CHANGES OF TADPOLE DOMINATION NUMBER UPON CHANGING OF GRAPHs

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***ABSTRACT*:** In this paper, the effect of tadpole domination change of the graph is examined in the network, by deleting a vertex from a graph, where this vertex may represent the network (representing the graph) error or the possibility of dispensing it to reduce costs. Based on that, change tadpole domination number is examined. The increase, decrease, and non-increase or non-decrease in domination number is also determined in a case of deletion, so some basic cases for this domination change have been proved.

***Keywords*:** dominating set, tadpole graph, tadpole domination number.

1. INTRODUCTION

The mathematical model for a networks structure such as social, electronic communication networks, telephone, radio, and control system are modeled by undirected or directed graph, in order to analyze and study these networks and how to control them. For that an example we take a computer network, in this network the computers are represented the vertices of the graph and the links between them represented the edges. When we want to control each of the computers by small number of computers in such a way that every of these computers are controlled by this number. In graph theory we call this dominant group a dominating set by vertices. When the effort of network management is to be reduced, then a smallest dominating set provides the perfect solution. In the case the computers of the network are subject to random failure, but also disrupting computers have to be monitored. We can ask for the probability that the whole network is monitored or not.

A set $D \subseteq V$ of vertices in a graph G is called a dominating set, if every vertex $v \in V$ is either an element of D or is adjacent to an element of D. The domination number of G $\gamma(G)$ is the minimum cardinality taken over all dominating sets in [1-2]. Connected domination plays a vital role in wireless networks where the communication channel is shared among each node and its neighbors. For example they can be used as a virtual backbone routing infrastructure in ad-hoc wireless networks "is a network that is composed of individual devices communicating with each other directly, in the windows operating system, ad-hoc is a communication mode (setting) that allows computers to directly communicate with each other without a router ", where only the connected dominators cells are serving the other cells. Some types of connected domination are given in [3], [4], [5], [6].

Let $G = (V, E)$ be finite, simple, connected and undirected graph where V denotes its vertex set and E its edge set. A number of vertex set $|V(G)|$ is order of graph G. A degree of a vertex v of any graph G is defined as the number of edges incident on v. It is denoted by $deg(v)$. A vertex of degree one also called an end-vertex, leaf or pendent. The set of neighbors of a vertex v in G is $N(v) = \{u \in V: uv \in E\}$, that is called the neighborhood of v. If $X \subseteq V(G)$ and $u \in X$ then the private neighbor of u with respect to X, is defined by $pn[u, X] = \{\{v: N[v] \cap X\} = \{u\}\}$. The obtained graph by joining cycle C_m to a path P_n with a bridge called Tadpole graph denoted by $T_{m,n}$. A subset D of V of a nontrivial connected graph G is said to be a tadpole dominating set, if D is a dominating set and the vertices of D forms a subgraph as a tadpole graph. The minimum cardinality taken over all tadpole dominating sets is called tadpole domination number and is denoted by $\gamma_{TP}(G)$. This definition were first introduced in [7]. Tadpole domination essentially made of two parts, a cycle and path graphs, where one of the cycle vertices is used as a entry to path graph. The cycle graph is a subset of a closed continues mesh graph and dominates all the mesh vertices in its neighbors, while the path is open continues mesh dominating the remaining part of the network. In any graph G of any network, the study of determining the effect of removal a vertex from the graph has several important applications such as how dealing with any damaged device. Some changes in domination number are given in references [8-12]. The dominating set may be materially expensive and have important and wearisome calculations so having to replace it is considered a change in system and storage for this reason in this work, the followed approach is to preserve the dominating set as much possible after deleting vertex, and avoid re-determining the dominating set from scratch.

2. Changing and unchanging for tadpole domination number with respect to vertex deletion in graphs

In tadpole dominating set, $\gamma_{TP}(G)$–set denotes to a minimum set of vertices (any device in some determined networks) that can communicate directly with all other processors in the network system and which have linkage among themselves. Shortcoming in a network can be resolved by determining the effect that removing a vertex (processor failure) from its graph G has on the failure likelihood measure.

It is needful that $\gamma_{TP}(G)$–set does not increase when the graph G is modified by deleting this vertex.

Let $G - v$, denote the graph formed by removing vertex v from G. Let G has a tadpole dominating set D and the vertices of the set D are labeled as follows, the vertices of the path are $V(P_n) = \{x_1, x_2, x_3, \dots, x_n\}$, and the vertices of the cycle are $V(C_m) = \{y_1, y_2, y_3, \dots, y_m\}$, and the edge in tadpole graph joined P_n and C_m is $e = y_1 x_1$.

When $G - v$ has a tadpole dominating set D, then the vertices set of the graph G are partitioned into three subsets, each vertex will belong to one of these sets according to its removal effect on $\gamma_{TP}(G)$ −set. So, v belongs to $(V^0 \cup V^+ \cup V^-)$ such that:

$V^0 = \{v \in V: \gamma_{TP}(G - v) = \gamma_{TP}(G)\}$,

$V^+ = \{v \in V: \gamma_{TP}(G - v) > \gamma_{TP}(G)\}$, and

- $V^- = \{ v \in V : \gamma_{TP}(G-v) < \gamma_{TP}(G)\}$.

Observation 2.1. *If G is a graph has $\gamma_{TP}(G) = 4$ and $G - v$, has tadpole dominating set, then $v \in V^0$.*

Example 2.2. *For the complete graph K_n, $n \geq 5$, $\gamma_{TP}(K_n - v) = \gamma_{TP}(K_n) = 4$.*

Theorem 2.3. *For any graph G with γ_{TP} –set, then $v \in V^-$, if the following holds:*

i) $v \in V - D$, $pn[v_i, D] = \{v\}$ such that, $v_i \in D$, and one of the following statements hold:

a) *IIf there exists an edge between the vertices v_k and v_m, $k = i-1, i-2$ and $m = i+1, i+2$, ($k = i-2, m = i+2$ together are not happened) such that the removed vertex v_z, $k < z < m$, from D satisfies the following condition $pn[v_z, D] = \emptyset$. Additionally, $k = i-2$ and $m = i+2$ whenever, v_i is dominated by a vertex v_w, $w \neq i-1, i+1$.*

b) *For $k < z < m$, $z \geq 3$, such that v_i and the vertices between v_k and v_m they are dominated by D and satisfy the condition in (a).*

c) *is adjacent to vertex x_n, $pn[x_n, D] = \{v\}$.*

d) *If $v_i = y_1$, and there exist the edges $y_m y_2$ and $y_m x_1$ or $y_2 x_1$.*

ii) *$v_i \in D - \{x_n, y_1\}$, $m \geq 6$, $n \geq 5$, the vertices $\{v_i, v_{i-1}, v_{i+1}\}$, $i \geq 2$, with $pn[v_j, D] = \emptyset$, $\forall j = i-1, i, i+1$ such that there exist a path of length less than or equal to 3. In general, if there exists some vertices $v_s \neq v_{i-1}, v_{i+1} \in D$, $s \geq 1$, with $pn[v_s, D] = \emptyset$, and they are adjacent to vertices in D, such that $D - \{v_j\} - \{v_s\}$ is also tadpole dominating set.*

Proof. *According to the number of the vertices between v_k and v_m we have the following:*

i. **a**

1) If the vertex v which is adjacent to v_i is deleted and since is an edge between the two vertices v_{i+1} and v_{i-1}, then v_i does not belong to any γ_{TP} –set in $G - v$, therefore, $v \in V^-$.

2) If the vertex v which is adjacent to v_i is deleted and the two vertices v_{i-1} and v_{i+2} are adjacent, the vertices v_{i+1} as well as v_i are excluded from γ_{TP} –set in $G - v$, because v_i and v_{i+1} are dominated by v_{i-1}, and v_{i+2} respectively. Therefore, $v \in V^-$.

3) The same proof as in (2) if vertices v_{i-2} and v_{i+1} are adjacent.

4) When $k = i-2$ and $m = i+2$, since v_i, v_{i-1} and v_{i+1} are dominated and have no private neighbors from $V - D$ then they excluded from γ_{TP} –set in $G - v$.

b. The proof is clear.

c. If v is adjacent to the vertex x_n and not adjacent to other vertices in D, then γ_{TP} –set in $G - v$ is decreased by exactly one vertex, and $v \in V^-$.

ii. When the vertex $v = v_i$ is deleted from D, then if there exists an edge $e = v_{i-2} v_{i+2}$ then the dominating completed by this edge, therefore, $v \in V^-$.

If the two vertices, v_{i-2} and v_{i+2} are not adjacent and they have no private neighbors from $V - D$ then these two vertices are excluded from D. If there are at most two vertices between v_{i-2} and v_{i+2}, then v_{i-1} and v_{i+1} are replaced by these two vertices . Also, $v \in V^-$. Furthermore, if there exists some vertices belong to D are dominated by D have no private neighbors from $V - D$ then when these vertices are excluded from D with the vertices v_i, v_{i-1} and v_{i+1} D is remain tadpole dominating set.

Observation 2.4. For any $\in V - D$, where the vertex v is adjacent to $u \in D$, if $|pn[u, V-D]| \geq 2$, *then* $v \in V^0$.

Observation 2.5. *For any pendent vertex $v = v_i \in D$ if it is deleted from G, then $v \in V^0$, if G has other pendent vertex.*

Theorem 2.6. *For any graph G with tadpole dominating set, $v \in D$ then $v \in V^0$ if the following conditions hold:*

Let $v = v_i \in D$, there exists a vertex $v_j \in V - D$, with $pn[v_i, D] = \{u : N[u] \cap D\} = \{v_i\}$, such that $u \in N(v_j)$, then $(D - \{v_i\}) \cup \{v_j\}$ is a tadpole dominating set when:

i) *$v_i \notin \{y_1, x_n\}$, such that v_j is adjacent to the two vertices v_{i+1} and v_{i-1}, therefore, $v_i \in V^0$.*

ii) *$v = y_1$ is deleted with the following:*

v_j is adjacent to y_2, and to y_k, $k = 3, \ldots, m-1$ such that y_m is adjacent to x_1 or x_n.

v_j is adjacent to y_2, and to y_m, then $C_m - y_1 \cup \{v_j\}$ is the cycle adjacent to x_1 or x_n. So $(D - y_1 \cup \{v_j\})$ is a tadpole dominating set for $G - y_1$, and $y_1 \in V^0$.

iii) *$v = x_n$ is deleted, and there is a vertex v_j that is adjacent to x_{n-1} and to the private neighbors of x_n from $V - D$, so $x_n \in V^0$.*

iv) *$v_i \neq x_n$ is deleted, (for $m \geq 3, n \geq 2$), and there is a vertex say $v_k \in V - D$ adjacent to the vertex x_n and to v_{i-1} (or v_{i+1}), and there is an edge between y_j, ($y_j \in C_m$), and any vertex from the cycle $C = \{x_n, v_k, v_{i-1}, \ldots, x_n\}$. Now if there is a path from y_j to v_{i+1} (or v_{i-1}) including all the vertices of C_m, we can take this path with C for a tadpole dominating set in $G - v_i$, such that $v_i \in V^0$.*

v) *$v_i \neq x_n$ is deleted,(for $m \geq 3, n \geq 2$), and $v_k \in V - D$ is adjacent to v_{i+1} (or v_{i-1}) and to any vertex $v \neq v_n \in D$, such that $C = \{ v_{i+1}, v_k, v, \ldots, y_1, \ldots, v_{i+1} \}$ with a path that is forms from $V(C_m - C)$ and its end adjacent to the vertex x_n is tadpole dominating set in $G - v_i$, such that $v_i \in V^0$.*

vi) *$y_i \neq y_1$ is deleted,(for $m \geq 3, n \geq 3$), and there exist $x_i \neq x_n \in P_n$ is adjacent to (y_{i+1} and y_{i-1}) such that $C_m - y_i \cup \{x_i\}$ cycle. And there exist the paths $\{x_1, \ldots, x_n, v_k\}$ or $\{x_1, \ldots, v_k \ldots, x_n\}$. In the same manner if y_i is replaced by $y_k \in C_m$. If $x_i = x_n$ then the paths are $\{x_1, \ldots, x_{n-1}, v_k\}$ or $\{x_1, \ldots, v_k \ldots, x_{n-1}\}$.*

Theorem 2.7. *For any graph G with unique γ_{TP} –set, then $v \in V^+$, when $v \in D$ and the following conditions hold:*

In $G - v$, $v_i = v$ if there is a path with at least four vertices from v_{i-1} to v_{i+1} does not contain v_i, and there is no path of order three join the vertices v_{i-1} and v_{i+1}.

a) *$v \in D$ such that v has a private neighborhood set from $V - D$.*

Proof.

a) If $v \neq x_n, y_1 \in D$, where v has no private neighbors from $V-D$. Then if $G[D-v]$ is isomorphic to a tadpole graph then $v \in V^-$. Also, if $G[D-v]$ is isomorphic to a tadpole graph only with one vertex from $V-D$, then $v \in V^0$. Therefore, if $G[D-v]$ is isomorphic to a tadpole graph with at least two vertices from $V-D$ then $v \in V^+$.

b) If v has a private neighbors from $V-D$. Then $v \notin V^-$. If v can be compensated by a vertex from $V-D$ this means γ_{TP} *−set is not unique, so* $v \notin V^0$. Therefore,$v \in V^+$.

Example 2.8. In the following figures Theorem 2.7 is illustrated, as we see in Fig.1, there is a unique γ_{TP} *−set* of cardinality value of 13 and only $D - v_i \cup \{v_r\}$ is isomorphic to tadpole graph, $G - v_i$ has γ_{TP} −set with cardinality value of 14.

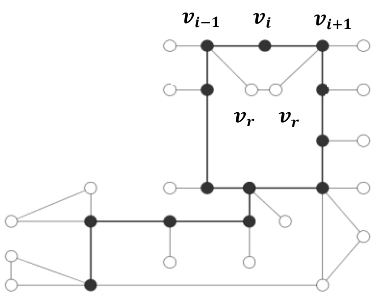

Figure 1. A unique minimum tadpole dominating set in G

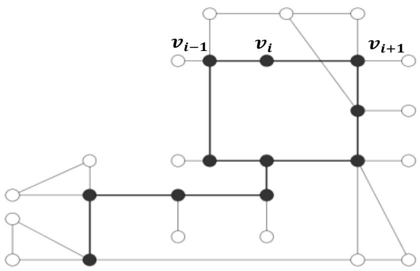

Figure 2. A unique minimum tadpole dominating sets in G

Also in Fig.2, in the graph $G - v_i$, (v_i has no private neighbors from $V-D$) since there is no edge between the two vertices (v_{i-1} and v_{i+1}) then $v_i \notin V^-$, but to complete the path of tadpole graph there are three vertices from $V-D$ between these two vertices, so $v_i \in V^+$, $\gamma_{TP}(G) = 11$, $\gamma_{TP}(G - v_i) = 14$.

In Fig.3, we can take a cycle $\{v_{i-1}, v_{i-2}, v_k, v_k\}$ and replace the vertex x_n by v_r, or the cycle $\{v_{i+1}, v_r, v_r\}$ with replace the vertex v_{i-1} by x_n, where $G - v_i$ has γ_{TP} *−set* of cardinality is 12, but G has γ_{TP} *−set* of cardinality is 10.

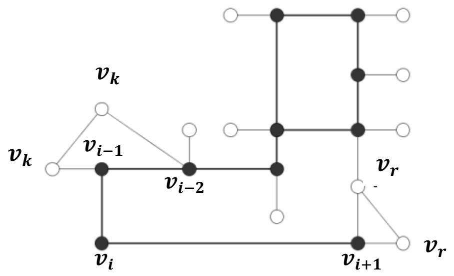

Figure 3. A unique minimum tadpole dominating sets in G

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