\}$, and A= { a } ⊂ X. Then, (A, δ_A) is a F_μ-R-retract of (X, δ), but not a F_μ- C retract.

Example 1.4 Let λ, β and μ be F- sets on $\text{X} = \{\text{a}, \text{b}\}$, defined by

$\lambda = a_{0.2} \vee b_{0.2}$
$\beta = a_{0.4} \vee b_{0.4}$
$\mu = a_{0.7} \vee b_{0.9}$

$\delta = \{\underline{0}, \underline{1}, \lambda, \beta\}$, and A= { a } ⊂ X. Then, (A, δ_A) is a F_μ-C - retract of (X, δ), but not a F_μ- P- retract.

Theorem 1.4 Let (X, δ) be a F-ts, A$\subset X$ and $r: (X, \delta) \longrightarrow (A, \delta_A)$ be a mapping such that $r(a) = a\ \forall a \in A$. if the graph $g: (X, \delta) \longrightarrow (X \times A, \theta)$ of r is F_μ-perfectelycontinuous (resp., F_μ-completelycontinuos) then f is a F_μ-retraction, where θ is the product topology generated by δ and δ_A

Proof. It follows directly from Theorem 1.3

Definition 1.5 Let (X, δ) be a F_μ-ts. Then (A, δ_A) is said to be a F_μ-neighbourhood perfectlyretract (F_μ-neighborhood completely retract, F_μ-neighborhood R-retract) (F_μ-nbd P-retract, F_μ-nbd R-retract, F_μ-nbd C-retract) of (X, δ) if (A, δ_A) is a F_μ- perfectlyretract (F_μ- completelyretract, F_μ- R-retract) of ((Y, δ_Y)), such that A$\subset$ Y $\subset$ X, $1_Y \in \delta$

Remark 1.3 Every F_μ-P- retract is a F_μ-nbd P-retract, but the converse is not true .

Example 1.5 Let X= { a, b, c }, A={ a } $\subset$ X , λ_1, λ_2 and μ be F-sets on X, defined by

$\lambda_1 = a_{0.2} \vee b_{0.2} \vee c_{0.4}$

$\lambda_2 = a_1 \vee b_1$

$\mu = a_{0.4} \vee b_{0.4} \vee c_{0.5}$

Consider $\delta = \{\underline{0}, \underline{1}, \lambda_1, \lambda_2, \lambda_1 \vee \lambda_2, \lambda_1 \wedge \lambda_2\}$. Then (A, δ_A) is a F_μ-nbd P-retract of (X, δ), but not a F_μ-P-retract of (X, δ).

Example 1.6 Let X = { a, b, c }, A= { a } $\subset$ X, λ_1, λ_2 and μ be F- sets on X, defined by

$\lambda_1 = a_{0.2} \vee b_{0.2} \vee c_{0.4}$

$\lambda_2 = a_1 \vee b_1$

$\mu = a_{0.8} \vee b_{0.8} \vee c_{0.5}$

Consider $\delta = \{\underline{0}, \underline{1}, \lambda_1, \lambda_2, \lambda_1 \vee \lambda_2, \lambda_1 \wedge \lambda_2\}$. Then (A, δ_A) is a F_μ-nbd C-retract of (X, δ), but not a F_μ-C-retract of (X, δ).

Example 1.7 in example 1.6 (A, δ_A) is a F_μ-nbd R-retract of (X, δ), but not a F_μ-R-retract. of (X, δ)

2- ON F_μ-SEMI CLOSURE AND F_μ-SEMI INTERIOR AND ON F_μ-IRRESOLUTE MAPPING

Definition 2.1 Let (X, δ) be a F-ts, $\mu, \lambda \in A_\mu$. Then ν is called

(i) (F .S .Mahmoud 2003) a F_μ-semiopen (briefly, $F_\mu$so) set if there exists $\lambda \in \delta_\mu$. such that $\nu \le \lambda \le Cl_\mu(\nu)$ (or $\nu \le Cl_\mu(Int_\mu(\nu))$.

(ii) [15] a F_μ-semiclosed (briefly, $F_\mu$sc) set if there exists $\nu \in \delta_\mu$. such that $Int_\mu(\nu) \le \lambda \le \nu$ (or, $\lambda \le Cl_\mu(Int_\mu(\lambda))$

(iii) The F_μ-semi-interior of λ, denoted by $SI_\mu(\lambda) = \vee\{\nu \in \delta_\mu : \nu \le \lambda,\ \nu \text{ is } F_\mu\text{so}\}$.

(iv) The F_μ-semi -closure of λ, denoted by $SC_\mu(\lambda) = \wedge\{\nu \in \delta_\mu : \nu \ge \lambda, \nu \text{ is} F_\mu\text{sc}\}$.

Theorem 2.1. Let (X, δ) be a F-ts, $\mu, \lambda \in \mathcal{A}_\mu$. The following statements are equivalent.

(i) λ is $F_\mu$so

(ii) $\lambda \le Cl_\mu(Int_\mu(\lambda))$.

(iii) $Cl_\mu(\lambda) = Cl_\mu(Int_\mu(\lambda))$.

(iv) $\mu - \lambda$ is $F_\mu$sc

(v) $Int_\mu(Cl_\mu(\mu - \lambda)) \le \mu - \lambda$

(vi) $t_\mu(Cl_\mu(\mu - \lambda)) = Int_\mu(\mu - \lambda)$

Proof (i) $\Rightarrow$ (ii) Let λ be $F_\mu$so There exists $\nu \in \delta_\mu$ such that $\nu \le \lambda \le Cl_\mu(\nu)$ by Theorem 1.3. $Int_\mu(v) = v$ since $v \le \lambda$, we have $Int_\mu(v) = v \le Int_\mu(\lambda)$. It implies $Cl_\mu(\nu) \le Cl_\mu(Int_\mu(\lambda)$. Since $\lambda \le Cl_\mu(\nu)$, we have $\lambda \le Cl_\mu(Int_\mu(\lambda)$. (ii) $\Rightarrow$ (iii) By the definition of Cl_μ and (ii), $Cl_\mu(\lambda) \le Cl_\mu(Int_\mu(\lambda)$. Since, $Int_\mu(\lambda) \le \lambda$, $Cl_\mu(Int_\mu(\lambda) \le Cl_\mu(\lambda)$. Thus, we have $Cl_\mu(\lambda) = Cl_\mu(Int_\mu(\lambda)$.

(iii) $\Rightarrow$ (i) Put $v = Int_\mu(\lambda)$. By the definition of t_μ, from Theorem 1.3, we have $\nu \le \lambda \le Cl_\mu(\lambda) = Cl_\mu(Int_\mu(\lambda) = Cl_\mu(\nu)$. Hence, λ is $F_\mu$so.

(iv) $\Rightarrow$ (i) It is easily proved from the following $\nu \le \lambda \le Cl_\mu(v) \Leftrightarrow \mu - Cl_\mu(v) \le \mu - \lambda \le \mu - v \Leftrightarrow Int_\mu(\mu - v) \le \mu - \lambda \le \mu - v$. (from Theorem 1.3)

(ii) $\Rightarrow$ (v) and (iii) $\Rightarrow$ (vi) are easily proved from Theorem 1.3

Theorem 2.1. (F.S. Mahmoud 2003) Let (X, δ) be a F-ts, $\mu \in \mathcal{A}_\mu$

(i) Any union of $F_\mu$so sets is $F_\mu$so

(ii) Any intersection of $F_\mu$sc sets is $F_\mu$sc

Theorem 2.2. Let (X, δ) be a F-ts, $\mu, \beta, \lambda \in \mathcal{A}_\mu$. Then,

(i) $Int_\mu(\lambda)$ is $F_\mu$so

(ii) $Cl_\mu(\lambda)$ is $F_\mu$sc

(iii) If λ is $F_\mu$so and $Int_\mu(\lambda) \leq \beta \leq Cl_\mu(\lambda)$, then β is $F_\mu$so.

(iv) If λ is $F_\mu$sc and $Int_\mu(\lambda) \leq \beta \leq Cl_\mu(\lambda)$, then β is $F_\mu$sc .

Proof we prove only (iii) and (iv).

(iii) Since λ is $F_\mu$so, then there exists $\nu \in \delta_\mu$ such that, $\nu \leq \lambda \leq Cl_\mu(\nu) \Rightarrow$ $\nu = Int_\mu(\nu) \leq Int_\mu(\lambda)$ and $Cl_\mu(\lambda) \leq Cl_\mu(\nu)$. Thus, $\nu \leq \beta \leq Cl_\mu(\nu)$. Hence, β is $F_\mu$so.

(iv) It is easily proved from (iii) and Theorem 2.1. And the following $Int_\mu(\lambda) \leq \beta \leq Cl_\mu(\lambda) \Leftrightarrow \mu - Cl_\mu(\lambda) \leq \mu - \beta \leq \mu - Int_\mu(\lambda) \Leftrightarrow Int_\mu(\mu-\lambda) \leq \mu - \beta \leq Cl_\mu(\mu-\lambda)$ by Theorem 2.1

Theorem 2.3. Let (X, δ) be a F-ts , $\mu, \nu, \lambda \in \mathcal{A}_\mu$. The following statements are valid :

(i) λ is $F_\mu$so iff $\lambda = sI_\mu(\lambda)$.

(ii) λ is $F_\mu$sc iff $\lambda = sC_\mu(\lambda)$.

(iii) $sC_\mu(\underline{0}) = \underline{0}$

(iv) $Int_\mu(\lambda) \leq sI_\mu(\lambda) \leq \lambda \leq sC_\mu(\lambda) \leq Cl_\mu(\lambda)$.

(v) $sC_\mu(\lambda) \vee sC_\mu(\nu) = sC_\mu(\lambda \vee \nu)$.

(vi) $sC_\mu\left(sC_\mu(\lambda)\right) = sC_\mu(\lambda)$

(vii) $Cl_\mu(sC_\mu(\lambda)) = sC_\mu(Cl_\mu(\lambda)) = Cl_\mu(\lambda)$

(viii) $sI_\mu(\mu - \lambda) = \mu - sC_\mu(\lambda)$.

Proof we prove only (vii) and (viii).

(vii) From (ii) and Theorem 2.2 $sC_\mu(Cl_\mu(\lambda)) = Cl_\mu(\lambda)$, we only show that $Cl_\mu(sC_\mu(\lambda)) = Cl_\mu(\lambda)$. Since $\lambda \leq sC_\mu(\lambda)$, $Cl_\mu(sC_\mu(\lambda)) \geq Cl_\mu(\lambda)$. Suppose that $Cl_\mu(sC_\mu(\lambda)) \nleq Cl_\mu(\lambda)$. By the definition of Cl_μ, there exists $\xi \in \delta_\mu$ with $\lambda \leq \xi$ such that, $Cl_\mu(sC_\mu(\lambda)) \geq \xi \geq Cl_\mu(\lambda)$. On the other hand, since $\xi \leq Cl_\mu(\xi)$, $\lambda \leq \xi \Rightarrow sC_\mu(\lambda) \leq sC_\mu(\xi) = sC_\mu(Cl_\mu(\xi)) = Cl_\mu(\xi) = \xi$. Thus, $Cl_\mu(sC_\mu(\lambda)) \leq \xi$. It is a contradiction. Hence $Cl_\mu(sC_\mu(\lambda)) \leq Cl_\mu(\lambda)$.

(viii) $\forall \lambda \in \delta_\mu$, we have the following:
$\mu - sC_\mu(\lambda) = \mu - \wedge\{\nu : \nu \geq \lambda,\ \nu \text{ is } F_\mu\text{sc}\} = \vee \{\mu - \nu : \mu - \nu \leq \mu - \lambda,\ \mu - \nu \text{ is } F_\mu\text{so}\} = sI_\mu(\mu - \lambda)$.

Definition 2.2 Let (X, δ) and (Y, γ) be a F-ts's , $\mu \in \mathcal{A}_\mu$. Let $f : (X, \delta) \longrightarrow (Y, \gamma)$ be a mapping.

(i) [Macho Stadler and M.A de Prada Vicente 1993] f is called F_μ-continuous mapping iff $f^{\leftarrow}(\upsilon) \in \delta_\mu$, for each $\upsilon \in \gamma_{f(\mu)}$.

(ii) [F . S . Mahmoud 2003] f is called F_μ-semi continuous mapping iff $f^{\leftarrow}(\upsilon)$ is $F_\mu$so $\in \delta_\mu$, for each $\upsilon \in \gamma_{f(\mu)}$.

(iii) f is called F_μ-irresolute mapping iff $f^{\leftarrow}(\upsilon)$ is $F_\mu$so $\in \delta_\mu$, for each $F_{f(\mu)}$ so , $\upsilon \in \gamma_{f(\mu)}$.

(iv) f is called F_μ-irresolute open mapping iff $f(\upsilon)$ is $F_\mu$so $\in \gamma_{f(\mu)}$. , for each $F_{f(\mu)}$ so $\upsilon \in \delta_\mu$.

(v) f is called F_μ-irresolute closed mapping iff $f(\upsilon)$ is $F_\mu$sc $\in \gamma_{f(\mu)}$. , for each $F_{f(\mu)}$ sc $\upsilon \in \delta_\mu$.

Remark 2.1 Every F_μ-continuous mapping is F_μ-irresolute mapping, but the converse is not true.

Example 2.1 Let $X = \{a, b, c\}$, $Y = \{y\}$, $\delta = \{\underline{0}, \underline{1}, \lambda\}$. and $\gamma = \{\underline{0}, \underline{1}, \theta\}$.λ and $\mu \in I^X$,$\theta \in I^Y$,defined by

$\lambda = a_{0.1} \vee b_{0.1}$

$\mu = a_{0.2} \vee b_{0.2}\ \ c_{0.3}$

$\theta = y_{0.1}$

Then, the constant function f is F_μ-irresolute mapping, but not F_μ-continuous.

Proposition 2.1 Let (X, δ) and (Y, γ)be a F-ts's , $\mu \in \mathcal{A}_\mu$ .Let $f : (X, \delta) \to (Y, \gamma)$ be a mapping. If f is F_μ-irresolute mapping, then For each $F_\mu$sc $\lambda \in \gamma_{f(\mu)}$, $f^{\leftarrow}(\lambda)$ is $F_\mu$sc $\in \delta_\mu$.

Proof For each F_μ sc set $\lambda \in \gamma_{f(\mu)} \Rightarrow f(\mu) - \lambda$ is $F_\mu$so set $\in \gamma_{f(\mu)}$, $f^{\leftarrow}(f(\mu) - \lambda) \wedge \mu \leq (\mu - f^{\leftarrow}(\lambda)) \wedge \mu$ is $F_\mu$so set $\in \delta_\mu$. $f^{\leftarrow}(\lambda) \wedge \mu$ is $F_\mu$sc set $\in \delta_\mu$.

Proposition 2.2 Let (X, δ) and (Y, γ) be a F-ts's , $\mu \in \mathcal{A}_\mu$ .Let $f : (X, \delta) \to (Y, \gamma)$ be a mapping. If For each $F_\mu$sc $\lambda \in \gamma_{f(\mu)}$, $f^{\leftarrow}(\lambda)$ is $F_\mu$sc $\in \delta_\mu$ then , $f\left(sC_\mu(\lambda)\right) \leq sC_{f(\mu)}\left(f(\lambda)\right)$, for each $\lambda \in \delta_\mu$.

Proof Suppose there exists $\lambda \in \delta_\mu$ such that , $f\left(sC_\mu(\lambda)\right) \nleq sC_{f(\mu)}\left(f(\lambda)\right)$

Since, $sC_{f(\mu)}\left(f(\lambda)\right) \leq \nu \in \gamma'_{f(\mu)}$. Moreover, $(f(\lambda) \leq \nu \Rightarrow \lambda \leq f^{\leftarrow}(\nu) \wedge \mu$.

$\Rightarrow f^{\leftarrow}(\nu) \wedge \mu$ is $F_\mu sc \in \delta'_\mu$, Thus, $SC_\mu(\lambda) \le f^{\leftarrow}(\nu) \wedge \mu \Rightarrow SC_\mu(\lambda) \le f^{\leftarrow}(\nu) \wedge \mu \ge \lambda$, then $f(SC_\mu(\lambda)) \le SC_{f(\mu)}(\nu \wedge f(\mu)) \ge SC_{f(\mu)}(f(\lambda))$. It is a contradiction

Proposition 2.3 Let (X, δ) and (Y, γ) be a F-ts's, $\mu \in \mathcal{A}_\mu$. Let $f: (X, \delta) \longrightarrow (Y, \gamma)$ be a mapping. If $f^{\leftarrow}(SI_{f(\mu)}(\lambda)) \wedge \mu \le SI_\mu(f^{\leftarrow}\lambda) \wedge \mu)$, for each λ is $F_\mu so \in \gamma_{f(\mu)} \in \gamma_{f(\mu)}$, then f is F_μ-irresolute mapping.

Proof Let λ is $F_\mu so \in \gamma_{f(\mu)}$ From theorem 2.3(i). $\lambda = SI_{f(\mu)}(\lambda)$. Since, $f^{\leftarrow}(\lambda)) \wedge \mu \le SI_\mu(f^{\leftarrow}(\lambda)) \wedge \mu)$. On the other hand, by Theorem 2.3(iv), $f^{\leftarrow}(\lambda)) \wedge \mu \ge SI_\mu(f^{\leftarrow}(\lambda) \wedge \mu)$. Thus, $f^{\leftarrow}(\lambda)) \wedge \mu = SI_\mu(f^{\leftarrow}(\lambda)) \wedge \mu)$, that is $f^{\leftarrow}(\lambda)) \wedge \mu$ is $F_\mu so \in \delta_\mu \Rightarrow f$ is F_μ-irresolute mapping.

Theorem 2.4 Let (X, δ) and (Y, γ) be a F-ts's, $\mu \in \mathcal{A}_\mu$. Let $f: (X, \delta) \longrightarrow (Y, \gamma)$ be a mapping. The following statements are equivalent.

(i) A map f is F_μ-irresolute open mapping

(ii) $f(SI_\mu(\lambda)) \wedge f(\mu) \le SI_{f(\mu)}(f(\lambda) \wedge f(\mu))$, for each λ is $F_\mu so \in \delta_\mu$.

(iii) $SI_\mu(f^{\leftarrow}\lambda) \wedge \mu \le (f^{\leftarrow}(SI_{f(\mu)}(\lambda)) \wedge \mu$, for each $\lambda \in \gamma_{f(\mu)}$

(iv) For any $\nu \in \gamma'_{f(\mu)}$ and any $F_\mu sc$ $\lambda \in \delta_\mu$ such that $f^{\leftarrow}(\nu) \wedge \mu \le \lambda$, there exists $F_\mu sc$ set $\rho \in \gamma_{f(\mu)}$ with $\nu \le \rho$ such that $f^{\leftarrow}(\rho) \wedge \mu \le \lambda$.

Proof (i) $\Rightarrow$ (ii) For each λ be $F_\mu so$ set $\in \delta_\mu$, since $SI_\mu(\lambda) \le \lambda$ from Theorem 2.3(iv). $f(SI_\mu(\lambda)) \wedge f(\mu) \le f(\lambda) \wedge f(\mu)$ by (i) $f(SI_\mu(\lambda)) \wedge f(\mu)$ is F_μ so set $\in \delta_\mu$, hence, $f(SI_\mu(\lambda)) \wedge f(\mu) \le SI_{f(\mu)}(f(\lambda) \wedge f(\mu))$.

(ii) $\Rightarrow$ (iii) for each $\lambda \in \gamma_{f(\mu)}$ from (ii) $f(SI_\mu(f^{\leftarrow}(\lambda))) \wedge f(\mu) \le SI_{f(\mu)}(f(f^{\leftarrow}(\lambda)) \wedge f(\mu)) \le SI_{f(\mu)}(\lambda) \wedge f(\mu) \Rightarrow SI_\mu(f^{\leftarrow}(\lambda))) \wedge \mu \le f^{\leftarrow}(SI_{f(\mu)}(\lambda)) \wedge \mu$.

(iii) $\Rightarrow$ (iv) Let λ be $F_\mu sc$ set $\in \delta'_\mu$ and $\lambda \in \gamma'_{f(\mu)}$ such that $f^{\leftarrow}(\nu) \wedge \mu \le \lambda$.

from Theorem 2.2 $\lambda = Int_\mu(CI_\mu((\lambda))$. Since $\mu - \lambda = f^{\leftarrow}(\mu - \nu) \wedge \mu$, we have $SI_\mu(\mu - \lambda) = \mu - \lambda \le SI_\mu\left(f^{\leftarrow}(\mu - \nu)\right) \wedge \mu$, by (iii) $\mu - \lambda \le SI_\mu\left(f^{\leftarrow}(\mu - \nu)\right) \wedge \mu \le f^{\leftarrow}(SI_{f(\mu)}(\mu - \nu)) \wedge \mu \Rightarrow \lambda \ge \mu - \left(f^{\leftarrow}\left(SI_{f(\mu)}(\mu - v)\right) \wedge \mu\right) = f^{\leftarrow}\left(\mu - (SI_{f(\mu)}(\mu - v)\right) \wedge \mu) = f^{\leftarrow}(SC_{f(\mu)}(v)) \wedge \mu$.

By Theorem 2.3 (viii), thus there exists $F_\mu sc$ set $\rho = SC_{f(\mu)}(v) \in \gamma'_{f(\mu)}$ with $v \le \rho$ such that $f^{\leftarrow}(\rho) \wedge \mu \le \lambda$

(iv) $\Rightarrow$ (i) Let σ be F_μ so set $\in \delta_\mu$, $\lambda = \mu - \sigma$ is F_μ sc set $\in \delta'_\mu$ put $v = f(\mu) - f(\sigma) \in \gamma'_{f(\mu)}$ we obtain $f^{\leftarrow}(v) \wedge \mu = f^{\leftarrow}(f(\mu) - f(\sigma)) \le \mu - (\sigma)) = \lambda$. by (iv) there exists $\rho \in \gamma'_{f(\mu)}$ with $v \le \rho$ such that $f^{\leftarrow}(\rho) \wedge \mu \le \lambda = \mu - \sigma \Longrightarrow \sigma = \mu - (f^{\leftarrow}(\rho) \wedge \mu) = f^{\leftarrow}(\mu - \rho) \wedge \mu$, Thus $f(\sigma) \wedge f(\mu) \le f(f^{\leftarrow}(\mu - \rho) \wedge \mu) \le (\mu - \rho) \wedge f(\mu))$

(1) On the other hand, since $v \le \rho$ From (1) $f(\sigma) \wedge f(\mu) = f(\mu) - v \ge f(\mu) - \rho$. Hence, $f(\sigma) \wedge f(\mu) = f(\mu) - \rho$ that is $f(\sigma)$ is F_μ so $\in \gamma_{f(\mu)}$. Then f is F_μ-irresolute open mapping

Definition 2.3 Let (X, δ) and (Y, γ) be a F-ts's, $\mu \in \mathcal{A}_\mu$. Let $f: (X, \delta) \to (Y, \gamma)$ be a mapping, then f is called F_μ-almost open mapping iff for each $\lambda \in \delta_\mu$, with $\lambda = Int_\mu(Cl_\mu(\lambda))$. $f(\lambda) \in \gamma_{f(\mu)}$.

Theorem 2.5 2 Let (X, δ) and (Y, γ) be a F-ts's, $\mu \in \mathcal{A}_\mu$. Let $f: (X, \delta) \to$ be a mapping. The following statments are equivalent.

(i) A map f is F_μ-almost open mapping

(ii) $f(Int_\mu(\lambda)) \le Int_{f(\mu)}\left(f(\lambda)\right)$, for each λ is F_μ sc $\in \delta_\mu$

(iii) For any $v \in \gamma'_{f(\mu)}$ and any $\lambda = Cl_\mu(Int_\mu(\lambda))$ such that $f^{\leftarrow}(v) \wedge \mu \le \lambda$ there exists $\rho \in \gamma_{f(\mu)}$ and $v \le \rho$ such that $f^{\leftarrow}(\rho) \wedge \mu \le \lambda$

Proof (i) $\Rightarrow$ (ii) Let λ be F_μ sc $\in \delta_\mu$ that is $Int_\mu(Cl_\mu(\lambda)) \le \lambda$. From Theorem 2.2, we easily prove the following $Int_\mu(Cl_\mu(\lambda)) = Int_\mu(Cl_\mu(Cl_\mu(\lambda)))$.

Since f is F_μ-almost open mapping, $Int_{f(\mu)}(f(Int_\mu(Cl_\mu(\lambda)) = f(Int_\mu(Cl_\mu(\lambda)) \in \gamma_{f(\mu)}$. (1)

On the other hand, $Int_\mu\left(Cl_\mu(\lambda)\right) \le \lambda \Rightarrow Int_\mu(Int_\mu(Cl_\mu(\lambda))) \le Int_\mu(\lambda)$,

Thus, $Int_\mu(\lambda) = Int_\mu(Cl_\mu(\lambda)) \le \lambda. \Rightarrow f\left(Int_\mu(\lambda)\right) = f(Int_\mu(Cl_\mu(\lambda))) = Int_{f(\mu)}(f(Int_\mu(Cl_\mu(\lambda))) \le Int_{f(\mu)}(f(\lambda))$ From(1)

(ii) $\Rightarrow$ (i) $\lambda = Int_\mu\left(Cl_\mu(\lambda)\right) \in \delta_\mu$. Since $Int_\mu(\lambda) = \lambda$ and λ is F_μ sc by (ii),

$f(\lambda) = f\left(Int_{\mu}(\lambda)\right) \leq Int_{f(\mu)}\left(f(\lambda)\right)$ From Theorem 2.2, $f(\lambda) = Int_{f(\mu)}\left(f(\lambda)\right) \in \gamma_{f(\mu)}$.

(i) ⇒ (iii) let $\lambda = Cl_{\mu}(Int_{\mu}(\lambda))$ and $v \in \gamma'_{f(\mu)}$ such that $f^{\leftarrow}(v) \wedge \mu \leq \lambda$. But $\rho = f(\mu) - f(\mu - \lambda)$ since $\mu - \lambda = Int_{\mu}\left(Cl_{\mu}(\mu - \lambda)\right)$, by (1) . Since $f^{\leftarrow}(v) \wedge \mu \leq \lambda$ iff $v \leq f(\mu) - f(\mu - \lambda)$ then, $v \leq \rho$, also, $f^{\leftarrow}(\rho) \wedge \mu = f^{\leftarrow}(f(\mu) - f(\mu - \lambda)) \leq \mu - (\mu - \lambda)) = \lambda \implies f^{\leftarrow}(\rho) \wedge \mu \leq \lambda$.

(iii) ⟹ (i) let σ be F_{μ} sc $\in \delta'_{\mu}$ such that $\sigma = Int_{\mu}(Cl_{\mu}(\sigma))$ put $v = f(\mu) - f(\sigma)$ and $\lambda = \mu - \sigma$ with $\lambda = Cl_{\mu}(Int_{\mu}(\lambda))$, we obtain

$f^{\leftarrow}(v) \wedge \mu = f^{\leftarrow}\left(f(\mu) - f(\sigma)\right) \leq \mu - (\sigma)) = \lambda$

by (iii) there exists $\rho \in \gamma_{f(\mu)}$ with $v \leq \rho$ such that $f^{\leftarrow}(\rho) \wedge \mu = \leq \lambda = \mu - \sigma \Rightarrow \sigma = \mu - (f^{\leftarrow}(\rho) \wedge \mu) = f^{\leftarrow}(\mu - \rho) \wedge \mu$, Thus $f(\sigma) \wedge f(\mu) \leq$

$f(f^{\leftarrow}(\mu - \rho) \wedge \mu) \leq (\mu - \rho) \wedge f(\mu))$ (1)

On the other hand, since $v \leq \rho$ From (1)

$f(\sigma) \wedge f(\mu) = f(\mu) - v \geq (\mu - \rho) \wedge f(\mu))$

(2)

Hence from (1) and (2) $f(\sigma) \wedge f(\mu) = (\mu - \rho) \wedge f(\mu)$

Theorem 2.6 Let (X, δ) and (Y, γ) be a F-ts's, $\mu \in \mathcal{A}_{\mu}$. Let $f : (X, \delta) \longrightarrow (Y, \gamma)$ is F_{μ}- semi continuous and F_{μ}-almost open mapping, then f is F_{μ}-irresolute apping

Proof By Proposition 2.1, we will show that $f^{\leftarrow}(\lambda) \wedge \mu$ is F_{μ} sc set, $\forall\, F_{\mu} -$ sc set $\lambda \in \gamma_{f(\mu)}$.

Since λ is $F_{f(\mu)}$ sc set $\in \gamma_{f(\mu)}$, we have $Int_{f(\mu)}\,(Cl_{f(\mu)}\,(\lambda) \leq \lambda$. Since f is $F_{\mu} -$ semi Continuous mapping, $f^{\leftarrow}$ $(f(\mu) - Cl_{f(\mu)}(\lambda)) \wedge \mu = (\mu\, f^{\leftarrow}\left(C\, l_{f(\mu)}(\lambda)\right) \wedge$ μ is F_{μ} sc set $\in \delta_{\mu}$, that is $f^{\leftarrow}\left(C\, l_{f(\mu)}(\lambda)\right) \wedge$ μ is F_{μ} sc set $\in \delta'_{\mu}$ so, $Int_{\mu}\,(Cl_{\mu}(f^{\leftarrow}\left(C\, l_{f(\mu)}(\lambda)\right) \wedge$ $\mu \leq f^{\leftarrow}\left(C\, l_{f(\mu)}(\lambda)\right) \wedge \mu \Rightarrow$

$Int_{\mu}\,(Cl_{\mu}(f^{\leftarrow}\left(C\, l_{f(\mu)}(\lambda)\right) \wedge$

$\leq Int_{\mu}\,((f^{\leftarrow}\left(C\, l_{f(\mu)}(\lambda)\right) \wedge \mu$ (1)

since f is F_{μ}-almost open mapping, and λ is F_{μ} sc set $\gamma_{f(\mu)}$. By propostion 2.2

$f(Int_{\mu}((f^{\leftarrow}\left(C\, l_{f(\mu)}(\lambda)\right) \wedge \mu)$

$\leq Inl_{f(\mu)}(f(f^{\leftarrow}\left(C\, l_{f(\mu)}(\lambda)\right) \wedge \mu)$

$Inl_{f(\mu)}\,(Cl_{f(\mu)}(\lambda) = Inl_{f(\mu)}\,(\lambda) \leq \lambda$.

$\Rightarrow Int_{\mu}\,((f^{\leftarrow}\left(C\, l_{f(\mu)}(\lambda)\right) \wedge \mu \leq f^{\leftarrow}(\lambda) \wedge \mu$

(2),

Thus, we have $Int_{\mu}\,\left(Cl_{\mu}\left(f^{\leftarrow}(\lambda)\right) \wedge \mu\right)$ $\leq Int_{\mu}\,(Cl_{\mu}(f^{\leftarrow}\left(C\, l_{f(\mu)}(\lambda)\right) \wedge \mu) \leq$ $Int_{\mu}\,(f^{\leftarrow}\left(C\, l_{f(\mu)}(\lambda)\right) \wedge \mu)$ by(1)

$\leq f^{\leftarrow}(\lambda) \wedge \mu$ by (2). Hence $f^{\leftarrow}(\lambda)$ *is* F_{μ} sc .

Conclusion

A new F-topological notions called F_{μ}-perfectly continuous , F_{μ}-completely continuous and F_{μ}- R- continuous, F_{μ}-perfectely retract are introduced and studied , Using these notions we define and study F_{μ}-completely retract, F_{μ}-R-retract , F_{μ}-neighbourhood perfectly retract, F_{μ}-neighborhood completely retract and F_{μ}-neighbourhood R-retract. The notions of F_{μ}-semi closure, F_{μ}-semi interior and F_{μ}-irresolute mapping are introduced.

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