THE CONCEPTUAL IUH HYDROLOGICAL MODEL OF RAINFALL RUNOFF TRANSFORMATION FOR SMALL WATERSHED OF PAKISTAN.

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ABSTRACT *The study has been carried out on Kanshi catchment of Pakistan near to Palote having area of 429 square mile located in sub humid to humid region of Pakistan. This procedure simulates the hydrograph concept given by NASH considering sub catchments as a set of linear reservoir in series. The excess rainfall is routed through the reservoir to get a resultant direct run off hydrograph. The validity of the developed procedure has been checked by comparing the observed direct run off hydrograph to the computed direct run off hydrograph of the same region. The results are significantly predictable and comparable with the observed ones. As a result it can be said that the developed procedure is equally applicable for both gauged and ungauged catchments this can be used with confidence in the forecasting, designing and planning of hydrological works.*

Key words: Rainfall, Runoff, Nash model, instantaneous unit hydrograph (IUH), Direct Runoff (DRO).

1. INTRODUCTION

The hydrological cycle of nature is simply describe by hydrological modeling. The mathematical rainfall runoff model represent the relation of rainfall and runoff of the catchment. The rainfall runoff model produce the runoff hydrograph by taking the rainfall as a input. The rainfall runoff model are classified into deterministic, stochastic, conceptual, black box or simplified [8]. The hydrological models of rainfall runoff are used to understand the impact of land use change and climatic change on the catchment. There are many hydrological models which varying in nature, complexity [17]. The most known models are rational method, Soil Conservation Services (SCS) method[10] and Conceptual model of linear reservoir[13]. The response of catchment to rainfall on the basis of hydrological model estimate the catchment yield and the data of runoff is very important for the purpose of water resource planning and flood forecasting and many other application[16]. The availability of discharge data for ungauged catchment is very limited therefore in this circumstance rainfall –runoff model are developed to simulate the rainfall runoff relation for catchment. The rainfall runoff transformation is very complex process which are influence by a number of implicit and explicit factors such as precipitation distribution evaporation, transpiration and soil type.[13] divide the catchment into identical reservoir and prepare the conceptual rainfall runoff models by route the inflow through reservoir. The Nash conceptual model were tested on many small catchment of world and its found to be good agreement with observe and actual runoff hydrograph [7].[16] developed a model to predict runoff volume from small watershed to simulate daily monthly and annual runoff volume quite accurately.[7] developed a mathematical model of the instantaneous unit hydrograph based on time area histogram for a small watershed at Pantnagar.

2. Study Area

The hydrological station of Kanshi river basin is at Palote and there are two meteorological stations. The geographical location is latitude 33o14’N and longitude 73o26’E. Length of river near to Palote is 38 miles. The area of the catchment is about 429 square miles. Figure 1 shows the map of the study area. Kanshi basin is completely rainfed area where most water available in wet period of monsoon. The Kanshi basin contributes the average annual flow 2 MAF of water into Mangla dam. The average annual effective rain fall of Kanshi basin is 4.5cm.

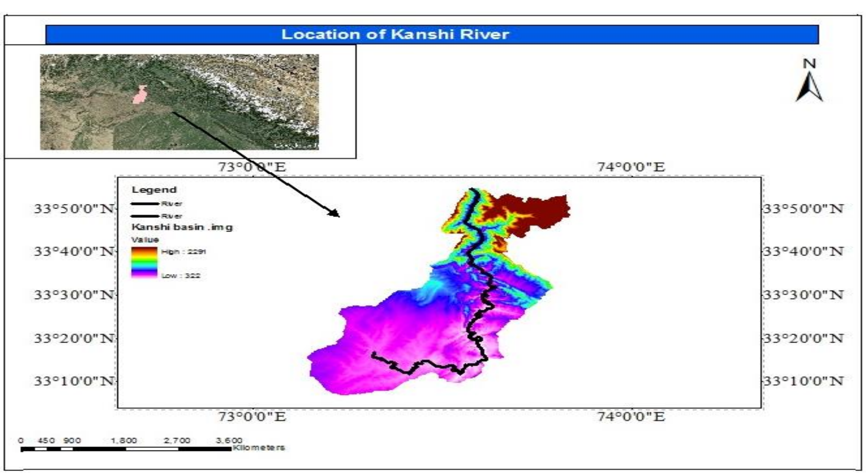

Figure 1: Map of study area

3. MATERIAL AND METHOD

The five large historic storms (1970,1972,1973) were consider to study the rain fall run off relation of Kanshi basin. Nash (1957) proposed conceptual model of a catchment to develop an equation for IUH. The catchment is assumed to be made up of a series of n identical linear reservoirs each having same storage constant K. The first reservoir receives a unit volume equal to 1 cm of effective rain from the catchment instantaneously. This inflow is routed through the first reservoir to get the outflow hydrograph. The outflow from the first reservoir is considered as the input to the second; the outflow from the second reservoir is the input to the third and so on for the "n" reservoirs. The conceptual cascade of the reservoirs as above and the shape of the outflow hydrographs from each reservoir of the cascade are shown in figure 2. The outflow hydrograph from the nth reservoir is taken as the IUH of the catchment.

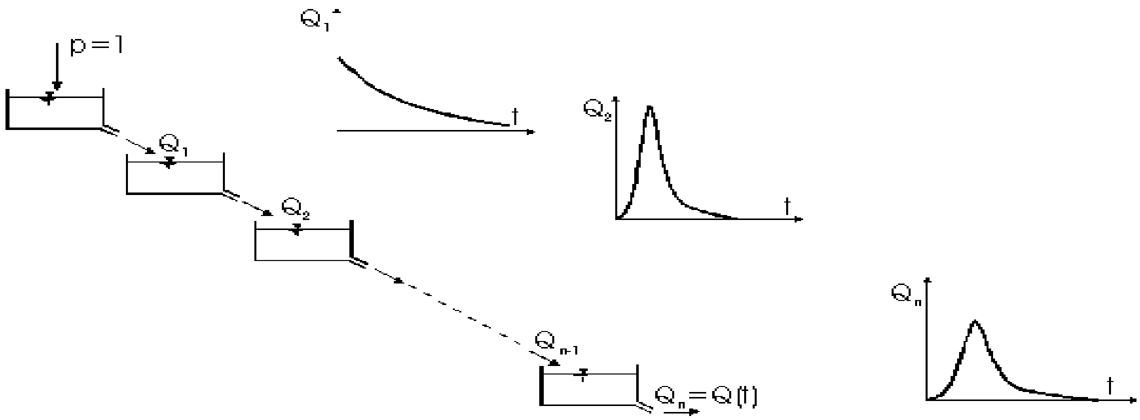

Figure 2: The Nash concept for deriving the instantaneous unit hydrograph

The equation of continuity,

$$I - Q = \frac{dS}{dt}$$

For a linear reservoir

$$S = KQ$$

And hence

$$\frac{dS}{dt} = K\, dQ/dt$$

Substituting in above equation and rearranging,

$$K\frac{dQ}{dt} + Q = 1$$

And the solution of this differential equation, where Q and I are the functions of time t, is

$$Q = \frac{1}{K}e^{-t/K}\int e^{t/K}\, I dt$$

Now the first reservoir, the input is applied instantaneously. Hence for t > 0, I = 0. Also at t =0, ∫ I dt = instantaneous volume inflow = 1 cm of effective rain. Hence for the first reservoir eq. becomes.

$$Q_1 = \frac{1}{K}e^{-t/K}$$

For the second reservoir

$$Q_2 = \frac{1}{K}e^{-t/K}\int e^{t/K}\, I dt$$

Here I = input = Q_1 given by equation. Thus,

$$Q_2 = \frac{1}{K}e^{-t/K}\int e^{t/K}\,\frac{1}{K}\,e^{-t/K}dt = \frac{1}{K^2}\,t\,e^{-t/K}$$

Similarly, for the hydrograph of outflow from the nth reservoir Qn is obtained as

$$Q_n = \frac{1}{(n-1)!\,K^n}\,t^{n-1}\,e^{-t/K}$$

As the outflow from the nth reservoir was caused by 1 cm of excess rainfall falling instantaneously over the catchment equation describes the IUH of the catchment. Using notation u (t) to represent the ordinate of the IUH, equation to represent the IUH of a catchment is written as

$$u\,(t) = \frac{1}{(n-1)!\,K^n}t^{n-1}e^{-t/K}$$

Here, if "t" is in hours, u (t) will have dimensions of cm/hr, k and n are constants for the catchment to be determined by effective rainfall and flood hydrograph characteristics of the catchment. It should be remembered that equation is based on a conceptual model and as such if n for a catchment happens to be a fraction; it is still all right to enable (n-1)! To be determined both for integer and fractional values of n, the gamma function Γ (n) is used to replace (n-1)! So that

$$u\,(t) = \frac{1}{K\Gamma(n)}\left(\frac{t}{K}\right)^{n-1} e^{-t/K} \quad \text{....(1)}$$

Where u (t) is the instantaneous unit hydrograph ordinate at time, t , K is the reservoir storage constant (hr), t is the time after beginning of direct runoff (hr), Γ(n) is the gamma function and n is the dimensionless shape parameter. Equation (1) representing the impulse response function of the watershed system composed of linear reservoir in terms of time to peak express as

$$u(0,t) = (n-1)^n/t_p\Gamma(n)\left[\left(t/t_p\right)e^{-t/t_p}\right]^{n-1}$$

Thus, the equation expressing the direct runoff as a function of time in Cumec, Q (t), watershed area in sq km (A), effective rainfall in cm (Re) and time to peak in hr (tp) is expressed as:

$$Q_p = \frac{2.78AR_e}{t_p}\left[\frac{(n-1)^n}{\Gamma(n)e^{n-1}}\right]$$

There are two parameters "n" and time of peak "tp" of the Nash model required to estimate peak flow QP.

3.1. Estimation of model Parameters

There is only one parameter "n" is required to determine the peak flow of peak flow The value of n will determine by using the recession curve of observe run off hydro graph.

$$\frac{K}{t_p} = \frac{\frac{\Delta t}{t_p}}{2.3\log_{10}(Q_0/Q_1)}$$

Where K is the recession constant for Q0 to Q1, Q0 and Q1 are the flow rate at time t0 and t1 respectively and Δt is the time difference between t1 and t0. which can be read from the semi log graph paper plot and the value of n can be evaluate by using the graphical relationship between K1/tp and n developed from graph developed by Wu et al . (1964) shown in figure 3.

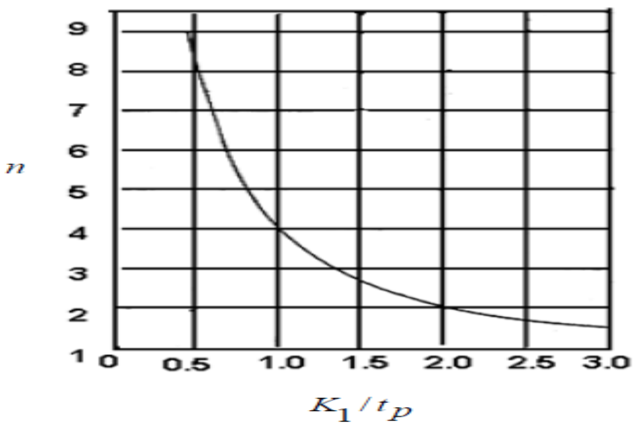

Figure 3: Weng.et.al graph

The average areal distribution of rain fall has been determined by thiessen polygon method Excess rain fall is calculated by using phi- index method. The time of peak is calculated by using following formula

tp =0.67*tc

Where tc is time of concentration which given by

$$t_c = 0.0078\left(\frac{L^{0.77}}{S^{0.385}}\right)$$

Where L = Travel length in meter and S = Slope of basin

3.2 Model Performance

The absolute relative error is used to estimate the performance of the predictable peak discharge. The absolute relative error measure the goodness of fit of computed peak discharge. The absolute relative error is given by

$$E_r = \frac{|Q_o - Q_c|}{Q_o} \times 100$$

Er = Absolute relative error in % age

Qo = Observed peak, Qc = computed peak

4. RESULT AND DISCUSSION

Five storm events were used to estimate the model parameter "n". The ratio of time of peak and recession constants of observed hydrograph of selected storms were calculated by plot the recession limb of observed hydrographs of storms on semi log graph paper with respect to time.The recession limb hydrograph of storm 14/9/1970 shown in figure 4. Recession constant of observed hydrograph of storms are given in table 1

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Table 1: Determination of recession constant of Kanshi River

STORMS	RECESSION CONSTANT (K)	TIME OF PEAK t_p (hr)	K/t_p
Storm 14/9/1970	2.8	8	0.36
Storm 8/7/1973	5.5	8	0.69
Storm 4/8/1973	4.7	8	0.59
Storm 6/81973	5.2	8	0.65
Storm 9/7/1972	4.2	8	0.52
AVERAGE			0.56

The average value of **k/tp** is **0.56**. The value of "n" for the model is calculated by Weng.et.al graph given in figure 3 is **7**.

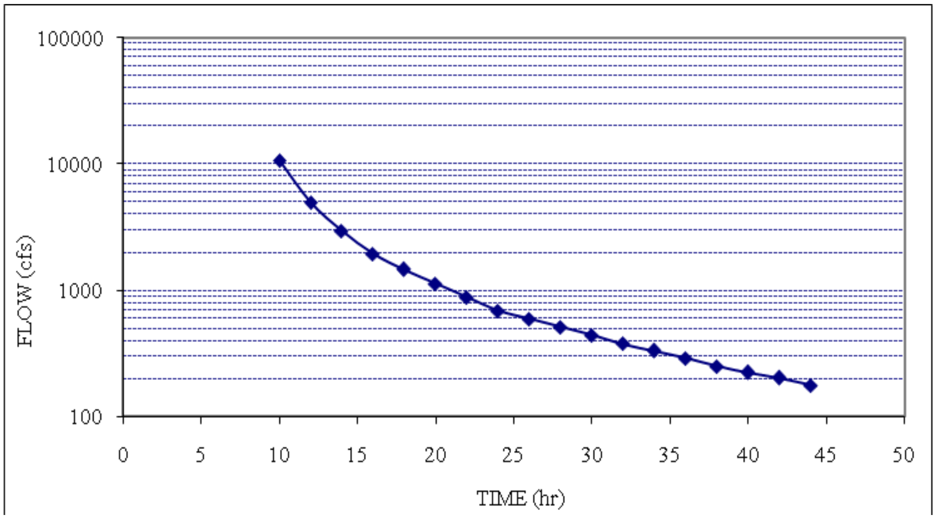

Figure 4: Log Plot of recession limb of hydrograph of Strom 14/9/1970.

The infiltration loss of water shed is calculated by using phi index method .The average phi loss is 0.20 in/hr shown in table 2.

Table2: Phi loss index

Storms	Volume of DRO	Depth of run off	Total Rain	Duration of storm	Phi loss
	m^3	cm	in	hr	In/hr
14/9/1970	26818891	2.41	4	4	0.24
9/7/1972	6779095	0.61	4.39	17	0.087
8/7/1973	9137005	0.82	3	3	0.21
4/8/1973	16095002	1.44	5.90	13	0.13
6/8/1973	25300675	2.27	5.4	15	0.087
Average					0.20

The value of "n" of Nash model is 7 and the time of peak of watershed is 8 hr therefore the ordinate of instantaneous unit hydrograph of Kanshi basin is given below:

U (t) = 48.6 * [t/8* exp (-t/8)] 6

The peak runoff rate is given below

Qp = 372 * Re

The result obtain by using the above rain fall runoff relation for determine the peak flow rate of the Kanshi River shown in table 3.

Table 3: Determination of peak flow of Kanshi River

STORM	OBSERVB PEAK	COMPUTED PEAK	EFFECTIVE RAIN	ABSOLUTE ERROR
	cms	cms	Cm	%
Storm 14/9/1970	928	930	2.5	0.21
Storm 8/7/1973	273	297	0.8	8.89
Storm 4/8/1973	488	483	1.3	0.90
Storm 6/8/1973	545	565	1.52	3.6
Storm 9/7/1972	119	148	0.4	24.04
AVERAGE				7

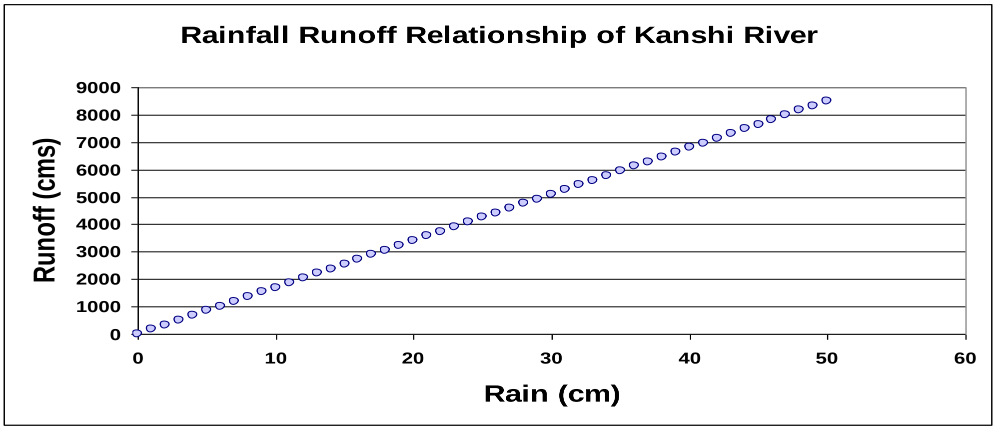

Figure 5: Rain fall Runoff relation for Kanshi River

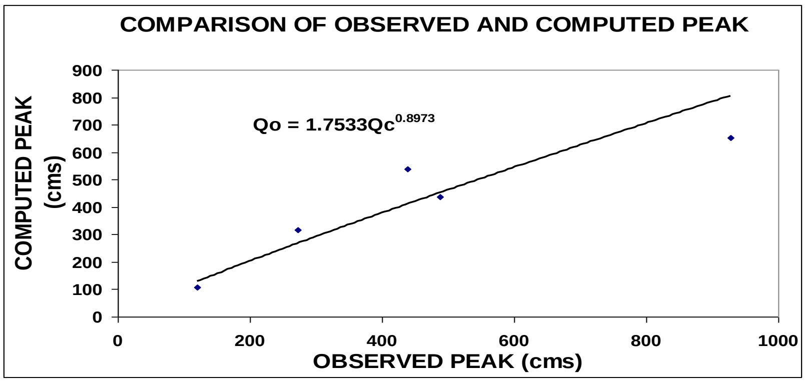

Figure 6: Comparison of observed and computed peak

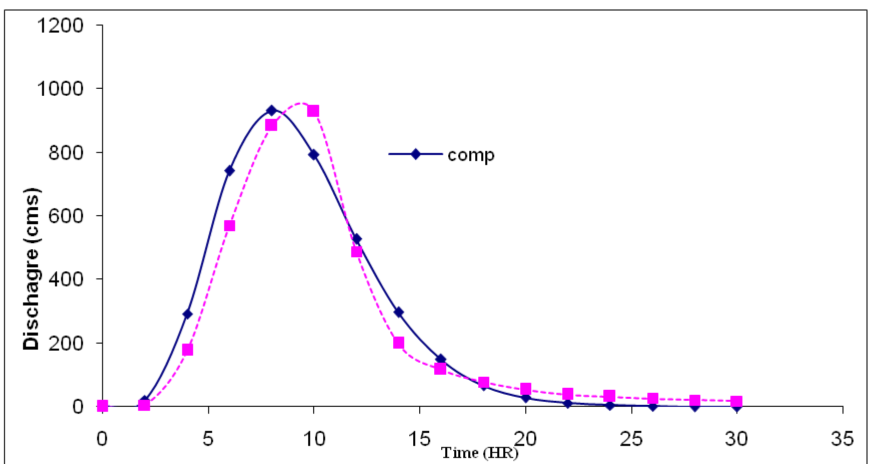

Figure 7: Unit hydrograph for the storm event of 14/9/1970

The average value of absolute relative error in estimated peak was found to be 7% which indicate that the Nash model generates closely comparable peak value to the observed peak value. The rain fall run off relarion shown in figure 5. The 96 % correlations were found between observed and computed peak flow rates. The observed and computed peaks of the model is shown in figure 6.

The observe and computed hydrograph of Storm 14/9/970 and the Storm of 8/7/1973 were found to be very close to each other as shown in the figure 7 and figure 8.

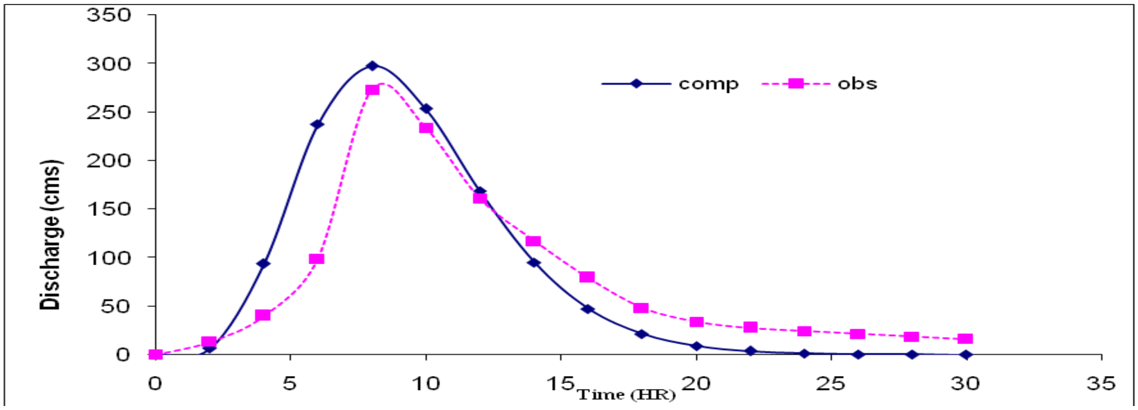

Figure 8: Unit hydrograph for the storm event of 8/7/1973

CONCLUSIONS:

The result indicates that the conceptual instantaneous unit hydrograph model can be utilized for the development of peak runoff rate for Small watershed of Pakistan having same geomorphologic characteristics. The relation between peak runoff rate and effective rainfall is linear which is shown in figure 5. The calculated instantaneous unit hydrograph is similar to observed unit hydrograph of the selected storms which are shown in figure 7 and figure 8. The value of the Nash model parameter "n" of Kanshi basin is "7". Therefore Nash model can be used for prediction of peak flow occurred due to the effective rainfall.

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