GENERALIZED N-DERIVATIONS IN PRIME NEAR–RINGS

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ABSTRACT: *The main purpose of this paper is to show that zero symmetric prime left near-rings satisfying certain identities on generalized n- derivation are commutative rings.*

1. INTRODUCTION

A near – ring is a set N together with two binary operations (+) and (.) such that (i) (N,+) is a group (not necessarily abelian). (ii) (N, .) is a semi group. (iii) For all a,b,c ϵ N ; we have. a.(b + c) = a.b + b.c. We will denote the product of any two elements x and y in N , i.e.; x.y by xy.. A nonempty subset U of N is called semigroup ideal if NU ⊆ U and UN ⊆ UN. N is called a prime near-ring if xNy = {0} implies that either x = 0 or y = 0 [7].

Throughout this paper, N will be a zero symmetric near – ring (i.e., N satisfying the property 0.x = 0 for all x ∈ N) and Z = {x ϵ N, xy = yx for all y ϵ N}. [x, y] = xy - yx and (x , y) = x + y – x - y while the symbol x∘y will denote xy + yx.

In [8] X.K. Wang derivations in near-rings and this concept has been stydied and in several ways by various authors. In [3] , [4] M. Ashraf defined n-derivations and generalized n-derivation in near-ring respectively.

Throught this paper, we show that prime near-rings havhng generalized n-derivation (as defined by M. Ashraf in [4]) and satisfying some identities are commutative rings.

2. Preliminary Results

We begin with the following lemmas which are essential for developing the proofs of
our main results.

Lemma 2.1[6] Let N be a prime near-ring. If z∈ Z \{0} and x is an element of N such that xz ∈ Z or zx∈ Z, then x∈ Z.

Lemma 2.2[6] Let N be a prime near-ring and U a nonzero semigroup ideal of N. If $x, y \in N$ and $xUy = \{0\}$ then $x = 0$ or $y = 0$.

Lemma 2.3 [6] Let N be a prime near-ring and *Z* contains a nonzero semigroup left ideal or nonzero semigroup right ideal, then N is a commutative ring.

Lemma 2.4 [4] Let d be an n-derivation of a near ring N. Then d(Z,N,…,N) ⊆ Z.

Lemma 2.5 [3] Let N be a prime near-ring admitting a nonzero n-derivation d such that d(N, N, . . ., N) ⊆ Z then N is a commutative ring.

Lemma 2.6 [4] Let N be a prime near ring, d a nonzero n-derivation of N, and $U_1,U_2,...,U_n$ be nonzero semigroup left ideals of N. If $d(U_1,U_2,...,U_n) \subseteq Z$, then N is a commutative ring.

Lemma 2.7 [5] Let N be a near-ring. Then f is a left generalized n-derivation of N associated with n-derivation d if and only if

$f(x_1x'_1,x_2,\dots,x_n) = x_1f(x'_1,x_2,\dots,x_n) + d(x_1,x_2,\dots,x_n)\, x'_1$
$f(x_1,x_2x'_2,\dots,x_n) = x_2f(x_1,x'_2,\dots,x_n) + d(x_1,x_2,\dots,x_n)x'_2$
$f(x_1,x_2,\dots,x_nx'_n) = x_nf(x_1,x_2,\dots,x'_n) + d(x_1,x_2,\dots,x_n)x'_n$
hold for all $x_1, x'_1, x_2, x'_2,\dots,x_n, x'_n \in N$.

Lemma 2.8 [5] Let N be a near-ring admitting a generalized n-derivation f with associated n-derivation d of N. Then

$(d(x_1 ,x_2,\dots,x_n)x_1'+x_1f(x_1',x_2,\dots,x_n))y= d(x_1 ,x_2,\dots,x_n)x_1'y+ x_1f(x_1',x_2,\dots,x_n)y,$
$(d(x_1 ,x_2,\dots,x_n)x_2'+x_2f(x_1 ,x_2',\dots,x_n))y= d(x_1 ,x_2,\dots,x_n)x_2'y + x_2f(x_1 ,x_2',\dots,x_n)y,$
⋮
$(d(x_1 ,x_2,\dots,x_n)x_n'+x_nf(x_1 ,x_2,\dots,x_n'))y= d(x_1 ,x_2,\dots,x_n)x_n'y + x_nf(x_1 ,x_2,\dots,x_n')y,$
for all $x_1, x'_1, x_2, x'_2,\dots, x_n, x'_n, y \in N$.

Lemma 2.9[5] Let N be a near-ring admitting a generalized n-derivation f with associated n-derivation d of N. Then

$(x_1f(x'_1,x_2,\dots,x_n) + d(x_1,x_2,\dots,x_n)\, x'_1)y = x_1f(x'_1,x_2,\dots,x_n)y + d(x_1,x_2,\dots,x_n)\, x'_1y,$
$(x_2f(x_1,x'_2,\dots,x_n) + d(x_1,x_2,\dots,x_n)x'_2)y = x_2f(x_1,x'_2,\dots,x_n)y +d(x_1,x_2,\dots,x_n)x'_2y,$
$(x_nf(x_1,x_2,\dots,x'_n)+d(x_1,x_2,\dots,x_n)x'_n)y = x_nf(x_1,x_2,\dots,x'_n)y +d(x_1,x_2,\dots,x_n)x'_n y$
for all $x_1, x'_1, x_2, x'_2,\dots, x_n, x'_n, y \in N$.

Lemma 2.10 [2] Let N be a prime near-ring admitting a left generalized n-derivation f with associated nonzero n-derivation d of N. Let $U_1, U_2, \dots,U_n$ be nonzero semigroup ideals of N. If $f(u_1u'_1, u_2, \dots, u_n) = f(u'_1 u_1 , u_2, \dots, u_n)$ for all $u_1, u'_1 \in U_1,u_2 \in U_2,\dots,u_n \in U_n$, then N is a commutative ring**.**

Lemma 2.11 [2] Let N be a prime near-ring admitting a nonzero generalized n-derivation f with associated n-derivation d of N. Let $U_1, U_2, \dots,U_n$ be nonzero semigroup right ideals of N. If $f(U_1, U_2, \dots, U_n) \subseteq Z$, then N is a commutative ring.

3. MAIN RESULT

Theorem 3.1 Let N be a prime near ring admitting a generalized n-derivation f associated with nonzero n-derivation d of N. Let $U_1,U_2,\dots,U_n$ be semigroup ideals of N. Then the following assertions are equivalent

(i) $f([x, y],u_2,\dots,u_n) = [f(x,u_2,\dots,u_n), y]$ for all $x,y \in U_1,u_2 \in U_2,\dots,u_n \in U_n$.

(ii) $[f(x,u_2,\dots,u_n), y] = [x, y]$ for all $x,y \in U_1,u_2 \in U_2,\dots,u_n \in U_n$.

(iii) N is a commutative ring.

Proof. It is easy to verify that (iii) ⇒(i) and (iii) ⇒ (ii)

(i) ⇒ (iii) Assume that

$f([x, y],u_2,\dots,u_n) = [f(x,u_2,\dots,u_n),y]$ for all $x, y \in U_1,u_2 \in U_2,\dots,u_n \in U_n$. (1)

If we take y = x in (1) we get

$f(x,u_2,\dots,u_n)x = xf(x,u_2,\dots,u_n)$ for all $x \in U_1 ,u_2 \in U_2,\dots,u_n \in U_n$. (2)

Replacing y by xy in (1) to get

$f([x, xy],u_2,\dots,u_n) = [f(x,u_2,\dots,u_n), xy]$ for all $x \in U_1 ,u_2 \in U_2,\dots,u_n \in U_n$.

Therefore

$f(x[x, y],u_2,\dots,u_n) = [f(x,u_2,\dots,u_n), xy]$ for all $x \in U_1 ,u_2 \in U_2,\dots,u_n \in U_n$.

Hence, we get

$d(x,u_2,...,u_n)[x, y] + xf([x, y],u_2,...,u_n) = [f(x,u_2,...,u_n), xy]$ for all $x\epsilon U_1, u_2\epsilon U_2,...,u_n\epsilon U_n$.

Using (1) again, previous equation implies that

$d(x,u_2,...,u_n)[x, y] + x[f(x,u_2,...,u_n), y] = [f(x,u_2,...,u_n), xy]$ for all $x\epsilon U_1, u_2\epsilon U_2,...,u_n\epsilon U_n$.

Which means that

$d(x,u_2,...,u_n)[x,y] + xf(x,u_2,...,u_n)y - xyf(x,u_2,...,u_n) = f(x,u_2,...,u_n) xy - xyf(x,u_2,...,u_n)$ for all $x\epsilon U_1, u_2\epsilon U_2,...,u_n\epsilon U_n$.

Using (2) previous equation can be reduced to

$d(x,u_2,...,u_n)xy = d(x,u_2,...,u_n)yx$ for all $x, y\epsilon U_1, u_2\epsilon U_2,...,u_n \epsilon U_n$. (3)

Replacing y by yr, where $r\epsilon N$, in (3) and using it again to get

$d(x,u_2,...,u_n)U_1[x, r] = 0$ for all $x\epsilon U_1, u_2\epsilon U_2,...,u_n\epsilon U_n, r\epsilon N$. (4)

Using Lemma 2.2 in (4), we conclude that

for each $x\epsilon U_1$ either $x\epsilon Z$ or $d(x,u_2,...,u_n) = 0$ for all $u_2\epsilon U_2,...,u_n \epsilon U_n$, but using Lemma 2.4 lastly, we get $d(x,u_2,...,u_n)\epsilon Z$ for all $x\epsilon U_1,u_2\epsilon U_2,...,u_n\epsilon U_n$. So we get $d(U_1,U_2,...,U_n) \subseteq Z$. Now by using Lemma 2.6 we find that N is a commutative ring.

(ii) ⇒ (iii) suppose that

$[f(x,u_2,...,u_n),y] = [x, y]$ for all $x, y\epsilon U_1,u_2\epsilon U_2,...,u_n \epsilon U_n$. (5)

If we take y = x in (5), we get

$f(x,u_2,...,u_n)x = xf(x,u_2,...,u_n)$ for all $x\epsilon U_1, u_2\epsilon U_2,...,u_n \epsilon U_n$. (6)

Replacing x by yx in (5) and using it again, we get

$[f(yx,u_2,...,u_n),y] = [yx, y] = y[x, y] = y[f(x,u_2,...,u_n), y]$ for all $x, y\epsilon U_1,u_2\epsilon U_2,...,u_n\epsilon U_n$.

So we have

$f(yx,u_2,...,u_n)y - yf(yx,u_2,...,u_n) = yf(x,u_2,...,u_n)y - y^2f(x,u_2,...,u_n)$

for all $x,y\epsilon U_1,u_2\epsilon U_2,...,u_n\epsilon U_n$.

In view of Lemmas 2.7 and 2.9 the last equation can be rewritten as

$yf(x,u_2,...,u_n)y + d(y,u_2,...,u_n)xy - (y^2f(x,u_2,...,u_n) + yd(y,u_2,...,u_n)x) = yf(x,u_2,...,u_n)y - y^2f(x,u_2,...,u_n)$ for all $x,y\epsilon U_1,u_2\epsilon U_2,...,u_n\epsilon U_n$.

So we have

$d(y,u_2,...,u_n)xy = yd(y,u_2,...,u_n)x$ for all $x, y\epsilon U_1,u_2\epsilon U_2,...,u_n\epsilon U_n$. (7)

Replacing x by xr in (7) and using it again to get

$d(y,u_2,...,u_n)xry = yd(y,u_2,...,u_n)xr = d(y,u_2,...,u_n)xyr$.

Therefore

$d(y,u_2,...,u_n)U_1[y,r] = 0$ for all $y\epsilon U_1, u_2\epsilon U_2,...,u_n\epsilon U_n, r\epsilon N$. (8)

Since equation (8) is the same as equation (4), arguing as in the proof of (i) ⇒ (iii) we find that N is a commutative ring.

Corollary 3.2 Let N be a prime near ring admitting a generalized n-derivation f associated with nonzero n-derivation d of N. Then the following assertions are equivalent

(i) $f([x_1, y],x_2,...,x_n) = [f(x_1,x_2,...,x_n), y]$ for all $x_1,x_2,...,x_n, y \epsilon N$.

(ii) $[f(x_1,x_2,...,x_n), y] = [x_1, y]$ for all $x_1,x_2,..., x_n, y \epsilon N$.

(iii) N is a commutative ring.

Corollary 3.3 [1, Theorem 2.1] Let N be a prime near ring which admits a nonzero n-derivation d, if $U_1,U_2, ...,U_n$ are semigroup ideals of N, then the following assertions are equivalent

(i) $d([x, y],u_2,...,u_n) = [d(x,u_2,...,u_n), y]$ for all $x, y\epsilon U_1, u_2\epsilon U_2,...,u_n\epsilon U_n$.

(ii) $[d(x,u_2,...,u_n), y] = [x, y]$ for all $x, y\epsilon U_1,u_2\epsilon U_2,...,u_n\epsilon U_n$.

(iii) N is a commutative ring.

Theorem 3.4 Let N be prime near ring admitting a nonzero right generalized n-derivation f associated with n-derivation d of N. If $U_1,U_2,...,U_n$ are nonzero semigroup ideals of N. Then the following assertions are equivalent

(i) $[f(u_1,u_2,...,u_n),y]\epsilon Z$ for all $u_1\epsilon U_1,u_2\epsilon U_2,...,u_n \epsilon U_n,y\epsilon N$.

(ii) N is a commutative ring.

Proof. It is clear that (ii) ⇒ (i)

(i) ⇒ (ii) Suppose that

$[f(u_1,u_2,...,u_n),y] \epsilon Z$ for all $u_1\epsilon U_1,u_2\epsilon U_2,...,u_n \epsilon U_n, y\epsilon N$. (9)

Replacing y by $f(u_1,u_2,...,u_n)y$ in (9) to get

$[f(u_1,u_2,...,u_n), f(u_1,u_2,...,u_n)y] \epsilon Z$ for all $u_1\epsilon U_1, u_2\epsilon U_2,...,u_n \epsilon U_n, y\epsilon N$.

Which means that

$[[f(u_1,u_2,...,u_n), f(u_1,u_2,...,u_n)y],t] = 0$ for all $u_1\epsilon U_1,u_2\epsilon U_2,...,u_n \epsilon U_n$ and $y, t\epsilon N$.

Therefore, we get

$[f(u_1,u_2,...,u_n) [f(u_1,u_2,...,u_n),y],t] = 0$ for all $u_1\epsilon U_1 ,u_2\epsilon U_2,...,u_n \epsilon U_n, y,t\epsilon N$.

Hence,

$f(u_1,u_2,...,u_n) [f(u_1,u_2,...,u_n),y]t = tf(u_1,u_2,...,u_n) [f(u_1,u_2,...,u_n),y]$

for all $u_1\epsilon U_1 ,u_2\epsilon U_2,...,u_n \epsilon U_n, y,t\epsilon N$.

Using (9) in previous equation implies that

$[f(u_1,u_2,...,u_n),y][f(u_1,u_2,...,u_n),t] = 0$

for all $u_1\epsilon U_1 ,u_2\epsilon U_2,...,u_n \epsilon U_n$ and $y, t \epsilon N$. (10)

In view of (9), equation (10) assures that

$[f(u_1,u_2,...,u_n),y]N[f(u_1,u_2,...,u_n) ,y] = 0$ for all $u_1\epsilon U_1 ,u_2\epsilon U_2,...,u_n \epsilon U_n ,y\epsilon N$.

Primeness of N shows that $[f(u_1,u_2,...,u_n),y] = 0$ for all $u_1\epsilon U_1 ,u_2\epsilon U_2,...,u_n \epsilon U_n, y \epsilon N$.

Hence $f(U_1,U_2,...,U_n)\subseteq Z$. By Lemma 2.11 we conclude that N is a commutative ring.

Corollary 3.5 Let N be prime near ring admitting a nonzero right generalized n-derivation f associated with n-derivation d of N. Then the following assertions are equivalent

(i) $[f(x_1,x_2,...,x_n),y] \epsilon Z$ for all $x_1 ,x_2,...,x_n ,y \epsilon N$.

(ii) N is a commutative ring.

Corollary 3.6 [1, Theorem 2.9] Let N be a prime near ring admitting a nonzero n-derivation d of N. If $U_1,U_2,...,U_n$ are nonzero semigroup ideals of N. Then the following assertions are equivalent

(i) $[d(u_1,u_2,..,u_n),y] \epsilon Z$ for all $u_1\epsilon U_1, u_2\epsilon U_2,..,u_n\epsilon U_n ,y\epsilon N$.

(ii) N is a commutative ring.

Theorem 3.7 Let N be a 2-torsion free prime near ring, then there exists no generalized n-derivation f associated with nonzero n-derivation d of N such that

$f(x_1,x_2,...,x_n)\circ y = x_1\circ y$ for all $x_1 ,x_2,...,x_n ,y\epsilon N$.

Proof. Suppose that

$f(x_1,x_2,...,x_n)\circ y = x_1\circ y$ for all $x_1 ,x_2,...,x_n, y\epsilon N$. (11)

Replacing x_1 by yx_1 in (15) and using it again, we get

$f(yx_1,x_2,...,x_n)\circ y = (yx_1)\circ y$
$= y(x_1\circ y)$
$= y(f(x_1,x_2,...,x_n)\circ y)$
Using Lemma 2.7 and Lemma 2.9 in previous equation, we obtain
$yf(x_1,x_2,...,x_n)y + d(y,x_2,...,x_n)x_1y + yd(y,x_2,...,x_n)x_1 + y^2f(x_1,x_2,...,x_n) =$
$yf(x_1,x_2,...,x_n)y + y^2f(x_1,x_2,...,x_n)$ for all $x_1,x_2,...,x_n,y\in N$.
Hence we get
$d(y,x_2,...,x_n)x_1y = - yd(y,x_2,...,x_n)x_1$ for all $x_1,x_2,...,x_n,y\in N$. (12)
Replacing x_1 by zx_1 in (12),where $z\in N$, we get
$d(y,x_2,...,x_n)zx_1y = - yd(y,x_2,...,x_n)zx_1$
$= (yd(y,x_2,...,x_n)z)(- x_1)$
$= d(y,x_2,...,x_n)z(- y)(- x_1)$ for all $x_1,x_2,...,x_n ,y, z\in N$.
Since $- d(y,x_2,...,x_n)zx_1y = d(y,x_2,...,x_n)zx_1(-y)$ for all $x_1 ,x_2,...,x_n ,y, z\in N$.
The last expression reduced to
$d(y,x_2,...,x_n)zx_1(- y) = d(y,x_2,...,x_n)z(- y)x_1$ for all $x_1, x_2,...,x_n, y, z\in N$.
Taking $- y$ instead of y in previous equation, we get
$d(-y,x_2,...,x_n)zx_1y = d(-y,x_2,...,x_n)zyx_1$ for all $x_1 ,x_2,...,x_n ,y, z\in N$.
So that $d(-y,x_2,...,x_n)z[x_1, y] = 0$ for all $x_1 ,x_2,...,x_n ,y, z\in N$.
Therefore, $d(-y,x_2,...,x_n)N[x_1, y] = \{0\}$ for all $x_1,x_2,...,x_n ,y\in N$.
Primeness of N yields that for each $y\in N$,
either $d(y,x_2,...,x_n) = - d(-y,x_2,...,x_n) = 0$ for all $x_2,...,x_n\in N$ or $y\in Z$.
Using Lemma 2.4 lastly, we get $d(y,x_2,...,x_n)\in Z$ for all $y,x_2,...,x_n\in N$. Hence we conclude that $d(N,N,...,N)\subseteq Z$ and using Lemma 2.5 implies that N is a commutative ring. Since N is 2-torsion free, therefore (11) assures that
$f(x_1,x_2,...,x_n)y = x_1y$ for all $x_1 ,x_2,...,x_n,y\in N$. (13)
Replacing x_1 by x_1t in (13) and using it again, we get
$d(x_1,x_2,...,x_n)ty = 0$ for all $x_1, x_2,...,x_n,y,t\in N$. Therefore $d(x_1,x_2,...,x_n)Ny = 0$ for all $x_1,x_2,...,x_n,y\in N$. Primeness of N implies that either $d = 0$ or $y = 0$ for all $y\in N$; a contradiction.

Corollary 3.8 [1, Theorem 2.13] Let N be a 2-torsion free prime near ring, then there exists no nonzero n-derivation d of N such that $d(x_1,x_2,...,x_n)\circ y = x_1\circ y$ for all $x_1,x_2,..., x_n,y\in N$.

Theorem 3.9 Let N be 2-torsion free a prime near ring which admits a nonzero right generalized n-derivation f associated with n-derivation d. If $f(x_1,x_2,...,x_n)\circ y \in Z$ for all $x_1 ,x_2,...,x_n ,y \in N$, then N is a commutative ring.

Proof. By our hypothesis, we have
$f(x_1,x_2,...,x_n)\circ y \in Z$ for all $x_1 , x_2,...,x_n ,y\in N$. (14)
(a) If $Z = \{0\}$, then equation (14) reduced to
$yf(x_1,x_2,...,x_n) = - f(x_1,x_2,...,x_n)y$ for all $x_1,x_2,...,x_n, y\in N$. (15)
Replacing y by ry, where $r\in N$, in (15) to get
$ryf(x_1,x_2,...,x_n) = - f(x_1,x_2,...,x_n)ry$
$= f(x_1,x_2,...,x_n)r(- y)$
$= rf(-x_1,x_2,...,x_n)(-y)$ for all $x_1,x_2,...,x_n ,y,r \in N$.
Thus we get
$r(yf(x_1,x_2,...,x_n) + f(-x_1,x_2,...,x_n) y) = 0$ for all $x_1,x_2,...,x_n ,y,r \in N$.
Replacing x_1 by $-x_1$ in last eqation we get
$r(-yf(x_1,x_2,...,x_n) + f(x_1,x_2,...,x_n) y) = 0$ for all $x_1,x_2,...,x_n ,y,r \in N$.
which implies that
$rN(-yf(x_1,x_2,...,x_n) + f(x_1,x_2,...,x_n)y) = \{0\}$ for all $x_1,x_2,...,x_n,y,r\in N$.
Primeness of N implies that $f(N,N,..,N) \subseteq Z$ and thus $f = 0$, which contradicts our hypothesis, consequently, there exists an element $z \in Z$ such that $z \neq 0$
$f(x_1,x_2,...,x_n)\circ y \in Z$ for all $x_1 ,x_2,...,x_n ,y \in N$. Then $f(x_1,x_2,...,x_n)\circ z = f(x_1,x_2,...,x_n)z + zf(x_1,x_2,...,x_n) \in Z$ for all $x_1,x_2,...,x_n,y\in N$, $z \in Z$. which implies that
$z(f(x_1,x_2,...,x_n) + f(x_1,x_2,...,x_n)) \in Z$, by Lemma 2.1 we find that
$f(x_1,x_2,...,x_n) + f(x_1,x_2,...,x_n) \in Z$ for all $x_1,x_2,...,x_n \in N$. (16)
Moreover from (14) it follows that
$f(x_1+ x_1,x_2,...,x_n)\circ y \in Z$ for all $x_1 ,x_2,...,x_n ,y \in N$.
Which means that
$f(x_1+ x_1,x_2,...,x_n)y + yf(x_1+ x_1,x_2,...,x_n) = (f(x_1,x_2,...,x_n) + f(x_1,x_2,...,x_n))y + y(f(x_1,x_2,...,x_n) + f(x_1,x_2,...,x_n)) \in Z$
for all $x_1 ,x_2,...,x_n ,y \in N$.
Which because of (16), yields that
$(f(x_1 + x_1,x_2,...,x_n) + f(x_1 + x_1,x_2,...,x_n))y \in Z$ for all $x_1 ,x_2,...,x_n, y \in N$.
Therefore, for all $x_1 ,x_2,...,x_n, y, t \in N$ we have
$(f(x_1 + x_1,x_2,...,x_n) + f(x_1 + x_1,x_2,...,x_n))ty$
$= y(f(x_1 + x_1,x_2,...,x_n) + f(x_1 + x_1,x_2,...,x_n))t$
$= (f(x_1 + x_1,x_2,...,x_n) + f(x_1 + x_1,x_2,...,x_n))yt$
So that
$(f(x_1 + x_1,x_2,...,x_n) + f(x_1 + x_1,x_2,...,x_n))N[t, y] = \{0\}$
for all $x_1 ,x_2,...,x_n, y, t \in N$.
In view of the primeness and 2-torsion freeness of N, the previous equation yields
either $f(x_1 + x_1,x_2,...,x_n) + f(x_1 + x_1,x_2,...,x_n) = 0$ and thus $f = 0$, a contradiction, or $N \subseteq Z$ and N is a commutative ring by Lemma 2.3.

Corollary 3.10 [1, Theorem 2.16] Let N be 2-torsion free a prime near ring which admits a nonzero n-derivation d. If $d(x_1,x_2,...,x_n)\circ y \in Z$ for all $x_1,x_2,...,x_n,y\in N$, then N is a commutative ring.

The following example proves that the hypothesis of primness in various theorems is not superfluous.
Let S be a 2-torsion free commutative near-ring. Let us define

$$:N = \left\{\begin{pmatrix} 0 & x & y \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}, x, y, 0 \in S\right\}$$ is zero symmetric near-ring

with regard to matrix addition and matrix multiplication. Define $d:\underbrace{N \times N \times \ldots \times N}_{n-times} \longrightarrow N$ such that

$$f\left(\begin{pmatrix} 0 & x_1 & y_1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}, \begin{pmatrix} 0 & x_2 & y_2 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}, \ldots, \begin{pmatrix} 0 & x_n & y_n \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}\right) = \begin{pmatrix} 0 & x_1x_2\ldots x_n & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

$$d\left(\begin{pmatrix}0 & x_1 & y_1\\ 0 & 0 & 0\\ 0 & 0 & 0\end{pmatrix},\begin{pmatrix}0 & x_2 & y_2\\ 0 & 0 & 0\\ 0 & 0 & 0\end{pmatrix},\dots,\begin{pmatrix}0 & x_n & y_n\\ 0 & 0 & 0\\ 0 & 0 & 0\end{pmatrix}\right)=$$
$$\begin{pmatrix}0 & 0 & y_1y_2\cdots y_n\\ 0 & 0 & 0\\ 0 & 0 & 0\end{pmatrix}$$

It is easy to verify that f is generalized n-derivation associated with a nonzero n-derivation d of N satisfying the following conditions for all $A,B,A_1,A_2,\dots,A_n \in N$,

(i) $f([A,B],A_2,\dots,A_n) = [f(A,A_2,\dots,A_n),B]$

(ii) $[f(A,A_2,\dots,A_n),B] = [A,B]$

(iii) $[f(A_1,A_2,\dots,A_n),B] \in Z$ for all $A_1, A_2,\dots,A_n, B \in N$.

(iv) $f(A_1,A_2,\dots,A_n)\circ B = A_1\circ B$

(v) $f(A_1,A_2,\dots,A_n)\circ B \in Z$

However, N is not a ring.

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