ON α-g$\tilde{I}$- CLOSED SOFT SETS

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***ABSTRACT**: In this research we will use the notions of soft sets and soft ideals to introduce a new concepts called α-g$\tilde{I}$-closed soft sets where we studied the most important properties of this concepts. The disconnection was studied by using these sets.*

Keywords and phrases: α-g$\tilde{I}$- closed soft, α-g$\tilde{I}$- closed soft disconnection, α-closed soft disconnection, Soft ideals

1. INTRODUCTION:

Soft sets have been studied by Molodtsov [4]. If $\mathcal{X}$ is any set, Ƀ is a set of parameters and ß ⊆ Ƀ. A pair (Ɽ, ß) symbolizes him by $Ɽ_{ß}$ is said a soft set on $\mathcal{X}$, such that Ɽ is a function; Ɽ: Ƀ→ Ƥ ($\mathcal{X}$). The collection of each soft sets on $\mathcal{X}$ symbolizes him by $SS(\mathcal{X})_{Ƀ}$.

The union of soft sets [3], [5] (Ɽ, ß) and (Ҩ, ȥ) on $\mathcal{X}$ denoted by (Ҩ, Ƀ)$\tilde{\sqcup}$ (Ɽ, Ƀ) =

Ɽ (ą) if ą∈ ß − ȥ,
Ҩ (ą) if ą∈ ȥ − ß,
Ɽ (ą) ∪ Ҩ (ą) if ą∈ ß ∩ ȥ.

Shabir and Naz [6] introduce soft topological spaces. Ʈ⊆ $SS(\mathcal{X})_{Ƀ}$ is said to be a soft topology on $\mathcal{X}$ if :

i. $\tilde{X}$, $\tilde{\emptyset}$ ∈Ʈ, such that $\tilde{\emptyset}$(ą) = Ø and $\tilde{X}$ (ą) =$\mathcal{X}$ ∀ ą∈ Ƀ.
ii. The intersection of two soft sets in Ʈ is an in Ʈ also.
iii. The union of number of soft sets in Ʈ is an in Ʈ also.

Every set in Ʈ is called open soft sets, the closed soft set in $\mathcal{X}$ is a complement of open soft set. The closure and interior of soft set is symbolizes by *cl* and *int* resp.,

If $\tilde{I}$⊆ $SS(\mathcal{X})_{Ƀ}$ then $\tilde{I}$ is said to be a soft ideal on $\mathcal{X}$ with the parameters Ƀ then, [1],[2]:

i. If (Ҩ,Ƀ), (Ɽ, Ƀ) ∈ $\tilde{I}$ then, (Ҩ, Ƀ)$\tilde{\sqcup}$ (Ɽ, Ƀ) ∈ $\tilde{I}$,
ii. If (Ɽ, Ƀ) ∈ $\tilde{I}$, and (Ҩ, Ƀ) ⊆ (Ɽ, Ƀ) then, (Ҩ, Ƀ) ∈ $\tilde{I}$.

Any soft set (Ɽ, Ƀ) is called α-soft set if (Ɽ, Ƀ) ⊆ *int*(*cl*(*int*(Ɽ, Ƀ))),the collection of each α-soft set symbolizes by $Ʈ_{\alpha}$ [2],[5].

2. α-g$\tilde{I}$- closed soft set

Definition 2.1: Let ($\mathcal{X}$, Ʈ, Ƀ, $\tilde{I}$) be a soft ideal topological space. A soft subset (Ɽ, Ƀ) of a space ($\mathcal{X}$, Ʈ, Ƀ, $\tilde{I}$) is said to be α-g- closed soft set via ideal symbolizes by α-g$\tilde{I}$-closed soft set if cl_{α} (Ɽ, Ƀ)- (Ҩ,Ƀ)∈$\tilde{I}$ whenever (Ɽ, Ƀ)- (Ҩ, Ƀ) ∈$\tilde{I}$ and (Ҩ,Ƀ) is an α-open soft set. The collection of α-g$\tilde{I}$-closed soft sets is symbolizes by α-g$\tilde{I}$- $CS(\mathcal{X})_{Ƀ}$.

The complement of α-g$\tilde{I}$- closed soft sets is called α-g$\tilde{I}$-open soft sets and the collection of α-g$\tilde{I}$- open soft sets is symbolizes by α-g$\tilde{I}$- $OS(\mathcal{X})_{Ƀ}$.

Remark 2.2: Let ($\mathcal{X}$, Ʈ, Ƀ, $\tilde{I}$) be a soft topological space. Then,

i. Each closed soft is α-g$\tilde{I}$- closed soft set.
ii. Each α-closed soft is α-g$\tilde{I}$- closed soft set.

Example 2.3: Suppose that ($\mathcal{X}$, Ʈ, Ƀ, $\tilde{I}$) is a soft topological space ; $\mathcal{X}$= {â, ê, ô}, Ƀ = {μ_1, μ_2, μ_3}, $\tilde{I}$= {$\tilde{\emptyset}$, F: Ƀ → Ƥ(X) such that F(μ)= {ô} for each μ∈ Ƀ }and Ʈ= {$\tilde{X}$, $\tilde{\emptyset}$, F_1: Ƀ → Ƥ($\mathcal{X}$) such that F_1(μ)= {â} for each μ∈ Ƀ, F_2: Ƀ → Ƥ($\mathcal{X}$) such that F_2(μ)= {ê, ô} for each μ∈ Ƀ}. Then;

A soft set (F, Ƀ); F: Ƀ → Ƥ($\mathcal{X}$) such that F (μ) = {ê} for every μ ∈ Ƀ, is α-g$\tilde{I}$- closed soft not (closed and α-closed) soft set.

Proposition 2.4: The union of any two α-g$\tilde{I}$- closed soft sets is α-g$\tilde{I}$- closed soft set also.

Proof: Let (F_1,Ƀ) and (F_2,Ƀ) are two α-g$\tilde{I}$- closed soft sets and (U,Ƀ) be an α-open soft set such that ((F_1,Ƀ) $\tilde{\sqcup}$ (F_2,Ƀ)) – (U,Ƀ)∈$\tilde{I}$, this implies that:
((F_1, Ƀ) – (U,Ƀ)) $\tilde{\sqsubseteq}$ ((F_1, Ƀ) $\tilde{\sqcup}$ (F_2, Ƀ)) – (U, Ƀ) ∈$\tilde{I}$ and ((F_2, Ƀ) – (U,Ƀ)) $\tilde{\sqsubseteq}$ ((F_1, Ƀ) $\tilde{\sqcup}$ (F_2, Ƀ)) – (U,Ƀ)∈$\tilde{I}$, then cl_{α} (F_1,Ƀ) – (U,Ƀ)∈$\tilde{I}$ and cl_{α} (F_2, Ƀ) – (U,Ƀ)∈$\tilde{I}$, so cl_{α} (F_1, Ƀ) – (U,Ƀ) $\tilde{\sqcup}$ cl_{α} (F_2, Ƀ) – (U,Ƀ)∈$\tilde{I}$. Hence *clα* ((F_1, Ƀ) $\tilde{\sqcup}$ (F_2, Ƀ)) – (U, Ƀ) ∈$\tilde{I}$, therefore (F_1, Ƀ) $\tilde{\sqcup}$ (F_2, Ƀ) is α-g$\tilde{I}$- closed soft set.

Corollary 2.5: The intersection of any two α-g$\tilde{I}$- open soft sets is α-g$\tilde{I}$- open soft set also.

Remark 2.6: The union of any family of α-g$\tilde{I}$- closed soft sets needs not to be α-g$\tilde{I}$- closed soft set.

Example 2.7: : Let ($\mathcal{X}$, Ʈ, Ƀ, $\tilde{I}$) be a soft topological space; $\mathcal{X}$=Ƀ= ℵ (the set of natural numbers), $\tilde{I}$= {$\tilde{\emptyset}$, F_i:Ƀ→ Ƥ(X) such that F_i(e)= W_i for each e ∈Ƀ such that $W_i \subseteq E^+$ (the set of even natural numbers)} and Ʈ = {$\tilde{X}$, $\tilde{\emptyset}$, F_i: Ƀ → Ƥ($\mathcal{X}$) such that F_i(e)= U_i for each e∈ Ƀ, such that ($\mathcal{X}$-U_i) is finite set for all i∈ℵ}= $Ʈ_{\alpha}$ .Then;
{(Q_i, Ƀ): Q_i: Ƀ → Ƥ($\mathcal{X}$) such that Q_i (e) = {2i} for all e∈ Ƀ and i∈ℵ} is a family of α-g$\tilde{I}$- closed soft sets, but $\tilde{\sqcup}$(Q_i, Ƀ) = (Q, Ƀ) such that Q: Ƀ → Ƥ(X), Q (e)= E^+ for each e∈ E not α-g$\tilde{I}$- closed soft set since, there exists α-open soft set (U, Ƀ); U: Ƀ → Ƥ($\mathcal{X}$), U (e)= ℵ-{1} for each e∈ Ƀ such that (Q, Ƀ)- (U, Ƀ) ∈$\tilde{I}$ but, cl_{α} (Q, Ƀ) - (U, Ƀ)= $\tilde{X}$- (U, Ƀ) = (V, Ƀ) ∉ $\tilde{I}$, where V: Ƀ → Ƥ($\mathcal{X}$) such that V (e) = {1} for all e∈ Ƀ.

Proposition 2.8: Any soft set (Ą, Ƀ) of a soft ideal topological space ($\mathcal{X}$, Ʈ, Ƀ, $\tilde{I}$) is α-g$\tilde{I}$-open soft $\Leftrightarrow$ (Ɽ, Ƀ) - int_{α} (Ą, Ƀ) ∈$\tilde{I}$ whenever (Ɽ, Ƀ) - (Ą, Ƀ) ∈$\tilde{I}$ and (Ɽ, E) is α-closed soft set.

Proof: $\Rightarrow$) Let (Ą, Ƀ) be any α-g$\tilde{I}$-open soft and (Ɽ, Ƀ) - (Ą, Ƀ) ∈$\tilde{I}$ where (Ɽ, Ƀ) is an α-closed soft set.
((Ɽ, Ƀ) – (Ą, Ƀ))^c = (Ą, Ƀ)^c - (Ɽ, Ƀ)^c = (Ą^c, Ƀ) - (Ɽ^c, Ƀ) ∈$\tilde{I}$, and since (Ą^c, Ƀ) is α-g$\tilde{I}$- closed soft set, then cl_{α} (Ą^c, Ƀ) - (Ɽ^c, Ƀ) = (int_{α} (Ą^c, Ƀ)^c)^c – (Ɽ^c, Ƀ) = (int_{α} (Ą, Ƀ))^c – (Ɽ^c, Ƀ) = (Ɽ, Ƀ) - int_{α} (Ą, Ƀ) ∈$\tilde{I}$.
$\Leftarrow$) Let (Ɽ, Ƀ) - int_{α} (Ą, Ƀ) ∈$\tilde{I}$ whenever (Ɽ, Ƀ) - (Ą, Ƀ) ∈$\tilde{I}$ and (Ɽ, Ƀ) is an α-closed soft set.

(Ɽ, Ƀ) - int_α (Ą, Ƀ) = (int_α (Ą, Ƀ)) ^{c} - (Ɽ,Ƀ) ^{c} = cl_α (Ą ^{c}, Ƀ) - (Ɽ ^{c}, Ƀ) ∈$\tilde{I}$, whenever (Ɽ, Ƀ) - (Ą, Ƀ) = ((Ą, Ƀ)) ^{c} - (Ɽ, Ƀ) ^{c} = (Ą ^{c}, Ƀ) - (Ɽ ^{c}, Ƀ) ∈$\tilde{I}$ and (Ɽ ^{c}, Ƀ) is an α-open soft set. So (Ą ^{c}, Ƀ) is α-g$\tilde{I}$- closed soft set. Hence (Ą, Ƀ) α-g$\tilde{I}$- open soft set.

Remark 2.9: If for every α-closed soft set (Ɽ, Ƀ) ∈$\tilde{I}$, then α-g$\tilde{I}$- $CS(\mathcal{X})_{Ƀ}$= $SS(\mathcal{X})_{Ƀ}$.

Proposition 2.10: If ($\mathcal{X}$, Ʈ, Ƀ, $\tilde{I}$) is a soft topological space and x_0 ∈X; Ʈ= {$\tilde{X}$,$\tilde{\emptyset}$, U: Ƀ → Ƥ($\mathcal{X}$) such that U (µ)= {x_0} for each µ∈Ƀ}, then :

i. If $\tilde{I}$= {$\tilde{\emptyset}$, V_i : Ƀ → Ƥ(X) such that V_i (µ) = W_i for each µ ∈Ƀ, x_0 ∉ W_i and i∈ ʌ}. Then α-g$\tilde{I}$- $CS(\mathcal{X})_{Ƀ}$= $SS(\mathcal{X})_{Ƀ}$.
ii. If $\tilde{I}$= {$\tilde{\emptyset}$}, then α-g$\tilde{I}$- $CS(\mathcal{X})_{Ƀ}$ = {V_i : Ƀ → Ƥ($\mathcal{X}$) such that V_i (µ) = W_i for each µ ∈ Ƀ, x_0 ∉ W_i and i∈ ʌ}.

Proof:

i. Let (A, Ƀ) be a soft set. Now, either x_0 ∈ A (µ) for some µ ∈Ƀ or x_0 ∉ A (µ) for all µ ∈Ƀ, if x_0 ∈ A (µ) for some µ ∈Ƀ and (A, Ƀ) – (U,Ƀ)∈$\tilde{I}$ such that (U, Ƀ) ∈τ = τ_α then, cl_α (A, Ƀ) – (U,Ƀ) = $\tilde{X} - (U,Ƀ)$∈$\tilde{I}$. If x_0 ∉ A (µ) for all µ ∈Ƀ then, cl_α (A, Ƀ) – (U,Ƀ) = (A, Ƀ)– (U,Ƀ) ∈$\tilde{I}$.
ii. For any soft set (A, Ƀ), if x_0 ∈ A (µ) for some µ ∈Ƀ and (A, Ƀ) – (U,Ƀ)= $\tilde{\emptyset}$ such that (U, Ƀ) ∈ τ_α, this implies (A, Ƀ) $\tilde{\sqsubseteq}$ (U,Ƀ), then cl_α (A, Ƀ) – (U,Ƀ) = $\tilde{X} - (U,Ƀ)$ ∉$\tilde{I}$. If x_0 ∉ A (µ) for all µ ∈Ƀ then, cl_α (A, Ƀ) – (U, Ƀ) = (A, Ƀ) – (U,Ƀ) = $\tilde{\emptyset}$ ∈ $\tilde{I}$.

3. Soft Disconnection Sets.

Definition 3.1: If ($\mathcal{X}$, Ʈ, Ƀ, $\tilde{I}$) is a soft topological spaces and (Ƥ, Ƀ), (Ɽ, Ƀ) are two non-null α-closed soft sets then, (Ƥ, Ƀ) $\tilde{\sqcup}$ (Ɽ, Ƀ) is called α-closed soft disconnection sets in ($\mathcal{X}$, Ʈ, Ƀ, $\tilde{I}$) if and only if (Ƥ, Ƀ) $\tilde{\sqcup}$ (Ɽ, Ƀ) = $\tilde{X}$ and (Ƥ, Ƀ) $\tilde{\sqcap}$ (Ɽ, Ƀ) =$\tilde{\emptyset}$.

Definition 3.2: Let ($\mathcal{X}$, Ʈ, Ƀ, $\tilde{I}$) be a soft topological spaces and (Ƥ, Ƀ), (Ɽ, Ƀ) are two non-null α-g$\tilde{I}$-closed soft sets then, (Ƥ, Ƀ) $\tilde{\sqcup}$ (Ɽ, Ƀ) is called α-g$\tilde{I}$-closed soft disconnection sets in ($\mathcal{X}$, Ʈ, Ƀ, $\tilde{I}$) if and only if (Ƥ, Ƀ) $\tilde{\sqcup}$ (Ɽ, Ƀ) = $\tilde{X}$ and (Ƥ, Ƀ) $\tilde{\sqcap}$ (Ɽ, Ƀ) =$\tilde{\emptyset}$.

Definition 3.3: The soft topological space ($\mathcal{X}$, Ʈ, Ƀ, $\tilde{I}$) is said to be (resp., α-g$\tilde{I}$-closed, α-closed) soft disconnection if and only if there is (Ƥ, Ƀ), (Ɽ, Ƀ) are two (resp., α-g$\tilde{I}$-closed, α-closed) disconnection sets.

Remark 3.4: Every α-closed soft disconnection sets is α-g$\tilde{I}$-closed soft disconnection sets.

Example 3.5: Suppose that ($\mathcal{X}$, Ʈ, Ƀ, $\tilde{I}$) is a soft topological space, where $\mathcal{X}$= {â, ê, ô},Ƀ= {μ_1, μ_2, μ_3}, $\tilde{I}$= $SS(\mathcal{X})_{Ƀ}$ and Ʈ= {$\tilde{X}$, $\tilde{\emptyset}$}= $Ʈ_\alpha$. Then;
The soft sets (Ƥ, Ƀ), (Ɽ, Ƀ) where, {Ƥ: Ƀ → Ƥ($\mathcal{X}$) such that Ƥ (µ) = {â} for every µ ∈ Ƀ} and {Ɽ: Ƀ → Ƥ($\mathcal{X}$) such that Ɽ (µ) = {ê, ô} for every µ ∈ Ƀ} are two α-g$\tilde{I}$-closed soft disconnection sets in ($\mathcal{X}$, Ʈ, Ƀ, $\tilde{I}$) not α-closed soft set.

Definition 3.6: If ($\mathcal{X}$, Ʈ, Ƀ, $\tilde{I}$) is a soft topological spaces, (Ọ, Ƀ) $\tilde{\sqsubseteq}$ $SS(\mathcal{X})_{Ƀ}$ and (Ƥ, Ƀ), (Ɽ, Ƀ) are two non-null (resp., α-g$\tilde{I}$-closed , α-closed) soft sets then, (Ƥ, Ƀ) $\tilde{\sqcup}$ (Ɽ, E) is said to be (resp., α-g$\tilde{I}$-closed , α-closed) soft disconnection sets in (Ọ, Ƀ) if and only if satisfy the following:

1) (Ọ, Ƀ) $\tilde{\sqcap}$ (Ƥ, Ƀ) ≠ $\tilde{\emptyset}$.
1) (Ọ, Ƀ) $\tilde{\sqcap}$ (Ɽ, Ƀ) ≠ $\tilde{\emptyset}$.
2) ((Ọ, Ƀ) $\tilde{\sqcap}$ (Ƥ, Ƀ)) $\tilde{\sqcap}$ ((Ọ, Ƀ) $\tilde{\sqcap}$ (Ɽ, Ƀ)) = $\tilde{\emptyset}$.
3) ((Ọ, Ƀ) $\tilde{\sqcap}$ (Ƥ, Ƀ)) $\tilde{\sqcup}$((Ọ, Ƀ) $\tilde{\sqcap}$ (Ɽ, Ƀ)) = (Ọ, Ƀ).

It's clear that every α-closed soft disconnection sets in (Ọ, Ƀ) is α-g$\tilde{I}$-closed soft disconnection sets not conversely, by example 3.4 if we consider that Ọ (µ) = {â, ê} for every µ ∈ Ƀ.

Definition 3.7: A soft topological space ($\mathcal{X}$, Ʈ, Ƀ, $\tilde{I}$) is called (resp., α-g$\tilde{I}$-closed, α-closed) soft totally disconnection if and only if for every two elements ɤ, ɳ ∈$\mathcal{X}$ there is (Ƥ, Ƀ), (Ɽ, Ƀ) are two (resp., α-g$\tilde{I}$-closed, α-closed) disconnection sets such that ɤ ∈ (Ƥ, Ƀ) and ɳ ∈ (Ɽ, Ƀ).

Proposition 3.8:

i. Each α-closed soft totally disconnection is an α-closed soft disconnection space.
ii. Each α-g$\tilde{I}$ -closed soft totally disconnection is an α-g$\tilde{I}$ -closed soft disconnection space.
iii. Each α-closed soft totally disconnection is an α- g$\tilde{I}$-closed soft totally disconnection space.

Proof: i. and ii.
For any (resp., α-closed, α-g$\tilde{I}$-closed) soft totally disconnection space there is (Ƥ, Ƀ), (Ɽ, Ƀ) are two (resp., α-closed, α-g$\tilde{I}$-closed) disconnection sets both contain an element for each different elements in $\mathcal{X}$. So it's (resp., α-closed, α-g$\tilde{I}$-closed) soft disconnection space.
iii. The proof is clear by using remark 2.2.

Example 3.9:
Suppose that ($\mathcal{X}$, Ʈ, Ƀ, $\tilde{I}$) is a soft topological space; $\mathcal{X}$= {â, ê, ô}, Ƀ = {μ_1, μ_2, μ_3}, $\tilde{I}$= {$\tilde{\emptyset}$ }and Ʈ= {$\tilde{X}$, $\tilde{\emptyset}$, F_1: Ƀ → Ƥ($\mathcal{X}$) such that F_1(µ)= {â} for each µ∈Ƀ, F_2: Ƀ → Ƥ($\mathcal{X}$) such that F_2(µ)= {ê, ô} for each µ∈Ƀ}= $Ʈ_\alpha$.
It's clear that ($\mathcal{X}$, Ʈ, Ƀ, $\tilde{I}$) α- closed soft disconnection space and α-g$\tilde{I}$- $CS(\mathcal{X})_{Ƀ}$ = $SS(X)_{Ƀ}$. So, ($\mathcal{X}$, Ʈ, Ƀ, $\tilde{I}$) is α-g$\tilde{I}$- closed soft totally disconnection space not α-closed soft totally disconnection since there is ê, ô ∈$\mathcal{X}$, but there is no two α-closed sets (Ƥ, Ƀ), (Ɽ, Ƀ) such that ê ∈ (Ƥ, Ƀ) and ô ∈ (Ɽ, Ƀ).

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