SOME RESULTS ON ARTIN CO KERNEL OF THE GROUP $(D_n \times C_{13})$, WHEN N IS AN ODD NUMBER

Samer Thaaban Abaas Al-Shibley
Education College, Kufa University, IRAQ
samir-alshebly@uokufa.edu.iq

ABSTRACT: *The problem of finding the cyclic decomposition of Artin co kernel$AC(D_n \times C_{13})$has been considered in this paper when n is an odd number , we find that if $n = J_1^{\rho_1}, J_2^{\rho_2} \cdots, J_m^{\rho_m}$where $J_1, J_2, \cdots, J_m$are distinct primes and not equal to 2 , then :*

$$AC(D_n \times C_{13}) = \bigoplus_{i=1}^{2(\rho_1+1)(\rho_2+1)\cdots(\rho_m+1)-1} C_2$$

$$= \bigoplus_{i=1}^{2} AC(D_n) \oplus C_2 .$$

And we give the general form of Artin's characters table Ar($D_n \times C_{13}$)when n is an odd number .

Keywords: Artin Co kernel , irreducible characters table, groups D_n and C_{13}

INTRODUCTION:

For a finite group G the finite abelain factor group $\bar{R}(G)/T(G)$ is called Artin cokernel of G and denoted by $AC(G)$ where $\bar{R}(G)$ denotes the abelian group generated by Z-valued characters of G under the operation of pointwise addition and $T(G)$ is a normal subgroup of $\bar{R}(G)$ which is generated by Artin's characters. induced characters Permutation from the principle charcters of cyclic Subgroups . A well-known theorem which is due to Artin asserted that $T(G)$ has a finite index is, i.e [:$T(G)$] is finite .Artin exponent of G represented $AC(G)$and denoted by $A(G)$. In 1968 , Lam . T .Y [5] "gave the definition of the group $AC(G)$ and the studied $AC(Cn)$".In 1976 , David .G [12] "studied $A(G)$ of arbitrary characters of cyclic subgroups". In 1996 , Knwabuez .K [11] " studied $A(G)$ of p-groups" . In 2000,H.R.Yassein [4] "found $AC(G)$ for the group$\bigoplus_{i=1}^{n} C_p$". In 2002, k.Sekieguchi [12]" studied the irreducible Artin characters of p-group" and in the Same year H.H.Abbass [10]"found $\equiv*$ (Dn)". In 2006, Abid . A .S [6] " found $Ar(Cn)$ when Cn is the cyclic group of order n" . In 2007 , Mirza .R .N [9] "found in her thesis Artin cokernel of the dihedral group". In this paper we find the general form of $Ar(D_n \times C_{13})$ and we study $AC(D_n \times C_{13})$ of the non abelian group$D_n \times C_{17}$ when n is an odd number .

Preliminaries:

We have to give basic concepts, notations , theorems about matrix representation, characters and Artin characters, in this section.

Definition (1.1): [2]

A matrix representation of a group G is a homomorphism of G into GL(n, F), n is called the degree of matrix representation T. In particular,T is called a unit representation (principal) if T(g) = 1, for all g $\in$G.

Definition (1.2): [2]

Let A is amatrix of the size $n \times n$,then the trace of A be the sum of the main diagonal elements ,denoted by tr(A) .

Definition (1.3): [1]

Let the field F contain a finite group G where G has a T matrix representation of degree n. The character χ of degree n of T is the mapping χ : $G \rightarrow$ F defined by χ(g) = tr(T(g)) for all g $\in G$. In particular, the character of the principal representation (χ(g) = 1, for all g $\in G$) is called the principal character.

Definition (1.4): [3]

The group$G$has twoarbitrary elements g and hare said to be conjugate if h = xgx^{-1}, for some x $\in G$.

Definition (1.5): [3]

Let G be a group has a subgroup H and ϕ be a character of H, the induced character on G is given by:

$$\phi'(g) = \frac{1}{|H|}\sum_{x \in G} \phi^{\circ}(xgx^{-1}) \qquad , \forall g \in G$$

Where $\emptyset^{\circ}$ is known by :

$$\phi^{\circ}(h) = \begin{cases} \phi(h) & if \quad h \in H \\ 0 & if \quad h \notin H \end{cases}$$

Theorem (1.6): [3]

Let$G$be group has a subgroup H be cyclic subgroup of G and h_1 , h_2 , ... , h_m are chosen representatives for the m-conjugate classes of H contained in CL_g, g $\in G$, then:

$$1- \quad \phi'(g) = \frac{|C_G(g)|}{|C_H(g)|}\sum_{i=1}^{m} \phi(h_i) \qquad if \quad h_i \in H \cap CL(g)$$

$$2- \quad \phi'(g) = 0 \qquad if \quad H \cap CL(g) = \phi$$

$\phi^{\uparrow G}(g) = \frac{|C_G(g)|}{|C_H(g)|}\sum_{i=1}^{m} \phi(h_i)$ if $h_i \in H \cap CL_g$

$\phi^{\uparrow G}(g) = 0$

Definition (1.7): [5]

Let G be a finite group, any inducedcharacter from the principal character of cyclic subgroup of G is called Artin character of G.

Proposition (1.8): [6]

the number of $\Gamma -$ classes on Gequal to the number of all distinct Artin characters on a group G.

Definition (1.9): [3]

A irreducible characters ϑ of G is a character whose values are in the set of integer numbers Z ,which is $\vartheta(g) \in$ Z,for all $g \in G$.

Proposition (1.10):[12]

The number of Γ-classes on G equals to the number of all distinct irreducible valued characters of a finite group G.

Theorem [Artin] (1.11): [9]

Let G be afinite group ,then every irreducible character of G can be written as a linearcombination of Artin's characters with coefficient rational numbers.

2. The Factor Group AC(G):

In this section, we use some concepts in linear Algebra are used in this sectionfor studying the factor group $AC(G)$.In addition to thatwe will give the general form of Ar $(D_n \times C_{13})$ when n is an odd number . We shall study Ac(G) dihedral group Dn and $\equiv^* (Dn)$ when n is an odd number.

Definition (2.1): [5]

Let $\bar{R}(G)$ be the group of Z –valued generalized characters of G under the operation pointwise addition and T(G) is a normal subgroup of $\bar{R}(G)$ generated by Artin's characters. The abelian factor group $\bar{R}(G)/T(G)$ is called Artin's Cokernel of G, denoted by $AC(G)$.

Definition (2.1): [8]

Let a principle domain R has aT matrix with entries. A K – minor of T is the determinate of K ×K.

Definition (2.2): [8]

The greatest common divisor (g.c.d) of all K –minor isa K –th determinant divisor of T, denoted by $D_K(T)$.

Theorem (2.3): [8]

Let T be an n ×n matrix with entries in a principle domain R, then there exist matrices J and F such that, J and F are invertible ,J . T . F = D, D is a diagonal matrix and If we denote D_{jj} by d_j then there exists a natural number m; 0 ≤m ≤m implies $d_j = 0$ and j < m implies $d_j \neq 0$ and 1 < j < m implies d_j / d_{j+1}.

Definition (2.4): [8]

Let a principle domain R has aT matrix with entries, and equivalent to matrixD = $\{d_1, d_2, \ldots, d_m, 0, 0, \ldots, 0\}$. Such that d_j/d_{j+1} for 1 < j < m, D is called the invariant factor matrix of M and $d_1, d_2, \ldots, d_m$ the invariant factors ofT.

Remark (2.5):

An invertible matrix $T^{-1}(G)$ with entries in Q such that$\equiv *(G) = T^{-1}(G).Ar(G)$. $AC(G)$,was existsd by using Artin theorem (1.18) where $T(G) = Ar(G).(\equiv^* (G))^{-1}$.By theorem (2.4) there exists two matrices $J(G)$ and $F(G)$ such that$J(G).T(G).F(G) =$ diag = $\{d_1, d_2, \ldots, d_l\} = D(G)$where$dj = \pm Dj(T(G))/Dj-1(T(G))$ and $l$is the number of Γ-classes.

Theorem (2.6): [6]

$$AC(G) = \bigoplus_{j=1}^{i} C_{d_j} \; where \; d_j = \pm D_j(T(G))/D_{j-1}(T(G)),$$

and i is the number of all distinct $\Gamma -$ classesand C_{d_j} is cyclic subgroup of order d_j.

Theorem (2.7): [8]

The irreducible character table of the cyclic group C_{q^δ} of the rank $\delta + 1$ where q is prime number which is denoted by($\equiv^* (C_{q^\delta})$) given by:

Table(2.1)

Γ-classes	[1]	$[r^{q^{\delta-1}}]$	$[r^{q^{\delta-2}}]$	$[r^{q^{\delta-3}}]$	⋯	$[r^q]$	[r]
ϑ_1	$q^{\delta-1}(q-1)$	$-q^{\delta-1}$	0	0	⋯	0	0
ϑ_2	$q^{\delta-2}(q-1)$	$q^{\delta-2}(q-1)$	$-q^{\delta-2}$	0	⋯	0	0
ϑ_3	$q^{\delta-3}(q-1)$	$q^{\delta-3}(q-1)$	$q^{\delta-3}(q-1)$	$-q^{\delta-3}$	⋯	0	0
⋮	⋮	⋮	⋮	⋮	⋯	⋮	⋮
⋮	$q(q-1)$	$q(q-1)$	$q(q-1)$	$q(q-1)$	⋯	$-q$	0
ϑ_S	$q-1$	$q-1$	$q-1$	$q-1$	⋯	$q-1$	-1
ϑ_{S+1}	1	1	1	1	⋯	1	1

<u>Proposition (2.8): [10]</u>

The Irreducible characters table of D_n when n is an odd number is given by:

$(\equiv^* D_n) =$

Table (2.2)

	Γ -Classes of C_n	[S]
ϑ_1		0
ϑ_1	$\equiv^* (Cn)$	0
⋮		⋮
ϑ_l	1 1 1 ⋯ 1	1
ϑ_{l+1}	1 1 1 ⋯ 1	-1

where$l$ is the number of Γ-classes of C_n.

<u>Theorem (2.9): [6]</u>

The general form of Artin characters table of C_q^δ when q is a prime number and δ is positive integer is given by the lower Triangular matrix:

Table (2.3)

Γ-classes	[1]	$[r^{q^{\delta-1}}]$	$[r^{q^{\delta-2}}]$	$[r^{q^{\delta-3}}]$	⋯	[r]
ϑ_1	q^{δ}	0	0	0	⋯	0
ϑ_2	$q^{\delta-1}$	$q^{\delta-1}$	0	0	⋯	0
ϑ_3	$q^{\delta-2}$	$q^{\delta-2}$	$q^{\delta-2}$	0	⋯	0
⋮	⋮	⋮	⋮	⋮	⋮	⋮
⋮	⋮	⋮	⋮	⋮	⋮	⋮
ϑ_S	q	q	q	q	⋯	0
ϑ_{S+1}	1	1	1	1	⋯	1

Corollary (2.10):[6]
Suppose $n = J_1^{\rho_1}, J_2^{\rho_2} \cdots, J_m^{\rho_m}$, be any positive integer of n, *where* $J_1, J_2, \cdots, J_m$ *are distinct primes, then*:
$Ar(C_n) = Ar(C_{J_1}^{\rho 1}) \otimes Ar(C_{J_2}^{\rho 2}) \otimes \dots \dots .. \otimes Ar(C_{J_m}^{\rho m})$
.Where ⊗ is the tensor product.

Theorem(2.11):[9]
If n is an odd number where $n = J_1^{\rho_1}, J_2^{\rho_2} \cdots, J_m^{\rho_m}$, where $J_1, J_2, \cdots, J_m$ are distinct primes , then :

$$AC(D_n \times C_{13}) = \bigoplus_{i=1}^{(\rho_1+1)(\rho_2+1)\cdots(\rho_m+1)-1} C_2$$

Theorem (2.12):[4]

AC(G) = $\bigoplus_{i=1}^{m} Z$ such that $d_i = \overset{+}{-} D_i(G) \,|\, D_{i-1}(G)$ where m is the number of all distinct Γ-classes.

Proposition (2.13):[13]
If δ is a positive integer and q is prime number, then:

$$U(C_{q^\delta}) = \begin{bmatrix} 1 & 1 & 1 & \cdots & 1 & 1 \\ 0 & 1 & 1 & \cdots & 1 & 1 \\ 0 & 0 & 1 & \cdots & 1 & 1 \\ \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & 0 & \cdots & 1 & 1 \\ 0 & 0 & 0 & \cdots & 0 & 1 \end{bmatrix}$$

Proposition (2.14):[13]
The general form of matrices $V(C_{q^\delta})$ and $F(C_{q^\delta})$ are:

$$V(C_{q^\delta}) = \begin{bmatrix} 1 & -1 & 0 & 0 & \cdots & 0 & 0 \\ 0 & 1 & -1 & 0 & \cdots & 0 & 0 \\ 0 & 0 & 1 & -1 & \cdots & 0 & 0 \\ \vdots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & 0 & 0 & \cdots & 1 & -1 \\ 0 & 0 & 0 & 0 & \cdots & 0 & 1 \end{bmatrix}$$

that is $2(\delta+1) \times (\delta+1)$ square matrix $.F(C_{q^\delta}) = I_{2(\delta+1)}$ *where* $I_{2(\delta+1)}$ is an identity matrix and $D(C_{q^\delta} \times C_{11}) = dig\{1,1,1,\dots,1\}$.

Remark (2.15) : From now *on if* $n = J_1^{\rho_1}, J_2^{\rho_2} \cdots, J_m^{\rho_m}$, *where* $J_1, J_2, \cdots, J_m$ *are* distinct primes then:

$$V(C_n) = V(C_{J_1}^{\rho 1}) \otimes V(C_{J_2}^{\rho 2}) \otimes \dots \dots .. \otimes V(C_{J_m}^{\rho m})$$

So, then we can write T(C_n)

$$\mathcal{M}(C_n) = \begin{bmatrix} & & & & & 1 \\ \beta & (C_n) & & & & 1 \\ & & & & \vdots & \vdots \\ & & & \cdots & & 1 \\ 0 & 0 & \cdots & 0 & 0 & 1 \end{bmatrix}$$

$\beta_1(C_n)$ is the matrix obtain by omitting the last two rows {0 , 0 , … , 1 , 1} and {0 , 0 , … , 0 , 0 , 1} and the last two columns {1 , 1 , … , 1 , 0} .
$U(C_{J_1}^{\rho 1}) \otimes U(C_{J_2}^{\rho 2}) \otimes \dots \dots .. \otimes U(C_{J_m}^{\rho m})$.

Definition (2.16):
The direct product of the group $D_n \times C_{13}$,such that D_n (The dihedral group is a non- abelian group of order 2n.D_n = $\{ S^j r^k : 0 \le k \le n-1 , 0 \le j \le 1\}$).The cyclic group C_{13} of order 13 consisting of elements $\{1, x', x^{2'}, x^{3'}, \dots, x^{12'}\}$) with $(x')^{13}$=1 . The group $D_n \times C_{13}$ has order 26n .

3. The main results:
We have to give the general form of Artin characters table of the group $D_n \times C_{13}$ and the cyclic decomposition of the factor group AC($D_n \times C_{13}$) when n is an odd numberin this section .

Theorem (3.1):
The Artin characters table of the group $D_n \times C_{13}$ when n an odd number is is given as follows:
Ar($D_n \times C_{13}$)=

Table (3.1)

Γ-Classes	$[1,1']$	$[1,x']$	$\Gamma - Classes\ of\ C_n \times C_{13}$					$[S,1]$	$[S,x']$
$\lvert CL_\alpha \rvert$	1	1	2	2	…..	….	2	n	n
$\lvert C_{D_n \times c_{13}}(CL_\alpha) \rvert$	$26n$	$26n$	$13n$	$13n$	…..	…..	$13n$	26	26
$\Phi_{(1,1)}$	$2Ar(C_n) \otimes Ar(C_{13})$							0	0
$\Phi_{(1,2)}$								⋮	⋮
:								⋮	⋮
$\Phi_{(l,1)}$								⋮	⋮
$\Phi_{(l,2)}$								0	0
$\Phi_{(l+1,1)}$	$13n$	0	0	…..	…..	…..	0	$13n$	0
$\Phi_{(l+1,2)}$	$13n$	0	0	…..	…..	…..	0	0	$13n$

where l is the number of $\Gamma-classes$ of C_n and $C_n =< x' >= \{1', x'\}$.

Proof: By theorem (2.8)

$Ar\ (C_{13}) =$

Table (3.2)

Γ- classes	$[1']$	$[x']$
$\lvert CL_\alpha \rvert$	1	1
$\lvert c_{13(CL_\alpha)} \rvert$	13	13
φ_1'	13	0
φ_2'	1	1

Each cyclic subgroup of the group $D_n \times C_{13}$ is either a cyclic subgroup of $C_n \times C_{13}$ or $< (S, x') >$ or $< (S, 1') >$ .If H is a cyclic subgroup of $C_n \times C_{13}$, then :

$H = H_i \times < 1' > or\ H_i \times < 1' >= H_i \times C_{13}$,$\forall 1 \le i \le l\ where\ l$ is the number of Γ-Classes of C_n .

If $H = H_i \times < 1' > and\ f \in C_n \times C_{13}$

if $f \notin H$ Then by theorem(1.6)

$\Phi(1,i)(f) = 0\ for\ all\ 0 \le i \le l\ [since\ H \cap CL(f) = \phi]$

If $f \in H$ then either $f = (1, 1')$ or $\exists s, 0 < s < n$ such that $f = (x^s, 1')$

If $f = (1, 1')$, then :

$\Phi(I,1)(f) = \frac{|c_{D_n \times C_{13}}(f)|}{|c_{H(f)}|} . \varphi'(f)\ [since\ H \cap CL(f) = \{(1,1')\}]$,

where φ is the principle character

$= \frac{26n}{|H_i|.\ |<1'>|} .1 = \frac{26\,n}{|H_i|} = 2. \frac{n}{|H_i|} .1.13 = 2 \frac{|C_{C_n}(1)|}{|C_{H_i}(1)|} \cdot \varphi(1) \cdot \varphi'(1')$

$= 2 \cdot \varphi_i(1) \cdot \varphi'(1')$

If $f = (x^s, 1')$ then

$\Phi(i,1)(f) = \frac{|c_{D_n \times C_{13}}(f)|}{|c_{H(f)}|} .\sum_1^2 \varphi'\ (f)$

[since $H \cap CL(f) = \{(x^s, 1'), (x^s, 1')\}$]

$= \frac{13n}{|H_{i \times <1'>}|} .(1+1)$

$= \frac{13n}{|H_{i \times <1'>}|}.2 = \frac{13n}{|H_i|}.2$

$= 2.\frac{n}{|H_{i(x^s)}|}.1.13 == 2\frac{|C_{C_n}(x^s)|}{|C_{H_i}(x^s)|}.\varphi\ (x^s).\varphi'(1')$

$= 2.\varphi_i(x^s).\varphi_1'(1')$

If $H = H_i \times < x' >= H_i \times C_{13}$

let $f \in D_n \times C_{13}$

if $f \notin H$ then

$\Phi(i,2)(f) = 0\ for\ all\ 1 \le i \le l\ [since\ H \cap CL(f) = \phi]$

If $f \in H$ then either

$g = (1,1')\ or\ f = (1,x')\ or \exists s, 0 < s < n\ such\ that\ f = (x^s, x')$

if $f = (1,1')$

$\Phi_{(i,2)} = \frac{|c_{D_n \times c_{13(f)}}|}{|c_{H(f)}|}.\ \varphi\ (f)$ [sir^{s}nce H ⋂ CL(f) = {$(1,1')$ }]

$= \frac{26n}{|H_{i \times C_{13}}|} = \frac{26n}{2|H_i|} = \frac{13n}{|H_i|} = 13\frac{|C_{C_n}(1)|}{|C_{H_i}(1)|} \cdot \varphi(1)$

$=13.\varphi_i(1).\varphi_2'(1)$

if $f = (1, = x')$

$\Phi(i,2)(f) = \frac{|c_{D_n \times c_{13(f)}}|}{|c_{H(f)}|}.\ \varphi\ (f)[since\ H \cap CL(f) = \{(1,x')\}$

$= \frac{26n}{|H_{i \times C_{13}}|}$

$= \frac{26n}{2|H_i|} = \frac{13n}{|H_i|} = 13\frac{|C_{C_n}(1)|}{|C_{H_i}(1)|} = 13.\varphi_i(1).\varphi'_2(r')$

If $f = (x^s, x')$ then

$\Phi(i,2)(f) = \frac{|c_{D_n \times C_{13}}(f)|}{|c_{H(f)}|} .\sum_1^2 \varphi'\ (f)[since\ H \cap CL(f) = \{(x^s,x'),(x^s,x')\}]$

$= \frac{13n}{|H_{i \times C_{13}}|}(1+1) = \frac{26n}{2|H_i|} = \frac{13n}{|H_i|} = 2\frac{|C_{C_n}(r^s)|}{|C_{H_i}(r^s)|} \cdot\ \varphi(r^s) \cdot \varphi'_2(r')$

$= 13 \cdot \varphi_i(r^s) \cdot \varphi'_2(r')$.

if $H =< (s,1') >= \{(1,1'),(s,1')\}$ then

$\Phi_{(l+1,1)}((1,1')) = \frac{|c_{D_n \times c_{13(1,1')}}|}{|c_{H(S,1')}|}.\ \varphi\ (f) = \frac{26n}{2} = 13n$

$\Phi_{(l+1,1)}\ ((S,1')) = \frac{|c_{D_n \times c_{13(1,1')}}|}{|c_{H(S,1')}|}.\ \varphi\ (f)$[since $H \cap CL((s,1'))$

$= \{(s,1')\}]$

$= \frac{26}{2} = 13$

Otherwise

$\Phi_{(l+1,1)}(f) = 0$ for all $f \in D_n \times C_{13}, [since\ f \notin H]$

if $H =< (s,x') >= \{(1,1'),(s,x')\}$ then

$\Phi_{(l+1,2)}\ ((1,1')) = \frac{|c_{D_n \times c_{13(1,1')}}|}{|c_{H(1,1')}|}.\ \varphi\ (1,1')$ [since $H \cap CL((1,1'))$

$= \{(1,1')\}] = \frac{26n}{2} .1 = 13n$

$\Phi_{(l+1,2)}((s,x')) = \frac{|c_{D_n \times c_{13(s,x')}}|}{|c_{H(s,x')}|}.\ \varphi\ (s,x')\ = \frac{26}{2}.1 = 13$.

Otherwise $\Phi_{(l+1,2)}(f) = 0$ for all $f \in D_n \times C_{13}$ since $H \cap CL(f)$

Proposition (3.2):

If $n = J_1^{\rho_1}, J_2^{\rho_2} \cdots, J_m^{\rho_m}\ such\ that\ J_1, J_2, \cdots, J_m$ are distinct primes and $J_m \neq 2$ for all $1 \le i \le m$ and ρ_i anypositiveintegers, then :

$$\mathcal{M}(D_{n \times C_{13}}) = \begin{pmatrix} 2\beta(C_n) \times \mathcal{M}(C_{13}) & \begin{matrix} 1\,1\,1\,1 \\ 1\,0\,1\,0 \\ 1\,1\,1\,1 \\ \vdots\,\vdots\,\vdots\,\vdots \end{matrix} \\ \begin{matrix} 0 & 0 & \cdots \\ 0 & 0 & \cdots \\ 1 & 1 & \cdots \\ 1 & 1 & \cdots \end{matrix} & \begin{matrix} 0\,1\,1\,1\,1 \\ 0\,1\,0\,1\,0 \\ 1\,1\,1\,0\,0 \\ 1\,1\,0\,0\,1 \end{matrix} \end{pmatrix}$$

that is $2[(\rho_1+1).(\rho_2+1)\cdots(\rho_m+1)+1]\times 2[(\rho_1+1).(\rho_2+1)\cdots(\rho_m+1)+1]$ square matrix.

Proof:

By theorem(3.1) we obtain the Artin characters table Ar($D_{n\times c_{13}}$) and from theorem(1.12) we find the rational valued characters table:

$\equiv^*(D_{n\times c_{13}})$.

Therefore by the definition of $\mathcal{M}(G)$ we can find the matrix $\mathcal{M}(D_{n\times c_{13}})$:

$$\mathcal{M}(D_n\times C_{13})=\mathrm{Ar}(D_n\times C_{13}).(\overset{*}{\equiv}(D_n\times C_{13}))^{-1}$$

$$=\begin{pmatrix} 2&2&2&2&\cdots&&1&1&1&1\\ 0&2&0&2&\cdots&&1&0&1&0\\ 0&0&2&2&\cdots&&1&1&1&1\\ 0&0&0&2&\cdots&&1&0&1&0\\ \vdots&\vdots&\vdots&\vdots&\ddots&&\vdots&\vdots&\vdots&\vdots\\ &&&&&\ddots&1&0&1&0\\ 0&0&0&0&\cdots&0&1&1&1&1\\ 0&0&0&0&\cdots&0&1&0&1&0\\ 1&1&1&1&\cdots&1&1&1&0&0\\ 1&1&1&1&\cdots&1&1&0&0&1 \end{pmatrix}$$

$$=\begin{pmatrix} 2\beta(C_n)\otimes\mathcal{M}(C_{13}) & \begin{matrix}1\,1\,1\,1\\1\,0\,1\,0\\1\,1\,1\,1\\\vdots\,\vdots\,\vdots\,\vdots\\1\,0\,1\,0\end{matrix}\\ \begin{matrix}0&0&\cdots\cdots\\0&0&\cdots\cdots\\1&1&\cdots\cdots\\1&1&\cdots\cdots\end{matrix} & \begin{matrix}0\,1\,1\,1\,1\\0\,1\,0\,1\,0\\1\,1\,1\,0\,0\\1\,1\,0\,0\,1\end{matrix} \end{pmatrix}$$

that is $2[(\rho_1+1).(\rho_2+1)\cdots(\rho_m+1)+1]\times 2[(\rho_1+1).(\rho_2+1)\cdots(\rho_m+1)+1]$square matrix.

Proposition (3.3):

If $n=J_1^{\rho_1},J_2^{\rho_2}\cdots,J_m^{\rho_m} for\ that\ J_1,J_2,\cdots,J_m$are distinct primes and$J_m\neq 2$ for all $1\leq i\leq m$ and$\rho_i$any positive integers, then :

$$\mho(D_{n\times c_{13}})=\begin{pmatrix} V(C_n)\otimes V(C_{13}) & \begin{matrix}0&0\\0&0\\\vdots&\vdots\\0&0\\-1&-1\\0&0\end{matrix}\\ \begin{matrix}0\,0\,0&\cdots\\0\,0\,0&\cdots\end{matrix} & \begin{matrix}0\;0&1&0\\0\;0&0&1\end{matrix} \end{pmatrix}$$

and

$$\mathcal{L}(D_n\times C_{13})=\begin{pmatrix} I_K &&&&&0&0&0\\ &&&&&0&0&0\\ &&&&&0&0&0\\ &&&\vdots&\vdots&\vdots&\vdots\\ &&&0&0&0&0\\ -1&\cdots&-1&1&0&0&0\\ 0&\cdots&0&-1&0&1&0\\ 1&\cdots&1&-1&1&0&0\\ 0&\cdots&0&1&0&0&1 \end{pmatrix}$$

Where $k=2[(\rho_1+1).(\rho_2+1)\cdots(\rho_m+1)-1]\times 2[(\rho_1+1).(\rho_2+1)\cdots(\rho_m+1)-1]$

They are$2[(\rho_1+1).(\rho_2+1)\cdots(\rho_m+1)+1]\times 2[(\rho_1+1).(\rho_2+1)\cdots(\rho_m+1)+1]$

square matrix .

Proof :

By theorem(2.5) and using the form $M(D_n\times C_{13})$ from proposition(3.2) and the above forms of $P(D_n\times C_{13})$ and $W(D_n\times C_{13})$ then we have:

$\mho(\boldsymbol{D_n}\times\boldsymbol{C_{13}}).\mathcal{M}(\boldsymbol{D_n}\times\boldsymbol{C_{13}}).\mathcal{L}(\boldsymbol{D_n}\times\boldsymbol{C_{13}})$

$$=\begin{pmatrix} 2&0&0&0&\cdots&0&0&0&0\\ 0&2&0&0&\cdots&0&0&0&0\\ 0&0&2&0&\cdots&0&0&0&0\\ 0&0&0&2&\cdots&0&0&0&0\\ \vdots&\vdots&\vdots&\vdots&\ddots&\vdots&\vdots&\vdots&\vdots\\ 0&0&0&0&\cdots&-2&0&0&0\\ 0&0&0&0&\cdots&0&1&0&0\\ 0&0&0&0&\cdots&0&0&1&0\\ 0&0&0&0&\cdots&0&0&0&1 \end{pmatrix}$$

$D(D_n\times C_{13})$=diag{2,2,2,…..,-2,1,1,1} ,which is $2[(\rho_1+1).(\rho_2+1)\cdots(\rho_m+1)+1]\times 2[(\rho_1+1).(\rho_2+1)\cdots(\rho_m+1)+1]$square matrix .

Which is$2[(\rho_1+1).(\rho_2+1)\cdots(\rho_m+1)+1]\times 2[(\rho_1+1).(\rho_2+1)\cdots(\rho_m+1)+1]$squarematrix .

Theorem (3.4):

If $n=J_1^{\rho_1},J_2^{\rho_2}\cdots,J_m^{\rho_m} when\ J_1,J_2,\cdots,J_m$ are distinct prime numbers such that $J_m\neq 2$andα_i any positive integers for all$i, 1\leq i\leq m$,then the cyclic decomposition AC($D_{n\times C_{13}}$) is :

$$AC(D_{n\times C_{13}})=\bigoplus_{i=1}^{2((\rho_1+1).(\rho_2+1)\cdots(\rho_m+1))-1} C_2$$

$$AC(D_{n\times C_{13}})=\bigoplus_{i=1}^{2} AC(D_n)\oplus C_2$$

Proof :

From proposition (3.3) we have

$\mho(D_{n\times C_{13}})\ .\mathcal{M}(D_{n\times C_{13}})\ .\mathcal{L}(D_{n\times C_{13}})$ =diag{2,2,2,…,-2,1,1,1}= {d_1,d_2,………..,

$d2((\rho_1+1).(\rho_2+1)\cdots(\rho_m+1))-1$,$d2((\rho_1+1).(\rho_2+1)\cdots(\rho_m+1))$,

$d2((\rho_1+1).(\rho_2+1)\cdots(\rho_m+1))+1$,$d2((\rho_1+1).(\rho_2+1)\cdots(\rho_m+1))+2$}.

By theorem (2.11) we get

$$AC(D_{n\times C_{13}})=\bigoplus_{i=1}^{2((\rho_1+1).(\rho_2+1)\cdots(\rho_m+1))-1} C_{d_i}$$

$$AC(D_{n\times C_{13}})=\bigoplus_{i=1}^{2((\rho_1+1).(\rho_2+1)\cdots(\rho_m+1))-1} C_2$$

From theorem(2.7) we have :

$$AC(D_{n\times C_{13}})=\bigoplus_{i=1}^{2} AC(D_n)\oplus C_2$$

Example (3.6):

To find the cyclic decomposition of the groups AC($D_{24389\times C_{13}}$) ,AC($D_{12901781\times C_{13}}$) and AC($D_{219330277\times C_{13}}$) .

We can use above theorem :

$$AC(D_{24389\times C_{13}})=AC(29^3\times c_{13})=\bigoplus_{i=1}^{2(3+1)-1} C_2$$

$$=\bigoplus_{i=1}^{7} C_2=\bigoplus_{i=1}^{7} AC(D_{29^3})\oplus C_2$$

$AC(D_{219330277}\times c_{13})\qquad =AC(D_{29^3.23^2}\times c_{13})$

$$= \bigoplus_{i=1}^{2(3+1).(2+1)-1} C_2 = \bigoplus_{i=1}^{23} C_2$$

$$= AC \bigoplus_{i=1}^{2} (D_{29^3.23^2}.) \oplus C_2.$$

3-AC($D_{219330277 \times C_{13}}$) =AC($D_{29^3.23^2.17} \times c_{13}$)

$$= \bigoplus_{i=1}^{2(3+1).(2+1).(1+1)-1} C_2$$

$$= \bigoplus_{i=1}^{47} C_2 = \bigoplus_{i=1}^{2} AC(D_{29^3.23^2.13}) \oplus C_2.$$

CONCLUSION :

We have to found a new method with a new results for the cyclic decomposition of the factor group $AC(D_n \times C_{13})$,in this paper. for that we can extend this paper in future work .

REFERENCES

[1] A . M , Basheer " Representation Theory of Finite Groups " ,AIMS, South Africa , 2006 .

[2] Culirits . C and.Reiner . I ," Methods of Representation Theory with Appcation to Finite Groups and Order", John wily & sons, New york, 1981.

[3] J Moori, " Finite Groups and Representation Theory " ,University of Kawzulu – Natal , 2006 .

[4] H. R Yassien ," On Artin Co kernel of Finite Group", M.Sc. Thesis, Babylon University, 2000.

[5] T .Y Lam," Artin Exponent of Finite Groups ", Columbia University, New York, 1968.

[6] A . S, Abid . "Artin's Characters Table of Dihedral Group for Odd Number " , MSc.Thesis, university of kufa,2006.

[7] J . P Serre, "Linear Representation of Finite Groups", Springer- Verlage,1977.

[8] M . S Kirdar .M . S, " The Factor Group of The Z-Valued Class Function Modulo the Group of The Generalized Characters ",Ph. D. Thesis, University of Birmingham, 1982.

[9] Mirza . R . N , " On Artin Co kernel of Dihedral Group Dn When n is An Odd Number ",M.Sc. thesis , University of Kufa ,2007.

[10] H .H Abass," On Rational of Finite Group$D_n$when n is anodd "Journal Babylon University,vol7,No-3,2002.

[11] Knwabusz . K, " Some Definitions of Artin's Exponent of Finite Group ", USA.National foundation Math.GR.1996.

[12] David .G," Artin Exponent of Arbitrary Characters of Cyclic Subgroups ", Journal of Algebra,61,pp.58-76,1976.

[13] R.N.Mirza , On Artin Co kernel of Dihedral Group Dn when n is an odd number " M.Sc. thesis, University of Kufa ,2006.