RADIATION AND MASS TRANSFER EFFECTS ON MHD OSCILLATORY FLOW FOR JEFFERY FLUID WITH VARIABLE VISCOSITY THROUGH POROUS CHANNEL IN THE PRESENCE OF CHEMICAL REACTION

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ABSTRACT: *The present paper deals with a radiation and mass transfer effects on MHD oscillatory flow for Jeffery fluid with variable viscosity through porous channel in the presence of chemical reaction. The fluid viscosity is assumed to vary as an exponential function of temperature. The perturbation technique series use to solve the momentum equation. The effects of thermal conductivity, Grashof number, Darcy number, Hartmann number, radiation parameter, Schmidth number and chemical reaction parameters on velocity, temperature and concentration has been discussed for variations in the governing parameters.*

Keywords: Jeffrey fluid, MHD, Oscillatory flow, Porous medium, Thermal radiation.

1. INTRODUCTION

Human structure, place of residence and work are examples for applications of fluid mechanics. Fluid mechanics also play a major role in the design of aircraft engines, jets and torpedoes, as well as many applications in the other sciences [1, 2]. Fluid flow through the porous medium of contemporary and important topics at present, where there are excellent applications for fluid transport through the permeability channel in Human body, geophysics and technology. During the second half of the last century, researchers were interested in studying the flow of liquids accompanied by thermal transfer. In 1960, Nigam and Singh [3], they studied the effect of heat transfer by laminar flow between parallel plates under the action of transverse magnetic field. In recent years, the requirements of modern technology have tended to study the fluid flow in detail, which has been interested in the interaction of many phenomena. One of these studies is related to the effect of free thermal flow through a porous medium where it plays an important role in the petroleum and engineering industries (e.g., magnetism generators) [4, 5]. The first experiments that showed the concept of fluid thermal instability attributed to Bénard [6]. Schmidt (1928) [7], gave the definition of thermal instability and later modified by Hutchinson (1957) [8], which has taken a major deal of attention and has been recognized as a problem of fundamental in many areas of fluid dynamics. The MHD field of was initiated by Hannes Alfvén [9], where the magnetic field can induction the current into the fluid moving connector, which in turn attracts the fluid and changes the magnetic field likewise. Some important applications of MHD are the removal of arterial blockage [10]. Al-Khafajy and Abdulhadi [11], studied the effects of the peristaltic transport of the Jeffrey fluid through a porous medium with MHD effect and wall thickness. Radiation is the process by which energy can be transferred from one object to another through electro-magnetic waves. Magnetic radiation and hydrogen continue to attract interest in engineering science and applied mathematics research owing to extensive applications of such flows. M. O. Ibrahim et al [12], gave the formula of the effect of radiation on the flow of MHD fluid in a vertical channel under the thick vertical approximation. There are several non-Newtonian fluid models proposed to describe the behavior of these vital fluids. Among these fluids, Jeffrey is circulating the Newtonian fluid. In the existing literature, many scientists studied Jeffrey fluid under different mechanical and thermal boundary conditions [13, 14]. The aim of the this paper is to discuss the influence of (MHD) oscillatory slip flow for Jeffrey fluid with variable viscosity through regularly channel with varying temperature and concentration.

2. Mathematical Formulation

Consider the oscillatory flow of an incompressible Jeffrey fluid with variable viscosity in a channel of width *d* under the influence of electrically applied magnetic field and radioactive heat transfer as depicted in Figure 1. It is assumed that the fluid has a very small electromagnetic force produced by the small electric conduction. We are considering Cartesian coordinate system such that, $(u(y,t),0,0)$ is a velocity vector in which u is the x-component of velocity and y is perpendicular to the x-axis.

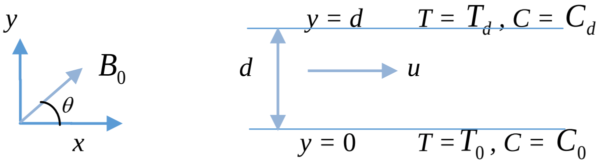

Figure 1. Geometry of the problem

The basic equations of continuity, momentum, energy and concentration governing such a flow, is:

$$\frac{\partial \bar{u}}{\partial \bar{x}}+\frac{\partial \bar{v}}{\partial \bar{y}}=0 \qquad (1)$$

$$\rho\left(\frac{\partial \bar{u}}{\partial \bar{t}}+\bar{u}\frac{\partial \bar{u}}{\partial \bar{x}}+\bar{v}\frac{\partial \bar{u}}{\partial \bar{y}}\right)=-\frac{\partial \bar{p}}{\partial \bar{x}}+\frac{\partial \bar{S}_{\bar{x}\bar{x}}}{\partial \bar{x}}+\frac{\partial \bar{S}_{\bar{x}\bar{y}}}{\partial \bar{y}}+\rho g\beta_T(T-T_0)$$
$$+\rho g\beta_C(C-C_0)-\sigma B_0^2\sin^2(\theta)\bar{u}-\frac{\mu(T)}{k}\bar{u}$$

(2)

$$\rho\left(\frac{\partial \bar{v}}{\partial \bar{t}}+\bar{u}\frac{\partial \bar{v}}{\partial \bar{x}}+\bar{v}\frac{\partial \bar{v}}{\partial \bar{y}}\right)=-\frac{\partial \bar{p}}{\partial \bar{y}}+\frac{\partial \bar{S}_{\bar{y}\bar{x}}}{\partial \bar{x}}+\frac{\partial \bar{S}_{\bar{y}\bar{y}}}{\partial \bar{y}}-\sigma B_0^2\cos^2(\theta)\bar{v}-\frac{\mu(T)}{k}\bar{v}$$

(3)

$$\rho\frac{\partial T}{\partial \bar{t}}=\frac{K}{c_p}\frac{\partial^2 T}{\partial \bar{y}^2}-\frac{1}{c_p}\frac{\partial q}{\partial \bar{y}}+\frac{Q_T}{c_p}(T-T_0)$$

(4)

$$\frac{\partial C}{\partial \bar{t}}=D_B\frac{\partial^2 C}{\partial \bar{y}^2}-K_C^*(C-C_0)+\frac{D_T K_T}{T_m}\frac{\partial^2 T}{\partial \bar{y}^2}$$

(5)

The boundary conditions are given by:

$\bar{u}=0$ at $\bar{y}=0$ and $\bar{u}=0$ at $\bar{y}=d$

(6)

$T = T_0$ at $\bar{y} = 0$ and $T = T_d$ at $\bar{y} = d$ (7)

$C = C_0$ at $\bar{y} = 0$ and $C = C_d$ at $\bar{y} = d$ (8)

where $\bar{u}$ is the axial velocity, *T* is the fluid temperature, *C* is the fluid concentration, $\mu(T)$ fluid viscosity dependent on temperature $\bar{p}$ is the pressure, ρ is the fluid density, B_0 is the magnetic field strength, σ is the conductivity of the fluid, *g* is the acceleration due to gravity, β_T is the coefficient of volume expansion due to temperature, β_C is the coefficient of volume expansion due to concentration, *k* is the permeability, c_p is the specific heat at constant pressure, *K* is the thermal conductivity and *q* is the radioactive heat flux, Q_T is heat generation, θ ($0 \le \theta < \pi$) is the angle between velocity field and magnetic field strength, K_C^* the chemical reaction rate on the concentration and K_T is the thermal diffusion ratio, T_m is the mean temperature, B_B and B_T the Brownian and thermophoretic diffusion coefficients, respectively.

The constitute equation of *S* for Jeffrey fluid with variable viscosity is

$$S = \frac{\mu(T)}{1+\lambda_1}\left(\dot{\gamma} + \lambda_2 \ddot{\gamma}\right)$$

where μ is the dynamic viscosity, λ_1 is the ratio of relaxation to retardation times, λ_2 is the retardation time, $\dot{\gamma}$ is the shear rate and dots over the quantities denote differentiation with time.

So $\bar{S}_{\bar{x}\bar{x}} = \bar{S}_{\bar{y}\bar{y}} = 0$, $\bar{S}_{\bar{x}\bar{y}} = \bar{S}_{\bar{y}\bar{x}} = \dfrac{\mu(T)}{1+\lambda_1}\dfrac{\partial \bar{u}}{\partial \bar{y}}$ (9)

Following Vinvent et al. [15], it is assumed that the fluid is optically thin with a relatively low density and the radioactive heat flux is given by:

$$\frac{\partial q}{\partial y} = 4R_m^2\left(T_0 - T\right)$$

(10)

here R_m is the mean radiation absorption coefficient.

Introducing the following non-dimensional variables:

$$x = \frac{\bar{x}}{d}, y = \frac{\bar{y}}{d}, u = \frac{\bar{u}}{U_m}, \Theta = \frac{T - T_0}{T_d - T_0}, t = \frac{\bar{t}U_m}{d}, p = \frac{\bar{p}d}{\mu_c U_m}, Da = \frac{k}{d^2},$$
$$\mu(\Theta) = \frac{\mu(T)}{\mu_c}, \mathrm{Re} = \frac{\rho d U_m}{\mu_c}, Pe = \frac{\rho d U_m c_p}{K}, R^2 = \frac{4R_m^2 d^2}{K}, Sc = \frac{U_m d}{D_B},$$
$$S_{xy} = \frac{d\bar{S}_{\bar{x}\bar{y}}}{\mu_c U_m}, \Phi = \frac{C - C_0}{C_d - C_0}, Q = \frac{Q_T d^2}{K}, Kc = \frac{K_C^* d}{U_m}, M^2 = \frac{\sigma d^2 B_0^2}{\mu_c}, \quad (11)$$
$$Gr = \frac{\rho g \beta_T d^2 (T_d - T_0)}{\mu_c U_m}, Gc = \frac{\rho g \beta_C d^2 (C_d - C_0)}{\mu_c U_m}, Sr = \frac{D_T K_T (T_d - T_0)}{U_m T_m d (C_d - C_0)}$$

where U_m is the mean flow velocity, *Da* Darcy number, Re Reynolds number, *M* magnetic parameter, *Pe* is the Peclet number and *R* is the radiation parameter. *Sc* is the Schmidt number, *Sr* is the Soret number, *Q* is the heat generation parameter, T_m is the mean temperature, *Gr* is Thermal Grashof number and *Gc* is Solutal Grashof number.

Substituting (10) and (11) into equations (1)-(5), we have:

$$\frac{\rho d U_m}{\mu_c}\frac{\partial u}{\partial t} = -\frac{\partial p}{\partial x} + \frac{\mu(\Theta)}{1+\lambda 1}\frac{\partial^2 u}{\partial y^2} + \frac{\rho g \beta_T d^2 (T_d - T_0)}{\mu_c U_m}\Theta + \frac{\rho g \beta_C d^2 (C_d - C_0)}{\mu_c U_m}\Phi - \frac{\sigma B_0^2 d^2 \sin^2(\theta)}{\mu_c}u - \frac{d^2}{k}\mu(\Theta)u \quad (12)$$

$$\frac{\rho d U_m c_p}{K}(T_d - T_0)\frac{\partial \Theta}{\partial t} = (T_d - T_0)\frac{\partial^2 \Theta}{\partial y^2} + \frac{4d^2 R_m^2}{K}(T_d - T_0)\Theta + \frac{Q_T d^2}{K}(T_d - T_0)\Theta \quad (13)$$

$$\frac{U_m}{d}(C_d - C_0)\frac{\partial \Phi}{\partial t} = \frac{D_B}{d^2}(C_d - C_0)\frac{\partial^2 \Phi}{\partial y^2} - K_C^*(C_d - C_0)\Phi + \frac{D_T K_T}{d^2 T_m}(T_d - T_0)\frac{\partial^2 \Theta}{\partial y^2}$$

(14)

After simplify, we obtain the following non-dimensional equations:

$$\mathrm{Re}\frac{\partial u}{\partial t} = -\frac{\partial p}{\partial x} + \frac{\mu(\Theta)}{1+\lambda_1}\frac{\partial^2 u}{\partial y^2} + Gr\Theta + Gc\Phi - M_\varphi^2 u - \frac{1}{Da}\mu(\Theta)u$$

(15)

$$Pe\frac{\partial \Theta}{\partial t} = \frac{\partial^2 \Theta}{\partial y^2} + (R^2 + Q)\Theta$$

(16)

$$\frac{\partial \Phi}{\partial t} = \frac{1}{Sc}\frac{\partial^2 \Phi}{\partial y^2} - K_C\Phi + Sr\frac{\partial^2 \Theta}{\partial y^2}$$

(17)

where $M_\varphi = M\sin(\theta)$.

With the boundary conditions:

$u(0) = u(1) = 0$ (18)

$\Theta(0) = 0$, $\Theta(1) = 1$ (19)

$\Phi(0) = 0$, $\Phi(1) = 1$ (20)

3. Solution of the Problem

By substituting equations $\Theta(y,t) = e^{\omega i t}\Theta_0(y)$ and $\Phi(y,t) = e^{\omega i t}\Phi_0(y)$ into energy equation (16) and concentration equation (17), respectively, with boundary conditions (19) and (20), we have the following solutions:

$$\Theta(y,t) = e^{\omega i t}\csc(A)\sin(Ay)$$

(21)

$$\Phi(y,t) = \frac{e^{\omega i t}}{\left(A^2 + B^2\right)}\left(\left(\frac{(A^2 + B^2 + ScSrA^2)}{\left(e^{2B} - 1\right)}\right)\left(e^{B(1+y)} - e^{B(1-y)}\right) - ScSrA^2\csc(A)\sin(Ay)\right)$$

(22)

where ω is the frequency of the oscillation, and

$A^2 = R^2 + Q - Pe\omega i$, $B^2 = Sc(Kc + \omega i)$.

To solve the momentum equation (15) for purely oscillatory flow, we defined Reynold's model and variation of viscosity with temperature as follows, [16]:

$$\mu(\Theta) = e^{-\alpha\Theta}$$

By using the Maclaurin series, Reynold's model can be written as:

$\mu(\Theta) = 1 - \alpha\Theta$, $\alpha << 1$ (23)

here α = 0 corresponds to the constant viscosity case. Let

$$-\frac{\partial p}{\partial x} = \lambda e^{\omega i t}$$

(24)

$$u(y,t)=e^{\omega it}u_0(y)$$
(25)

where λ is a real constant.

Substituting the equations (23 - 25) into the equations (15), we have:

$$\operatorname{Re}\omega i u_0(y)=\lambda+\frac{1}{1+\lambda_1}(1-\alpha\Theta)\frac{\partial^2 u_0(y)}{\partial y^2}-\left(M_\varphi^2+\frac{1}{Da}(1-\alpha\Theta)\right)u_0(y)+Gr\Theta_0+Gr\Phi_0$$
(26)

Small α suggests the use of perturbation technique to solve equation (26). Accordingly, we write:

$$u_0(y)=u_{00}(y)+\alpha u_{01}(y)+\alpha^2 u_{02}(y)+O(\alpha^3)$$
(27)

Substituting equation (27) into equation (26) with boundary conditions (18), then equating the like powers of α, we have:

3.1 Zeros-order system (α^0)

$$\frac{\partial^2 u_{00}}{\partial y^2}-(1+\lambda 1)\left(M_\varphi^2+\operatorname{Re}\omega i+\frac{1}{Da}\right)u_{00}=-(1+\lambda 1)(\lambda+Gr\Theta_0+Gc\Phi_0)$$
(28)

The associated boundary conditions are

$$u_{00}(0)=u_{00}(1)=0$$
(28)

3.2 First-order system (α^1)

$$\frac{\partial^2 u_{01}}{\partial y^2}-(1+\lambda 1)\left(M_\varphi^2+\operatorname{Re}\omega i+\frac{1}{Da}\right)u_{01}=\left(\frac{\partial^2 u_0}{\partial y^2}-\frac{(1+\lambda 1)}{Da}u_0\right)\Theta$$
(29)

The associated boundary conditions are

$$u_{01}(0)=u_{01}(1)=0$$
(30)

Therefore, the fluid velocity is

$$u(y,t)=\left\{\begin{array}{l}\dfrac{Be^{iwt}}{A}\left(1-\left(\dfrac{e^{\sqrt{A}y}+e^{\sqrt{A}(1-y)}}{1+e^{\sqrt{A}}}\right)\right)+\alpha\dfrac{Be^{2iwt}\theta_0}{4A^2\left(1+e^{\sqrt{A}}\right)}\\ \left[\begin{array}{l}e^{\sqrt{A}(y-1)}\left(\begin{array}{l}\left(\begin{array}{l}(A+H\sqrt{A})e^{2\sqrt{A}(1-y)}+\\2(A\sqrt{A}+AH)e^{2\sqrt{A}(1-y)}y\end{array}\right)\\-2(A\sqrt{A}-AH)e^{\sqrt{A}}y+(A-H\sqrt{A})e^{\sqrt{A}}\end{array}\right)\\+\dfrac{1}{\left(e^{2\sqrt{A}}-1\right)}\left(\begin{array}{l}e^{\sqrt{A}y}\left(\begin{array}{l}(A-H\sqrt{A})-2(A\sqrt{A}+AH)e^{\sqrt{A}}\\-(A-2A\sqrt{A}-H\sqrt{A}+2AH)e^{2\sqrt{A}}\end{array}\right)\\-e^{\sqrt{A}(1-y)}\left(\begin{array}{l}-(A+H\sqrt{A})(1+2\sqrt{A})\\+2(A\sqrt{A}-AH)e^{\sqrt{A}}\\+(A+H\sqrt{A})e^{2\sqrt{A}}\end{array}\right)\end{array}\right)\end{array}\right]\end{array}\right\}$$

where $H=\dfrac{1+\lambda 1}{Da}$, $A=(1+\lambda_1)\left(M^2\sin^2(\theta)+\operatorname{Re}\omega i+\dfrac{1}{Da}\right)$

and $B=(1+\lambda_1)(\lambda+Gr\Theta_0+Gc\Phi_0)$.

4. RESULTS AND DISCUSSION

This item includes a discussion of the solutions we have obtained from MATHEMATICA program. We discuss the numerical and arithmetical results of the effects of radiation and mass transfer on the flow of the MHD of the Jeffrey fluid with the variable viscosity through the porous channel in the presence of a chemical reaction. Numerical evaluations of analytical results and some of the graphically significant results are presented in Figures (2 - 13).

The figures (2-5), illustrate the effects of the parameters *Pe*, ω, *R* and *Q* on the temperature and concentration. Figure (2) shows increased temperature and decreased concentration with increasing *Pe*. Figure (3) shows a decrease in temperature and concentration with an increase of ω. Figures (4, 5) illustrate the increasing of *R* and *Q* lead to rise up temperature and reduction concentration.

We observed that in figure (6) the concentration decreases with increase *Sc* and *Sr*, respectively.

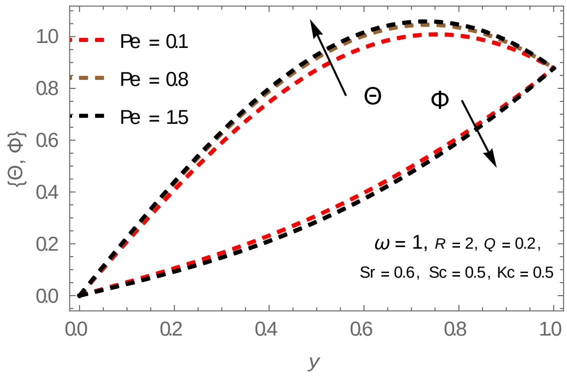

Figure 2. Influence of *Pe* on temperature and concentration.

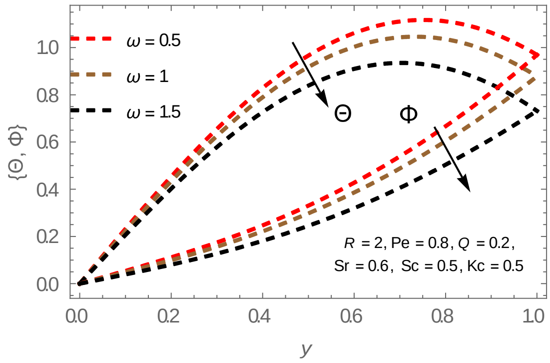

Figure 3. Influence of ω on temperature and concentration.

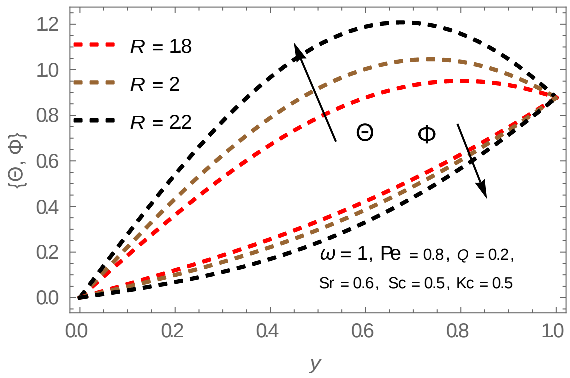

Figure 4. Influence of *R* on temperature and concentration.

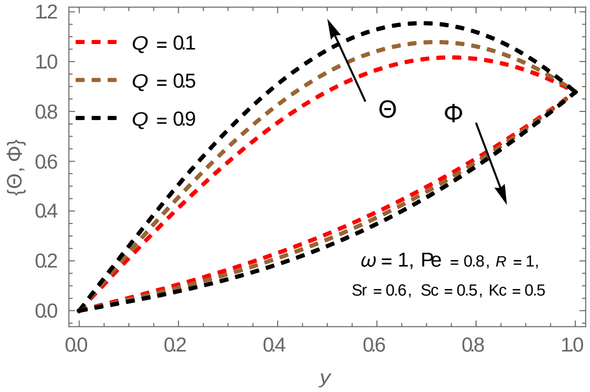

Figure 5. Influence of*Q* on temperature and concentration.

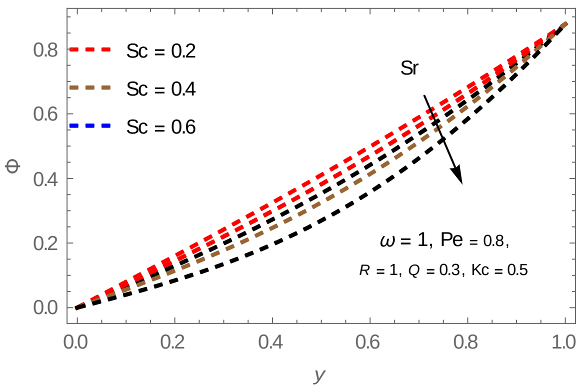

Figure 6. Concentration profile for various values of *Sr* and *Sc*.

The objective of this study is to determine the parameters most influencing the movement of fluid. Figures (7-13) shows the change of the velocity field under the effects of parameters *Da* Darcy number, α viscosity parameter, *Gr* Thermal Grashof number, *Gc* Solutal Grashof number, *Re* Reynolds number, *R* radiation parameter, *Pe* Peclet number, *Q* heat generation parameter, *Kc* chemical reaction, ω frequency of oscillation, *Sc* Schmidt number, *Sr* Soret number, *M* Hartman number and θ angle between velocity field and magnetic field strength. The profile of velocity was plotted in two dimensions by giving a constant value of time (t=0.5), and we determined the value of (M =1) and ($\theta = \pi/4$) in the figures (7-12).

Figure 7, shows that the value of velocity increases with *Da* and α, respectively. Figure 8, illustrates the effects of the parameters *Gr* and *Gc* on the velocity distribution function *u* vs. *y*. It is found that the velocity profile increases with increasing both *Gr* and *Gc*. and attains its maximum height near the center line of the channel. The fluid velocity starts increasing and tends to be constant at the walls, as specified by the boundary conditions. From figure 9, one can depict here that velocity decreases with increasing of *Kc* and ω, respectively. Figure 10, illustrates the increasing of *R* and *Re* lead to rising up the velocity field. Figure 11 contains the behavior of *u* under the variation of *Pe* and *Q*, one can depict here that u go up with the increasing effects of both the parameters *Pe* and *Q*. We note that in figure 12 the increasing *Sr* and *Sc* tend to decreases in velocity field. The influence of *M* and θ on velocity field is analyzed in Figure 13. It is found that the velocity profile go down with increasing both *M* and θ.

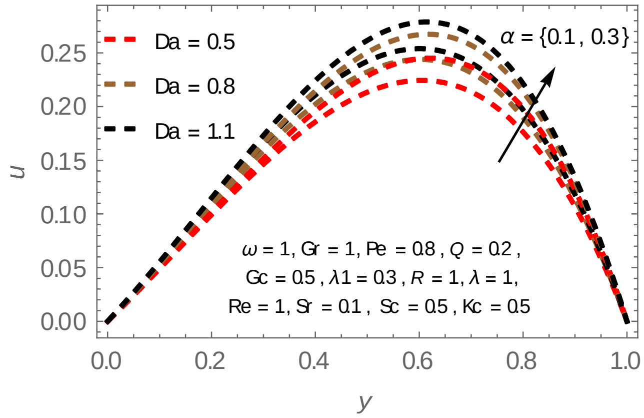

Figure 7. Velocity profile for various values of α and *Da*.

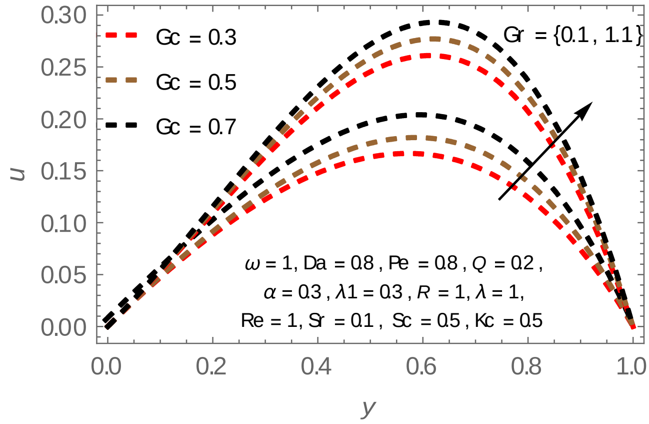

Figure 8. Velocity profile for various values of *Gr* and *Gc*.

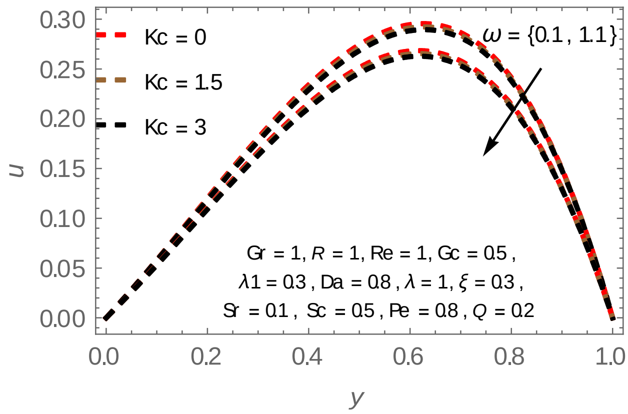

Figure 9. Velocity profile for various values of α and *Kc*.

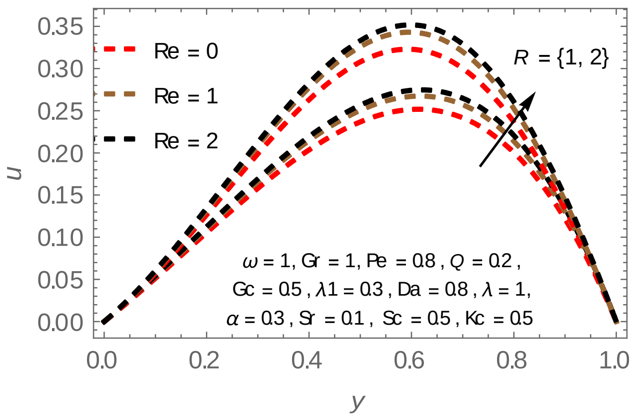

Figure 10. Velocity profile for various values of *R* and *Re*.

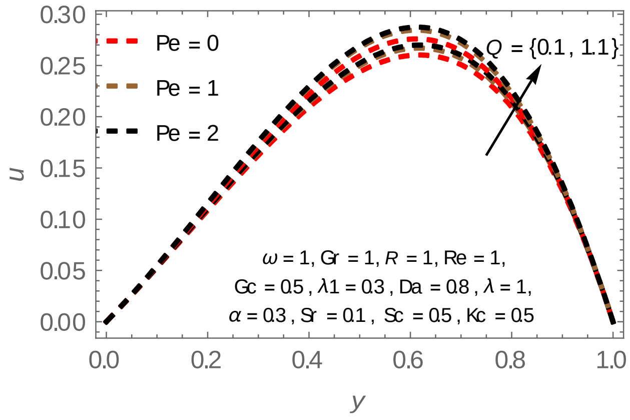

Figure 11. Velocity profile for various values of *Q* and *Pe*.

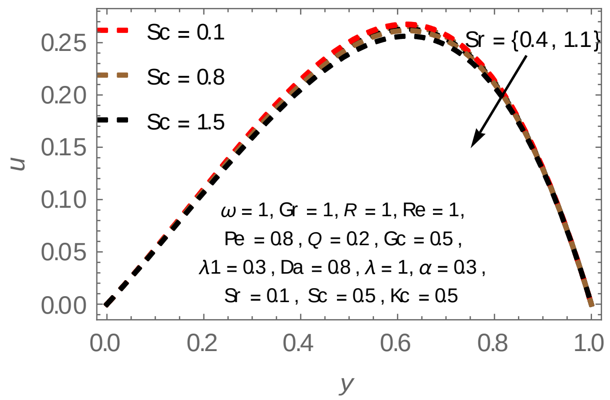

Figure 12. Velocity profile for various values of *Sr* and *Sc*.

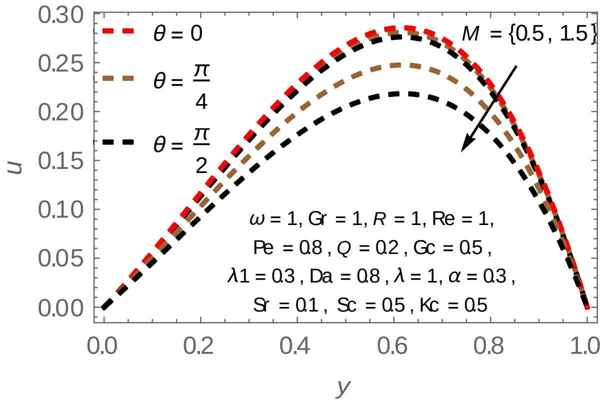

Figure 13. Velocity profile for various values of *M* and θ.

5. CONCLUSIONS

The present study deals with radiation and mass transfer effects on MHD oscillatory flow for Jeffery fluid with variable viscosity through porous channel in the presence of chemical reaction. The perturbation series in the viscosity parameter (α <<1) was used to obtain evident forms for velocity field. We have obtained an analytical solution to the problem.

The results are analyzed for different values of pertinent parameters namely Darcy number *Da*, viscosity parameter α, Thermal Grashof number *Gr*, Solutal Grashof number *Gc*, Reynolds number *Re*, radiation parameter *R*, Peclet number *Pe*, heat generation parameter *Q*, chemical reaction *Kc*, frequency of oscillation ω, Schmidt number *Sc*, Soret number *Sr*, Hartman number *M* and angle between velocity field and magnetic field strength θ .The main findings are:

- The temperature profile rises up with the increasing *Pe*, *R* and *Q*, while go down with the increasing ω.
- All parameters ω, *Sc*, *Sr*, *Pe*, *R* and *Q* adversely affect concentration.
- The velocity of fluid increases with increasing parameters *Da*, *Re*, *R*, *Pe* and *Q*, while the velocity go down with increasing *Kc*, *Sc*, *Sr* and *M*.
- The parameters with the greatest effect on velocity are *Gr*, *Gc*, α, ω and θ, where the velocity of the fluid is directly proportional to *Gr*, *Gc*, α and inversely proportional to ω, θ.

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