ON SHRINKAGE ESTIMATORS FOR THE RELIABILITY FUNCTION OF WEIGHTED RAYLEIGH DISTRIBUTION

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ABSTRACT:: *This paper concerned with estimate the reliability function of weighted Rayleigh distribution using different estimation methods. The proposed estimators were compared using Monte Carlo simulation based on Mean Squared Error (MSE) criteria.*

Keywords: Weighted Rayleigh Distribution, Maximum likelihood Method, Moment Method, Shrinkage Technique, Mean Squared Error.

1. INTRODUCTION:

Weighted statistical distribution delivers a style to treat with model requirement and data explanation problems. Weighted distribution appear recurrently in investigation associated with reliability, analysis of data, Biological Medicine, ecology, agriculture engineering and forking procedure which may be perceived in Gupta and Kirmani [9], patil and Rao [10], Cupta and keating [8]. Fisher [7] calculated in what way approaches of ascertainment which can be effect the form of scattering of recorded interpretations and then Rao [11] presented the concept and expressed it in common expressions in joining with demonstrating statistical data.

In Rao's research, he recognized numerous conditions that can be showed by weighted statistical distributions. These conditions mention to occurrences where the recorded explanations cannot be measured as a random sample from the original distributions.

Das and Roy [6] studied the biased length and properties for Generalized Weighted Rayleigh distribution .Shi et al [13] studied weighted Generalized Rayleigh properties and associated distributions. Rashwan [12] introduced the double weighted Rayleigh distribution properties and estimation, Al-Kadim and Hussein [5] published research for estimation the reliability of Weighted Rayleigh distribution through five methods using simulation and a comparison between the considered estimators were made. Ahmed and Ahmed [1] presented the characterization and estimation of double weighted Rayleigh distribution.

The aim for this paper is to estimate the reliability function of weighted Rayleigh distribution using shrinkage estimation methods depends on classical estimators; maximum likelihood (ML) and moment (Mom) methods.

The comparisons among the proposed estimators were done using Monte Carlo simulation based on (MSE) criteria to obtain the best estimator.

2. Weighted Rayleigh Distribution (WRD)

Let T be anon-negative random variable follows Rayleigh Distribution with one parameter β {T~ RD(β)}, the probability density function (pdf) of T is as follows.

$f(t;\theta) = 2t\beta e^{-t^2\beta}$; $t > 0$, $\beta > 0$ …(1)

And assume that ω(t) be a non-negative weighted function such that the mean of ω(t) exist. Then the random variable T_W which is defined on interval (l, U) having probability density function as below :

$f\omega(t) = \frac{\omega(t) f(t)}{E(t)}$, $l < t < U$ …(2)

Where, $\omega(t) = e^{t^2}$ *and* $E(\omega(t)) = \int_0^\infty \omega(t) f(t) dt$

From equations (1) and (2), the probability density function the random variable Tω will be, [5].

$f\omega(t;\theta) = 2t(\beta - 1)e^{-t^2(\beta-1)}$; $t > 0$, $\beta > 1$ …(3)

And the cumulative distribution function (cdf) of Tω will be

$F\omega(t;\theta) = 1 - e^{-t^2(\beta-1)}$ …(4)

Suppose that Tω be anon-negative random variable follows Weighted Rayleigh Distribution { Tω ~ WRD(θ)}, then the probability density function (pdf) of Tω became

$f_w(t;\theta) = 2t\theta e^{-t^2\theta}$; $t > 0$, $\theta > 0$ …(5)

And the cumulative distribution function (cdf) of T_W will be

$F_W(t;\theta) = 1 - e^{-t^2\theta}$ …(6)

Accordingly, the reliability and hazard functions will be respectively as follows:

$R\omega(t;\theta) = 1 - F\omega(t;\theta) = e^{-t^2\theta}$ …(7)

$h\omega(t;\theta) = \frac{f_\omega(t;\theta)}{R_\omega(t;\theta)} = 2t\theta$ …(8)

3. Estimation Methods of Reliability Function

3.1 Maximum Likelihood Estimation (MLE)

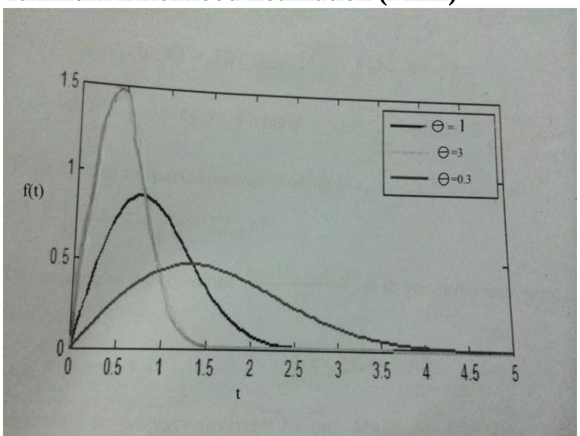

Figure (1):The plot of pdf of WRD

In this subsection, one can find the likelihood function $L(t_1,t_2,t_3,\ldots,t_n;\theta)$ based on the following

$l = L(t_1,t_2,t_3,\ldots,t_n;\theta) = \prod_{i=1}^{n} f\omega(t,\theta)$

$l = \theta^n \prod_{i=1}^{n} 2t_i e^{-\theta\sum_{i=1}^{n} t_i^2}$

$\ln l = n\ln\theta + \sum_{i=1}^{n} ln2t_i - \theta\sum_{i=1}^{n} t_i^2$

$\frac{\partial lnl}{\partial\theta} = \frac{n}{\theta} - \sum_{i=1}^{n} t_i^2$

Put $\frac{\partial lnl}{\partial\theta} = 0$

$\therefore \hat{\theta}_{mle} = \frac{n}{\sum_{i=1}^{n} t_i^2}$ …(9)

Then the maximum likelihood estimator for reliability function will became

$\hat{R}_{mle}(t) = e^{-t^2\theta_{mle}^n}$ …(10)

3.2 Method of Moments

Let $t_1, t_2, \ldots, t_n$ refer to a random sample of size n follows the WRD with p.d.f (1), then the moment estimator of $\hat{\theta}$ is obtained by setting the mean of the distribution equal to the sample mean,

i.e., $E(T^K) = \sum_{i=1}^{n} t^k / n$

The Moment estimator $\hat{\theta}_{mom}$ of θ obtained as below

$$\frac{\Gamma(1+\frac{k}{2})}{(\theta)^{k/2}} = \frac{\sum_i^n t_i^k}{n}$$

For k=1 we have

$$\frac{\Gamma(1+\frac{1}{2})}{\theta^{1/2}} = \frac{\sum_{i=1}^n t_i}{n}$$

$$\therefore \hat{\theta}_{mom} = \frac{\pi}{4(\bar{t})^2} \quad \ldots(11)$$

Hence the moment estimator for reliability function will take the following formula

$$\hat{R}_{mom}(t) = e^{-t^2(\hat{\theta}_{mom})} \quad \ldots(12)$$

3.3 Shrinkage Method

The problem of shrink a usual estimator $\hat{\theta}_{mle}$ of the parameter θ studied by Thompson in 1968 which related with previous estimate concerning the value of definite parameter θ from earlier knowledges or prior revisions as initial estimate θ_o through merge the usual estimator $\hat{\theta}_{mle}$ and initial estimate θ_o as a linear mixture via shrinkage weight factor $\emptyset(\hat{\theta}_{mle})$, $0 \le \emptyset(\hat{\theta}_{mle}) \le 1$, and the result estimator is so called shrinkage estimator which has the form below

$$\tilde{\theta}_{sh} = \emptyset(\hat{\theta}_{mle})\,\hat{\theta}_{mle} + \left(1 - \emptyset(\hat{\theta}_{mle})\right)\theta_o \quad \ldots(13)$$

Where, $\emptyset(\hat{\theta}_{mle})$ denotes to the trust of $\hat{\theta}_{mle}$ and $\left(1 - \emptyset(\hat{\theta}_{mle})\right)$ signifies the trust of θ_o, which might be constant or a function of $\hat{\theta}_{mle}$, function of sample size or could be create by reducing the mean square error for $\hat{\theta}_{sh}$.Thompson refers to the significant reasons to usage initial value.

1- Supposing the initial value θ_o near to the true value and then it is essential to use it.
2- If the initial value θ_o near to the actual value of the parameter θ , we get bad situation if not used θ_o, see [2, 3, 4, 13, 15].

In this work, there is no doubt to take the moment method as initial value, consequently the equation (13) becomes:

$$\tilde{\theta}_{sh} = \emptyset_1(\hat{\theta}_{mle})\hat{\theta}_{mle} + \left(1 - \emptyset_1(\hat{\theta}_{mle})\right)\hat{\theta}_{mom} \quad \ldots(14)$$

3.3.1 The Shrinkage Weight Function (sh1):

In this section we suggest shrinkage estimation method using shrinkage weight factor as a function of sample size n, as below.

$\emptyset_1(\hat{\theta}_{mle}) = e^{-n}$

Consequently, shrinkage estimator of θ, which is defined in (14) will be

$$\hat{\theta}_{sh1} = e^{-n}\hat{\theta}_{mle} + (1 - e^{-n})\hat{\theta}_{mom}$$

.

Therefor the estimation for reliability function using shrinkage weight function will be

$$\hat{R}_{sh1}(t) = e^{-t^2\hat{\theta}_{sh1}} \quad \ldots(15)$$

3.3.2 Constant Shrinkage Weight Factor (sh2):

In this subsection, we assume shrinkage estimation method using constant weight fader as below

$\emptyset_2(\hat{\theta}_{mle}) = 0.01$

Accordingly, shrinkage estimator of θ will be :

$$\hat{\theta}_{sh2} = (0.01)\hat{\theta}_{mle} + (0.99)\hat{\theta}_{mom} \quad \ldots(16)$$

Therefor the estimation for reliability function using specific constant weight factor will be as follows

$\hat{R}_{sh2}(t)$ …(17)

3.3.3 Modified Thompson Type Shrinkage Weight Function (sh3):

In this subsection, we consider the modified shrinkage weight factor introduced by Thompson as follows.

$$\emptyset_3(\hat{\theta}_{mle}) = \frac{(\hat{\theta}_{mle} - \hat{\theta}_{mom})^2}{(\hat{\theta}_{mle} - \hat{\theta}_{mom})^2 + var(\hat{\theta}_{mle})} * (0.001) \quad \ldots(18)$$

Where, $var(\hat{\theta}_{mle}) = \frac{n^2\theta^2}{(n-1)^2.(n-2)}$ …(19)

Hence, shrinkage estimator of θ became:

$$\hat{\theta}_{sh3} = \emptyset_3(\hat{\theta}_{mle})\hat{\theta}_{mle} + \left(1 - \emptyset_3(\hat{\theta}_{mle})\right)\hat{\theta}_{mom} \quad \ldots(20)$$

Accordingly, the estimation for reliability function using the modified shrinkage weight factor will be as follows:

$$\hat{R}_{sh3}(t) = e^{-t^2\hat{\theta}_{sh3}} \quad \ldots(21)$$

4. Simulation Study

In this section, Monte Carlo simulation study were employed for comparing the performance of the considered estimators for reliability function which were gained via unlike sample size (n=10,30,50,100), based on 1000 replication through MSE criteria as steps below [6] .

Step1: Generate random samples follows the continuous uniform distribution from interval (0,1) as $u_1, u_2, \ldots, u_n$.

Step2: Transform mention uniform random samples to random samples follows WRD using the cumulative distribution function (c.d.f) as follows:

$u \sim Uinf(0,1)$

$F(t) = 1 - R(t)$

$u_i = 1 - e^{-t_i^2\theta}$

$t_i = \sqrt{-\frac{1}{\theta}\ln(1 - u_i)}$

Step3: Calculate maximum likelihood estimator of $\hat{R}(t)$ from equation (10)

Step4: Apply moment method on $\hat{R}(t)$ via equation (12).

Step5: Compute the three shrinkage estimators of $\hat{R}(t)$ by equations (15), (17), (20).

Step6: Based on (L=1000) replication, the MSE for all proposed estimation methods of R(t) is utilized by:

$$MSE(\hat{R}) = \frac{1}{L}\sum_{i=1}^{L}\left(\hat{R}_i(t) - R_{real}(t)\right)^2$$

Where, $\hat{R}$ (t) denotes to suggested estimator of actual R(t).

5. Numerical Results

We put all the results in the tables below

Table (1): Shown the estimation methods and real value of R (t) w.r.t n and θ

n	θ	R real	$\hat{R}$ mle	$\hat{R}$ mom	$\hat{R}$ sh1	$\hat{R}$ sh2	$\hat{R}$ sh3
10	2	0.5176	0.4836	0.4970	0.4970	0.4969	0.4836
	3	0.5176	0.4836	0.4970	0.4970	0.4969	0.4836
	4	0.5176	0.4836	0.4970	0.4970	0.4969	0.4836
	5	0.5176	0.4836	0.4970	0.4970	0.4969	0.4836
30	2	0.5178	0.5162	0.5068	0.5068	0.5069	0.5162
	3	0.5178	0.5162	0.5068	0.5068	0.5069	0.5162
	4	0.5178	0.5162	0.5068	0.5068	0.5069	0.5162
	5	0.5178	0.5162	0.5068	0.5068	0.5069	0.5162
50	2	0.5247	0.5007	0.4952	0.4952	0.4953	0.5007
	3	0.5247	0.5007	0.4952	0.4952	0.4953	0.5007
	4	0.5247	0.5007	0.4952	0.4952	0.4953	0.5007
	5	0.5247	0.5007	0.4952	0.4952	0.4953	0.5007
100	2	0.5335	0.4986	0.5000	0.5000	0.5000	0.4986
	3	0.5335	0.4986	0.5000	0.5000	0.5000	0.4986
	4	0.5335	0.4986	0.5000	0.5000	0.5000	0.4986
	5	0.5335	0.4986	0.5000	0.5000	0.5000	0.4986

Table (2): Shown the MSE of estimation methods w.r.t n and θ

n	θ	MSEmle	MSEmom	MSEsh1	MSEsh2	MSEsh3	best
10	2	0.010770	0.006500	0.006501	0.006502	0.010702	Sh1
	3	0.010770	0.006500	0.006501	0.006502	0.010702	Sh1
	4	0.010770	0.006500	0.006501	0.006502	0.010702	Sh1
	5	0.010770	0.006500	0.006501	0.006502	0.010702	Sh1
30	2	0.001898	0.010441	0.010441	0.010311	0.001510	Sh3
	3	0.001898	0.010441	0.010441	0.010311	0.001510	Sh3
	4	0.001898	0.010441	0.010441	0.010311	0.001510	Sh3
	5	0.001898	0.010441	0.010441	0.010311	0.001510	Sh3
50	2	0.037956	0.046600	0.046600	0.046500	0.037800	Sh3
	3	0.037956	0.046600	0.046600	0.046500	0.037800	Sh3
	4	0.037956	0.046600	0.046600	0.046500	0.037800	Sh3
	5	0.037956	0.046600	0.046600	0.046500	0.037800	Sh3
100	2	0.110551	0.106001	0.106001	0.106001	0.110500	Sh1&Sh2
	3	0.110551	0.106001	0.106001	0.106001	0.110500	Sh1&Sh2
	4	0.110551	0.106001	0.106001	0.106001	0.110500	Sh1&Sh2
	5	0.110551	0.106001	0.106001	0.106001	0.110500	Sh1&Sh2

6. RESULT ANALYSIS

1- For n=10 (small sample size), the mean squared error (MSE) of the reliability estimator $\hat{R}_{sh1}$ is minimum than the other estimators and follows by $\hat{R}_{sh2}$ and $\hat{R}_{sh3}$, hence the best estimator in this case is $\hat{R}_{sh1}$ for all θ = 2,3,4,5.

2- For n = 30,50 (medium sample size), the mean squared error (MSE) of the reliability estimator $\hat{R}_{sh3}$ has minimum value than the other estimators follows by $\hat{R}_{sh2}$ and $\hat{R}_{sh1}$, consequently the best estimator in this case is $\hat{R}_{sh3}$ for all θ = 2,3,4,5.

3- For n=100 (large sample size), the minimum mean squared error approved by the estimators $\hat{R}_{sh1}$ and $\hat{R}_{sh2}$ follows by $\hat{R}_{sh3}$. Therefor the estimators $\hat{R}_{sh1}$ and $\hat{R}_{sh2}$ are the best for all θ=2,3,4,5.

4- For all n, the mean squared error (MSE) for all proposed estimators are approximately fixed with respect to θ.

7. CONCLUSIONS

From the results analysis, the shrinkage estimator suitable for estimate the reliability function of the weight Rayleigh distribution especially when employee different shrinkage weigh factors where the estimator performance good behavior in the sense of MSE.

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