EVALUATION OF OPTIMAL INITIAL CONDITIONS FOR ORBITAL ELEMENTS OF SATELLITE

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ABSTRACT *: This work deals with the change in the orbital elements that include semi-major axis, eccentricity and parameter represent ratio of mass to area contain the perturbations to show effects on the trajectory of satellite through developed algorithm in MATLAB program that using Cowell method for accelerations procedure as well as the propagation of orbit was numerical integration utilizations Runge-Kutta for 4th order, the results show the semi-major axis when altitude is 1000 km is more stable from other altitudes. Also, change in A/m ratio from (0.0052 to 0.0072) reduces the lifetime of the satellite about (3days) while other parameters remain constant. Drop in the eccentricity value from (0.1 to 0.01) causes fasting in an average change of amplitude when the orbit is elliptical while amplitude change is less when the orbit is semicircular.*

Keywords : orbital elements, perturbations ,lifetime, altitude.

1. INTRODUCTION.

The satellite moves around the earth and its motion can be described by the Newtonian equation in the centered coordinate of the earth. The rotation of two bodies about each other is described mathematically that is called analytically solution of two-body problem. The satellite motion dictated perturbations come from gravitational and non-gravitational forces that effect on orbital elements [1]. The orbital elements are the parameters required for identifying any orbit. The orbital elements are considered in classical two body systems. In this respect six parameters are used in astronomy and orbital mechanics. The main of these elements are the eccentricity (e) and the semi-major axis (a) that represent the shape and size of the orbit, inclination (i) is define as vertical tilt of the ellipse orbit with respect to the reference plane which is measured at the ascending node, longitude of the ascending node (Ω) that represents the horizontal orientation of the node of the ellipse orbit. Argument of perigee (W) is defined as the orientation of the ellipse in the orbital plane which represents the angle measured from the ascending node to the semi major axis, the mean anomaly presents the position of the orbiting along with the ellipse at a specific time [2].

2. METHODS.

Table (1) represents the cases of different initial conditions of orbital elements used in this work.

2.1 Determination the state vectors of satellite orbit from orbital elements.

When the state vectors are calculated at specified time (t) the time of perigee passage should be known to find mean anomaly (M) and then eccentric anomaly (E) can be calculated [3].

The perigee radius of orbit can be calculated as

$$r_p = h_p + R_e \quad (1)$$

Where

h_p , represents the height of satellite in perigee, R_e is show the radius of Earth = 6378.165 Km

The semi major axis (*a*) Ares calculated as,

$$r_p = a(1-e) \quad (2)$$

The mean motion (*n*) can be stated as

$$n = \frac{2\pi}{T} = \sqrt{\frac{\mu}{a^3}} \quad (3)$$

The orbital period is calculated as,

$$T = 2\pi\sqrt{\frac{a^3}{\mu}} \quad (4)$$

The mean anomaly M in describing the position of the satellite in an orbit is represented as,

$$M = n\,(t - t_p) \quad (5)$$

The eccentric anomaly E determined as,

$$E = M + e_o \sin E \quad (6)$$

Where

t_p is the time that passage in perigee

e is an eccentricity, M is mean anomaly , E is eccentric anomaly. Exist some methods to find (E), like the Newton-Raphson method [4].

Finding root of the equation by

$f(E_i) = E_i - e_0 \sin E_i - M$

Finding derivative of the equation (6) from the equation

E_{li} $f'(E_i) = 1 - e\cos E_i$

by applying the Newton-Raphson method based on an approximation $\Delta E_i = -\frac{f(E_i)}{f'(E_i)}$.

To calculate a new (E) from $E_{i+1} = E_i + \Delta E_i$

To calculate Cartesian coordinate (x_w ,y_w and z_w) of the satellite as the follow relation applied,.

$$x_w = a(\cos E - e),\; y_w = a\sqrt{1-e^2}\sin E$$

$$z_w = 0$$

To calculate the displacement radius (r) by the relation,

$$r = a(1 - \mathrm{e}\cos E) \quad (7)$$

Through the direct differentiation for (x_w ,y_w and z_w) we got.

$$\dot{x}_w = \frac{\sqrt{\mu a}}{r}\sin E$$

$$\dot{y}_w = \frac{\sqrt{\mu a\,(1-e^2)}}{r}\cos E$$

$$\dot{Z}_w = 0$$

Also the velocity $\dot{r}$ can calculated as

$$\dot{r} = \frac{\sqrt{\mu a}}{r}\, e\sin E \quad (8)$$

Converted position and velocity of the satellite from the orbital plane to the Earth equatorial plane can be calculated by Gaussian matrix, which contains the Euler angle (i, Ω, ω) [5].

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = R^{-1}\begin{bmatrix} x_w \\ y_w \\ z_w \end{bmatrix} \quad (9)$$

Where R^{-1} represents inverse of Gaussian matrix

$$R^{-1}\begin{bmatrix} P_x & Q_x & W_x \\ P_y & Q_y & W_y \\ P_z & Q_z & W_z \end{bmatrix} \quad (10)$$

In which

$$P_x = \cos\omega\cos\Omega - \sin\omega\sin\Omega\cos i$$
$$P_y = \cos\omega\sin\Omega + \sin\omega\cos\Omega\cos i \quad (11)$$
$$P_z = \sin\omega\sin i$$
$$Q_x = -\sin\omega\cos\Omega - \cos\omega\sin\Omega\cos i$$
$$Q_y = -\sin\omega\sin\Omega + \cos\omega\cos\Omega\cos i \quad (12)$$
$$Q_z = \cos\omega\sin i$$
$$W_x = \sin\Omega\sin i$$
$$W_y = -\cos\Omega\sin i \quad (13)$$
$$W_z = \cos i$$

Components of position are:

$$x = P_x x_w + Q_x y_w + W_x z_w$$
$$y = P_y x_w + Q_y y_w + W_y z_w$$
$$z = P_z x_w + Q_z y_w + W_z z_w$$
$$r = (x^2 + y^2 + z^2)^{1/2} \quad (14)$$

Components of velocity are:

$$\dot{x} = P_x \dot{x}_w + Q_x \dot{y}_w + W_x \dot{z}_w$$
$$\dot{y} = P_y \dot{x}_w + Q_y \dot{y}_w + W_y \dot{z}_w$$
$$\dot{z} = P_z \dot{x}_w + Q_z \dot{y}_w + W_z \dot{z}_w$$
$$\dot{r} = (\dot{x}^2 + \dot{y}^2 + \dot{z}^2)^{1/2} \quad (15)$$

2.2 Converting the state vectors to the orbital elements.

The orbital elements can be determined from the component of position, velocity and angular momentum as follows [6].

$$\vec{h} = \vec{r} \times \vec{v} \quad (16)$$

$$h = \begin{bmatrix} h_x \\ h_y \\ h_z \end{bmatrix} = \begin{bmatrix} i & j & k \\ x & y & z \\ \dot{x} & \dot{y} & \dot{z} \end{bmatrix} \quad (17)$$

where

$$h_x = y\dot{z} - z\dot{y}$$
$$h_y = z\dot{x} - x\dot{z}$$
$$h_z = x\dot{y} - y\dot{x} \quad (18)$$

also

$$h = \sqrt{h_x^2 + h_y^2 + h_z^2} \quad (19)$$

Inclination (i) is calculated by

$$\tan i = \frac{\sqrt{h_x^2 + h_y^2}}{h_z} \quad (20)$$

Longitude of ascending node (**Ω**) is defined by

$$\tan \Omega = \frac{h_x}{-h_y} \quad (21)$$

Semi-major axis is represented as,:

$$a = \left(\frac{2}{r} - \frac{v^2}{\mu}\right)^{-1} \quad (22)$$

Eccentricity is represented by the equation below

$$e = \sqrt{\left(1 - \frac{r}{a}\right)^2 + \frac{x\dot{x}+y\dot{y}+z\dot{z}}{\sqrt{\mu a}}} \quad (23)$$

Eccentric anomaly (E) is calculated as,

$$\tan E = \left(\frac{1-\frac{r}{a}}{x\dot{x}+y\dot{y}+z\dot{z}}\right)\sqrt{a\mu} \quad (24)$$

Mean anomaly (M) is represented as follows,

$$M = E - \frac{x\dot{x}+y\dot{y}+z\dot{z}}{\sqrt{a\mu}} \quad (25)$$

True anomaly (f) is represented by the relation

$$\tan\frac{f}{2} = \sqrt{\frac{1+e}{1-e}}\tan\frac{E}{2} \quad (26)$$

Argument of the latitude (u) is stated as,

$$\tan u = \frac{zh}{-xh_y + yh_x} \quad (27)$$

Argument of perigee (ω) can be found as

$$\omega = u - f \quad (28)$$

3. Theory.

3.1 The Orbital Perturbations.

The perturbations in this work are defined as perturbing accelerations on satellite orbit:

$$\mathbf{a_p} = \mathbf{a_g} + \mathbf{a_{DRAG}} + \mathbf{a_{srp}} + \mathbf{a_{3rdb}}$$

where

$\mathbf{a_g}$: is representing the geopotential acceleration.
$\mathbf{a_{DRAG}}$: is the atmospheric drag acceleration.
$\mathbf{a_{srp}}$: represents the solar radiation pressure acceleration.
$\mathbf{a_{3rdb}}$: showing the 3rd body gravity acceleration.

3.2 The Geopotential acceleration.

The perturbing acceleration that effects on satellite according to the Earth's gravity potential can be represented as [7],

$$a_g = -\frac{\mu}{r}\left[\sum_{n=0}^{\infty}\left(\frac{r_E}{r}\right)\sum_{m=0}^{n}(C_{nm}\cos(m\lambda) + S_{nm}\sin(m\lambda)).P_{nm}(\sin(\phi)\right] \quad (29)$$

Where

- μ : is the constant of Earth's gravitational pull.
- r_E : is the radius of the Earth.
- C_{nm} ,S_{nm} are normalized geopotential coefficients.
- P_{nm} : is a normalized associated Legendre function.
- r: is an object distance from the Earth's center.
- (ϕ, λ): are the geocentric coordinates (latitude,longittude)

3.3 Atmospheric Drag Acceleration.

Velocity of the satellite relative to atmospheric is represented by the relation,

$$\vec{V}_r = \overrightarrow{V_{in}} - \vec{W}_{Earth} \times \vec{r} \quad (30)$$

Where

- $\vec{V}_r$:is the satellite velocity vector relative to atmospheric.
- $\overrightarrow{V_{in}}$: is the velocity for satellite.
- $\vec{W}_{Earth}$: is the rotational velocity of Earth.
- $\vec{r}$: is the satellite position vector.

The perturbing acceleration due to atmospheric drag is:

$$a_{Drag} = -\frac{1}{2} C_D \frac{A}{M} \rho \frac{V_r^3}{v_r} \quad (31)$$

- V_r : is the relative speed between satellite and the atmosphere.
- C_D : is the drag coefficient.
- A: is the satellite cross sectional area.
- M : is representing the mass of satellite.
- ρ : is the air density at satellite altitude.

To calculate the density by NLRMSISE-00 model in which the Ballistic coefficient *B* is defined as

$$B = C_D \frac{A}{M} \quad (32)$$

Therefore, the equation (31) can be written as:

$$a_{Drag} = -\ \frac{1}{2}\ B\ \rho\ \frac{V_r^{\ 3}}{v_r} \qquad (33)$$

3.4 The Solar radiation pressure.

Solar radiation pressure is d as a force on the satellite according to the momentum flux from the sun, stated as,

$$P_{\odot} = \frac{\Phi_{\odot}}{c} = \frac{L_{\odot}}{4\pi\, r^2\, c} \qquad (34)$$

Where

- $\Phi_{\odot}$: is the intensity of the solar flux.
- c : is the speed of light.
- $L_{\odot}$: is the solar luminosity.
- r : is the distance from the sun.

Perturbing acceleration can be defined as:

$$a_{\odot} = \frac{L_{\odot}}{4\pi\ c}\ \frac{A}{M}\ C_R\ \ \frac{r_{sat-}r_{\odot}}{\|r_{sat-}r_{\odot}\|^3}\ v$$

$$C_R = (1+\epsilon) \qquad (35)$$

Where

- A: represents the satellite area.
- M: representing the satellite mass.
- C_R: representing the radiation pressure coefficient.
- ϵ : representing of the body reflectivity.
- v : is representing the shadow function

$r_{\odot}$: is represents the position of the sun.

3.5 The third body perturbation.

The third body refers to any other body in space besides the earth which has a gravitational influence on the satellite, obviously, the sun and the moon are the main sources of perturbations. On the other hand, the gravitational pull force exerted on satellite causes orbital variation in respect of all orbital elements. The equation that describes the perturbing acceleration due to the third body can be stated as follows:

$$a_{3rdB} = GM_{3rdB}\left(\frac{S}{S^3} - \frac{r_{3rdB}}{{r_{3rdB}}^3}\right) \qquad (36)$$

$$S = r_{3rdB} - r_{sat} \qquad (37)$$

Where

- S : presents relative position for satellite
- r_{3rdB} : the position vector for the 3rd body.
- r_{sat} ; the position vector for the satellite.

4. THE RESULTS AND DISCUSSION.

In fig.1, the variation of semi major axis in different initial conditions, the behavior of semi major axis is affected according to change in the altitude that is obvious in fig.1. When the altitude is about 200 km in case 1, (a) is reduced with time and when the altitude is about 500 km and 1000 km, in case 2 and case 3, the behavior becomes periodic while the semi major axis become secular and decreased when eccentricity is changed to 0.01. as in case 4. When change the ratio of mass to area changes the amplitude of change is reduced as in case 5. Finally, when altitude reaches to 36000 km the behavior become periodic as represented in case 6, that means the effect by the third body attraction is predominant. The results shown in case 3 are more stable as compared to other cases. The change A/m from (0.0052 to 0.0072) causes decrease in the lifetime of satellite about (3days) when other parameters remain constant. The change (e) from (0.1 to 0.01) affects the fasting in average change of amplitude when the orbit is elliptical, while the amplitude change is less when the orbit is semicircular. In fig.2, as a result of variation of eccentricity in different initial conditions, the behavior of eccentricity is affected according to change in the altitude; obvious in fig.2. When the altitude is about 200 km in case 1, the secular decrease is observed and when the altitude about in the range of 500km and 1000 km as in case 2 and case 3, the behavior become periodic, while the eccentricity becomes secular and decreases with fast change when eccentricity is in the range of 0.1, as in case 4. When the ratio of mass to area changes the amplitude is reduced, as in case 5. Finally, when altitude reaches to 36000 km, where the behavior become periodic. In fig.3, the variation the inclination in different in initial conditions, the behavior of inclination is effect according to change the altitude that obvious in fig.3, when the altitude about 200 km in case 1 its periodic decrease with time and when the altitude is in the range of about 500 km and 1000 km as in case 2 and case 3, the behavior become periodic, while the inclination becomes periodic, decreasing with fast change when eccentricity is changes to 0.1 as in case 4. When change in ratio of mass to area occurs, the amplitude of change is reduced, as in case 5, finally when altitude reaches to 36000 km the behavior observed is periodically increasing. In fig.4, the variation in the argument of perigee in different initial conditions, the behavior of it is affected according to change in the altitude, that obvious in fig.4. When the altitude is about 200 km in case 1, it is periodic with long effects with time and when the altitude is in the range about 500 km and 1000 km in case 2 and case 3 the behavior becomes periodic and increases while the argument of perigee becomes secular, increasing with short effect as in case 4. When eccentricity changes to 0.1, then the change in the ratio of mass to area, the change in amplitude is reduced as in case 5. Finally when altitude reaches to 36000 km, the behavior becomes periodic. In fig.5, the variation RAAN in different initial conditions, the behavior is same with different amplitude in most cases except the case 6 where the amplitude is very small. In fig.6, the variation true anomaly in different initial conditions, the behavior is shape as saw tooth according to the mean motion of satellite. Tables (2,3,4,5,6 and 7) represent the state vectors for different cases of orbital elements while Table (8) represents the value of orbital elements with perturbations after one revaluation.

Table (1): Represent the initial conditions of orbital elements for satellite.

Parameters	Case 1	Case 2	Case 3	Case 4	Case 5	Case 6
h (km)	200	500	1000	200	200	36000
a (km)	6644.5828	6947.6131	7452.663	7309.04	6644.582	42806.19
e	0.01	0.01	0.01	0.1	0.01	0.01
i (deg)	63	63	63	63	63	63
Ω (deg)	40	40	40	40	40	40
W (deg)	30	30	30	30	30	30
M (deg)	0	0	0	0	0	0
C_D	2.1	2.1	2.1	2.1	2.1	2.1
C_R	1.31	1.31	1.31	1.31	1.31	1.31
A/m	0.0052	0.0052	0.0052	0.0052	0.0072	0.0052
T_P (min)	89.84	96.05	106.71	103.64	89.837	1468.98

Table (2): The values of state vectors for satellite in Case 1.

state vectors	initial value	final value
X (km)	3404.21360390378	3421.62410555591
Y (km)	4182.01962517370	4167.38245490035
Z (km)	3767.48292843535	3767.80303876496
r_{mag} (km)	6578.13697200000	6578.06413651152
V_x (km/s)	-5.71522488736075	-5.71616713098755
V_y (km/s)	-0.158105942020134	-0.134095348549124
V_z (km/s)	5.33965218832787	5.33924937154355
V_{mag} (km/s)	7.82308625123898	7.82305132818475

Table (3): The values of state vectors for satellite in Case 2.

state vectors	initial value	final value
X (km)	3559.46488262238	3576.29696811859
Y (km)	4372.74323126292	4358.79449822588
Z (km)	3939.30133064118	3939.51394965012
r_{mag} (km)	6878.13700000000	6878.13634943553
V_x (km/s)	-5.58919638137245	-5.58994331165675
V_y (km/s)	0.154619490296294	-0.132998846934683
V_z (km/s)	5.22190557274318	5.22170136498389
V_{mag} (km/s)	7.65057652644378	7.65057645317637

Table (4): The values of state vectors for satellite in Case 3.

state vectors	initial value	final value
X (km)	3818.21698966986	3832.40174954278
Y (km)	4690.61587841017	4677.58065197624
Z (km)	4225.66530759025	4227.27318526601
r_{mag} (km)	7378.13700000000	7378.13679112214
V_x (km/s)	-5.39649057148609	-5.39818506417633
V_y (km/s)	-0.149288478095495	-0.132444254490539
V_z (km/s)	5.04186331373448	5.04051682000513
V_{mag} (km/s)	7.38679789978120	7.38679587374652

Table (5): The values of state vectors for satellite in Case 4.

state vectors	initial value	final value
X (km)	3404.21310088968	3415.73797443015
Y (km)	4182.01900722927	4169.64413519213
Z (km)	3767.48237174378	3770.76437148261
r_{mag} (km)	6578.13600000000	6578.13515134411
V_x (km/s)	-5.96443090440378	-5.96782766982551
V_y (km/s)	-0.164999975563558	-0.147007412540467
V_z (km/s)	5.57248177604735	5.56928390271604
V_{mag} (km/s)	8.16420321585166	8.16415956881838

Table (6): The values of state vectors for satellite in Case 5.

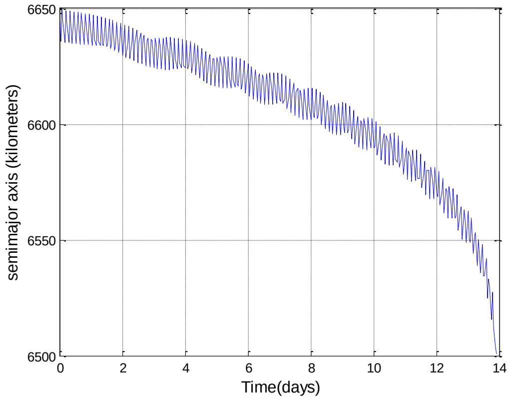
state vectors	initial value	final value
X (km)	3404.21360390378	3415.90554624272
Y (km)	4182.01962517370	4167.24544449983
Z (km)	3767.48292843535	3773.13964333724
r_{mag} (km)	6578.13697200000	6578.06415776552
V_x (km/s)	-5.71522488736075	-5.72094992761546
V_y (km/s)	0.158105942020134	0.139925408157705
V_z (km/s)	5.33965218832787	5.33395831770925
V_{mag} (km/s)	7.82308625123898	7.82304023568786

Table (7): The values of state vectors for satellite in Case 6.

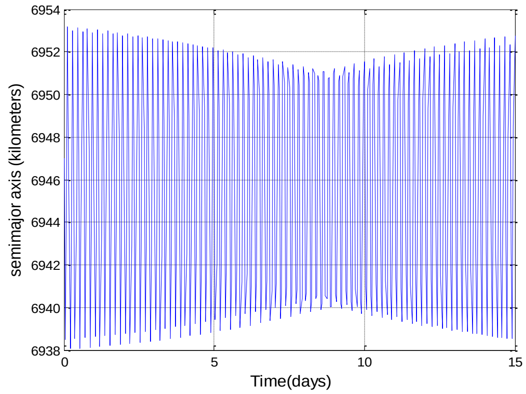
state vectors	initial value	final value
X (km)	21930.8598772055	-35805.1749880177
Y (km)	26941.6955205847	-23515.4924250134
Z (km)	24271.1385967468	-5314.55984305017
r_{mag} (km)	42378.1281000000	43165.1883609789
V_x (km/s)	-2.25171746038479	1.02334238689061
V_y (km/s)	-0.622914963546998	-0.983757107846140
V_z (km/s)	2.10374714937806	-2.67237629742364
V_{mag} (km/s)	3.08218491016295	3.02598789869961

Table (8): The final values of orbital elements for satellite with all cases.

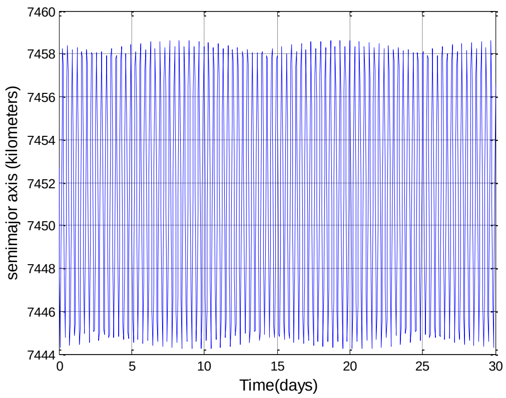
Cases	a (km)	e	i (deg)	Ω (deg)	w (deg)	M (deg)
Case 1 final	6644.354474	0.000997694235	62.999912113	40.02303583	29.75513224	0.004975896
Case 2 final	6947.611668	0.00999988516	62.99998134	40.00773124	29.77608404	359.9948762
Case 3 final	7452.659039	0.00999941711	62.999970093	40.00673293	29.80540440	0.011577062
Case 4 final	7.308.94238	0.099988111565	62.999947266	40.00726319	29.79357797	0.003466007
Case 5 final	6644.354474	0.09976942355	62.999912113	40.02303583	29.75513224	0.004975896
Case 6 final	42805.01255	0.00995578650	63.005019186	39.94849911	29.67185104	147.9938281



Case 1



Case 2



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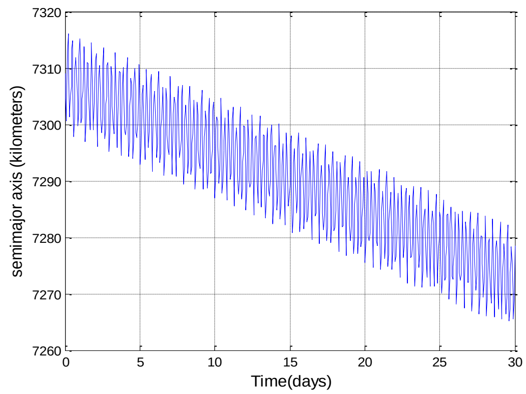

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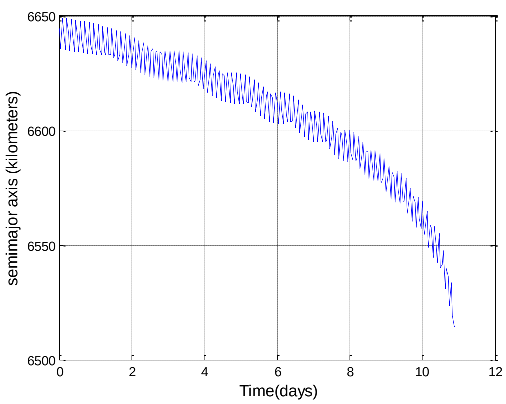

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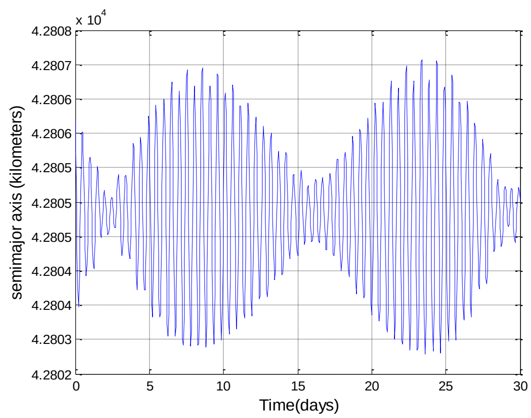
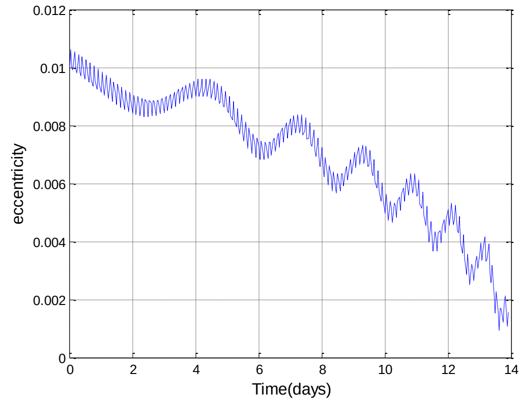
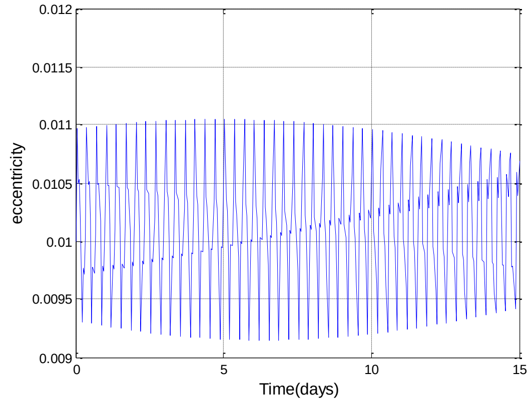


Case 6

Figure (1): Semi major axis with different conditions of orbital elements.



Case 1



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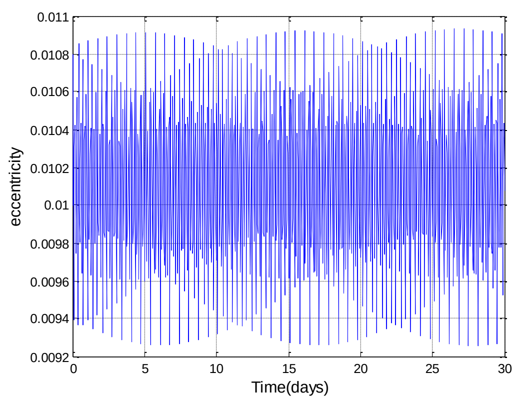
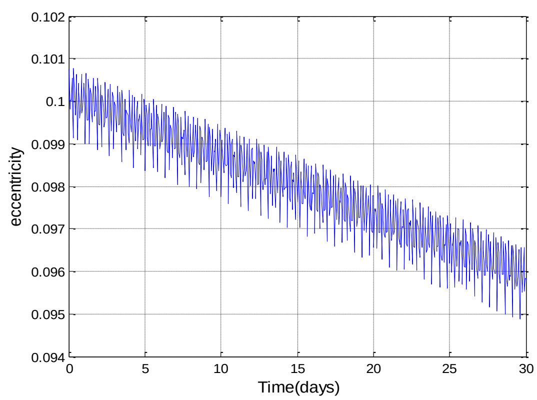
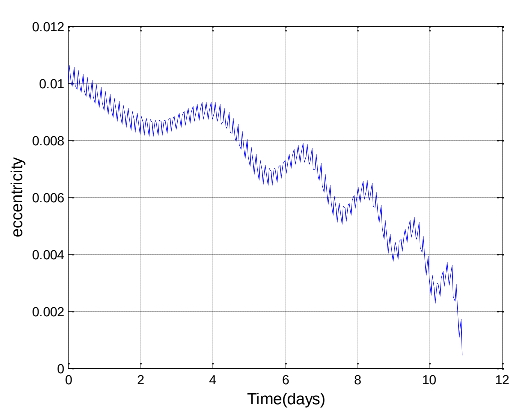


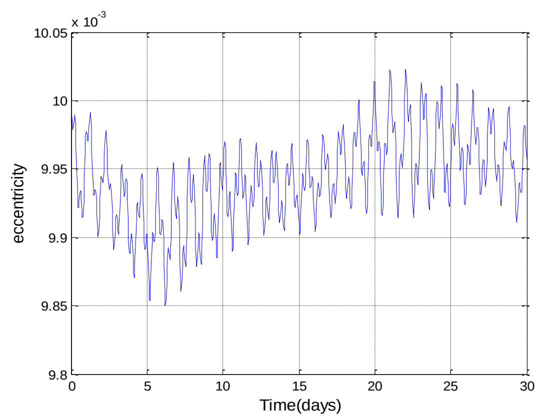
Case 3



Case 4

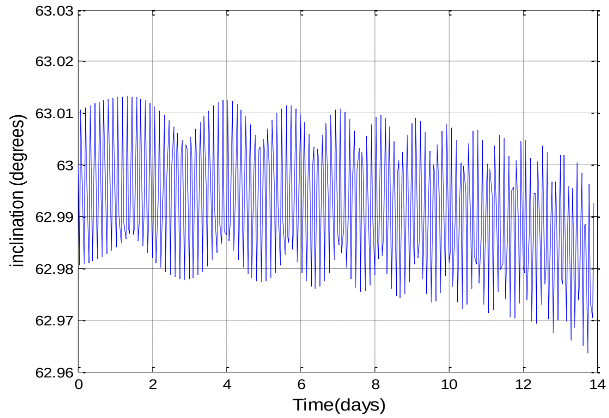


Case 5

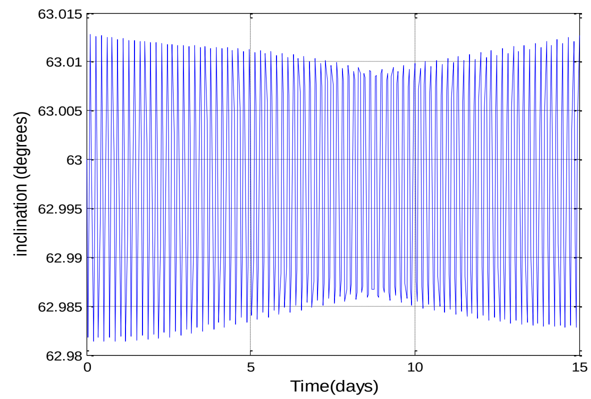


Case 6

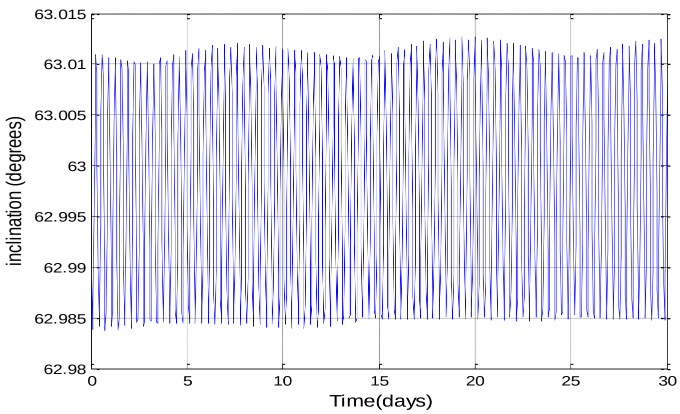
Figure (2): Eccentricity with different conditions of orbital elements.



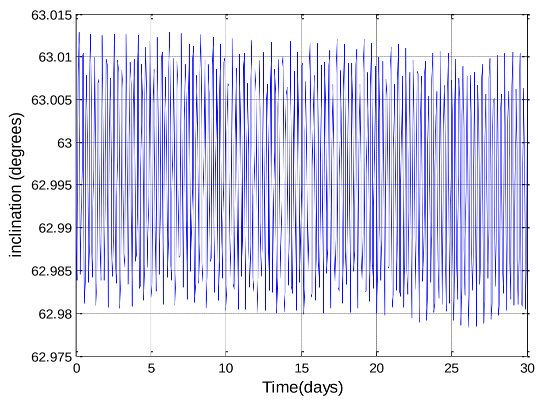
Case 1



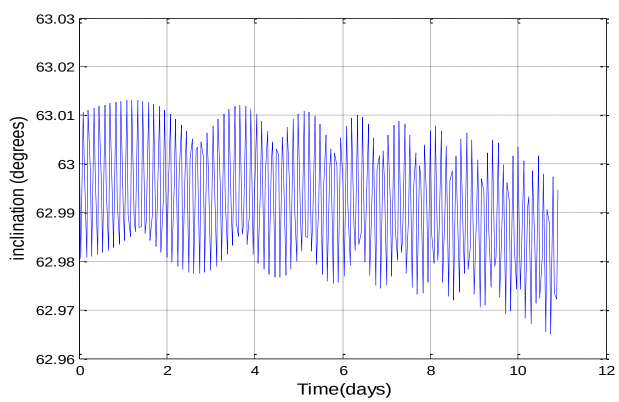
Case 2



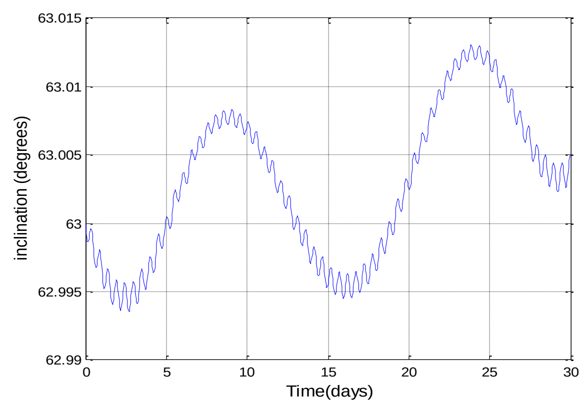
Case 3



Case 4

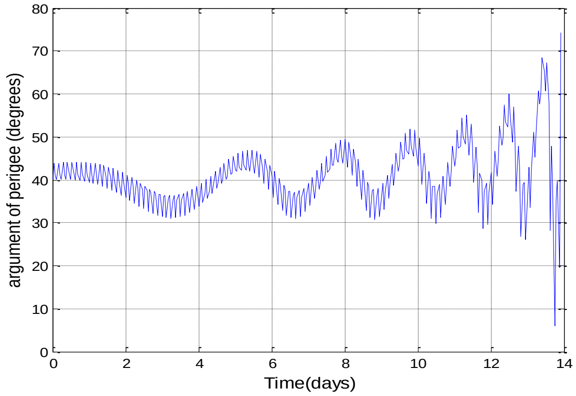


Case 5

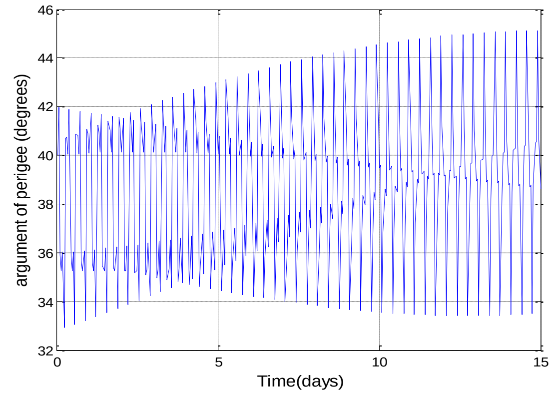


Case 6

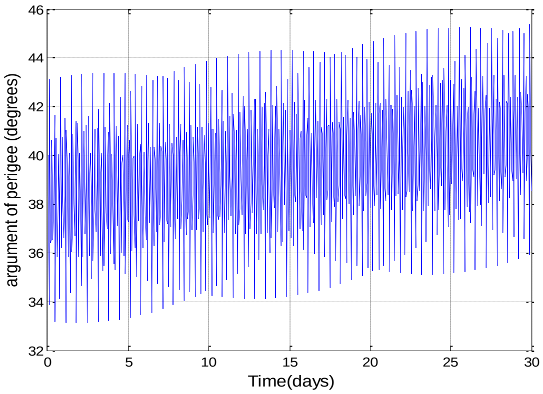
Figure (3): Inclination with different conditions of orbital elements.



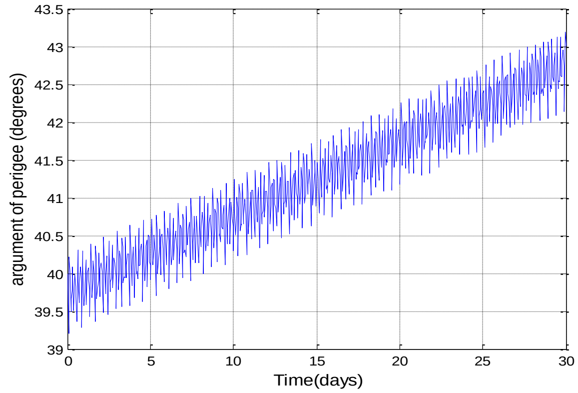
Case 1



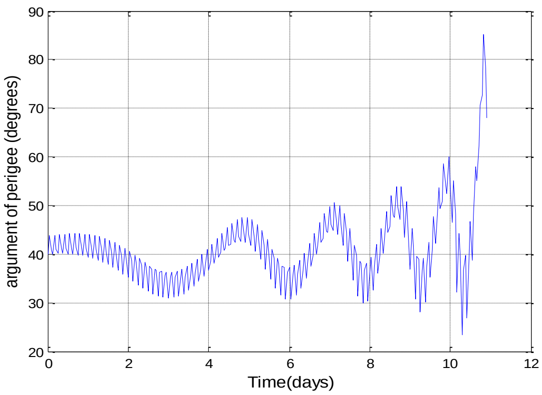
Case 2



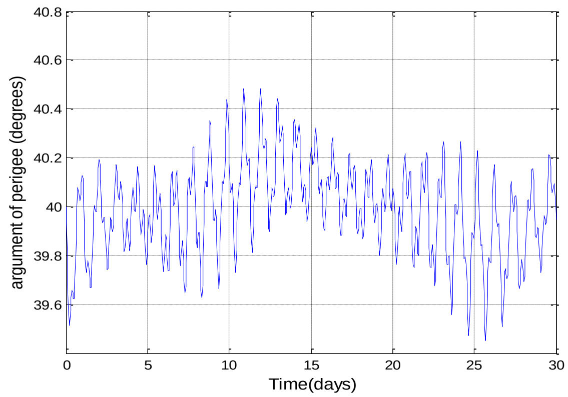
Case 3



Case 4



Case 5



Case 6

Figure (4): Argument of perigee with different conditions of orbital elements.

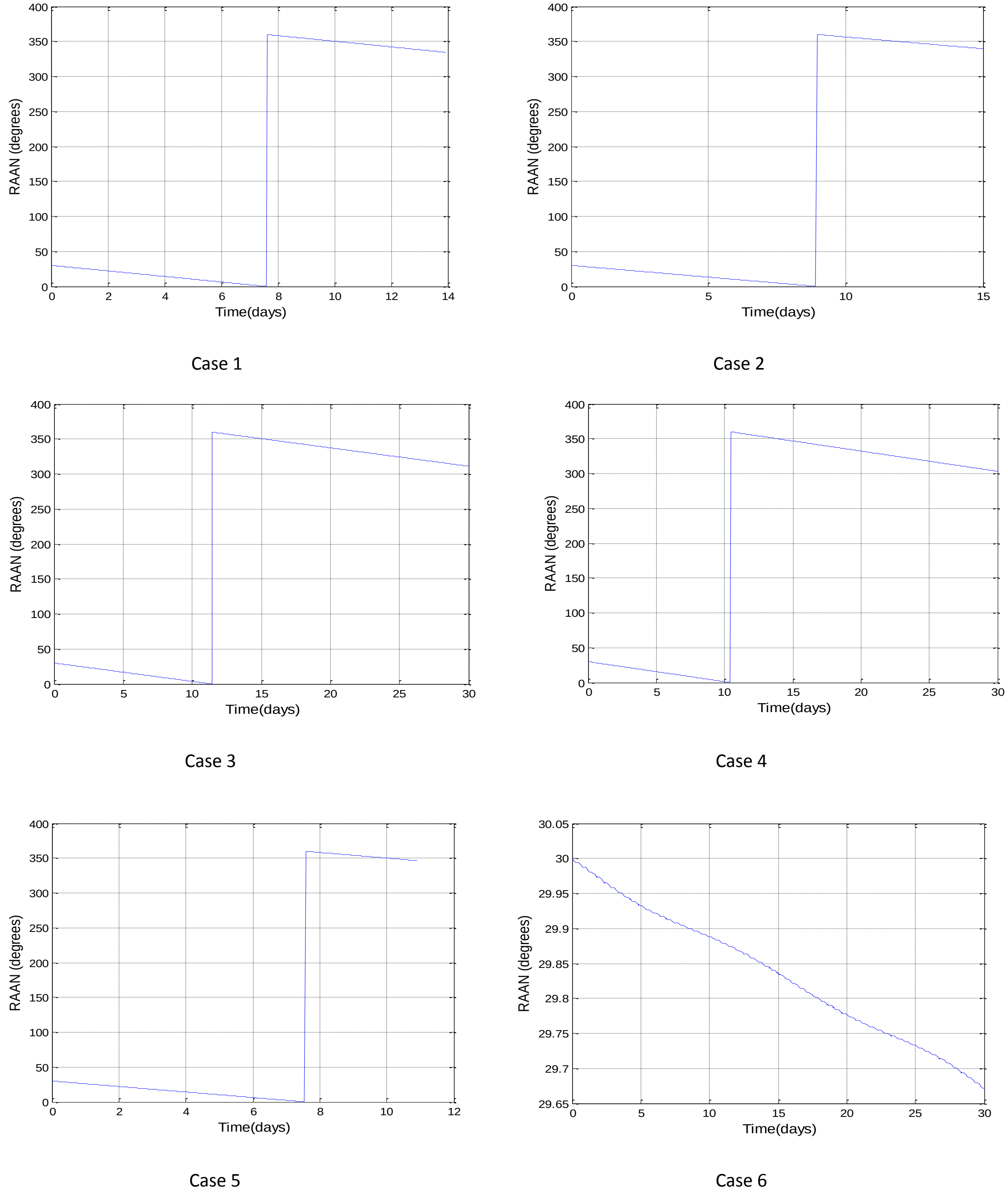
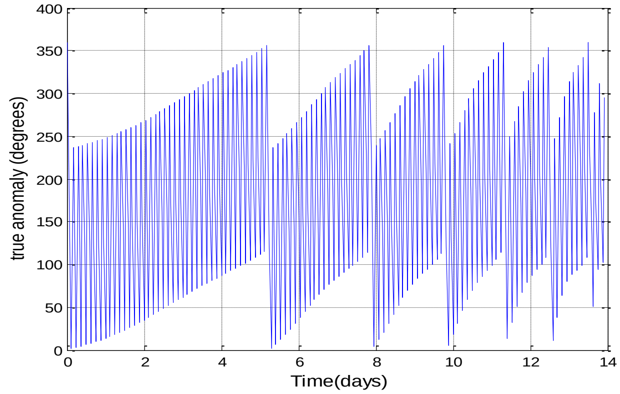
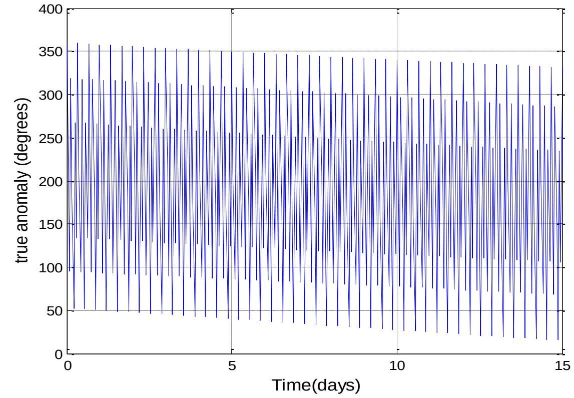


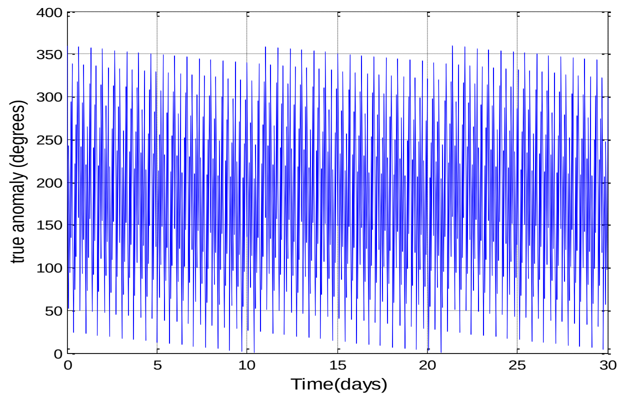
Figure (5): RAAN with different conditions of orbital elements.



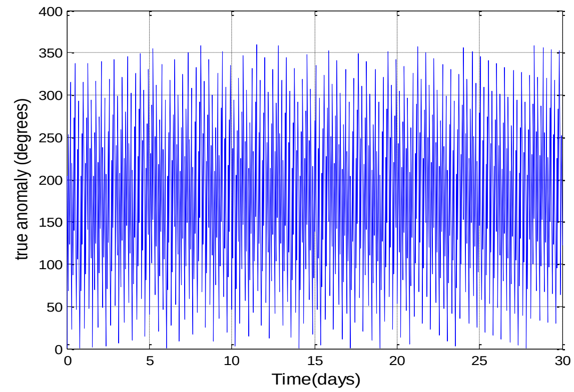
Case 1



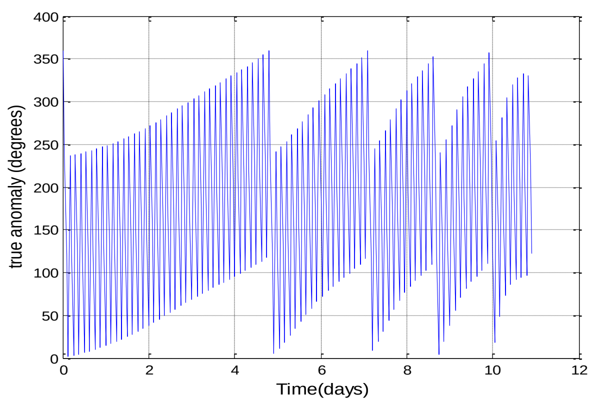
Case 2



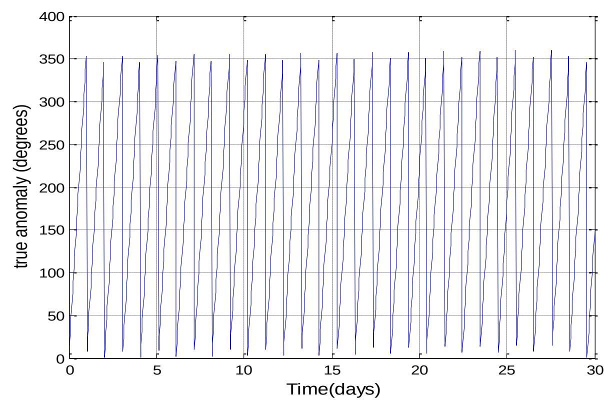
Case 3



Case 4



Case 5



Case 6

Figure (6): True anomaly with different conditions of orbital elements.

5. CONCLUSION.

The results evidently show that the semi major axis is more stable when altitude is 1000 km as compared to other altitudes. When the altitude drops to 200 km the effect by atmospheric drag as well as the semi major axis becomes periodic effected by third body attraction. When altitude reaches about 36000 km, the change in A/m from (0.0052 to 0.0072) reduces the lifetime of satellite about (3days) other parameters remaining constant. The drop of (e) value from (0.1 to 0.01) causes fasting in average change of amplitude when the orbit is elliptical while the amplitude change is less, when the orbit is semicircular.

6. REFERENCES.

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