A NOVEL S-CURVE ON CAPITAL ASSET PRICING MODEL

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ABSTRACT: *A capital asset pricing model is a theoretical mean-variance equilibrium model of market prices. Traditionally, this linear Sharpe-Lintner-Mossin model has been used to estimate adequate return expected from a certain level of risk consumption. In this paper, the linear model has been extended into a dynamic rate of return on a continuous risk span. The efficient frontier of market risk and return has been used as the basis to develop a novel S-curve model. This S-curve model has been proposed to give a more practical view on the relationship surrounding systematic market risk and expected market return. Dynamically, the rate of increase on return of investment assets has been broadened on its first and second derivatives, instead of static gradients. These observations from calculus interpretations of the S-curve relating to both risk and return have been used to build the model. The S-curve model is an alternative tool, offering realistic and improved forecasts on the expected returns from an investment fund portfolio, which in turn significantly aids investment making decisions.*

Keywords: S-curve Model, Capital Asset Pricing Model, Risk and Return, Efficient Frontier, Risky Asset

1. INTRODUCTION

A capital asset pricing model has been used in many empirical research studies in relation to investment on market funds and equity portfolios. It is a theoretical mean-variance equilibrium model of market prices, that is, it is a methodology to determine a rate of risk for future cash flow. The risk is measured by the standard deviation which is given in order to be able to determine the appropriate required rate of return for an investment. Thus, all variations in equity returns depend on standard deviation or risk only.

The concept of risk-free assets and the Keynesian model [1] were proposed by Tobin [2]. Sharpe continued the study by developing one of the foundation models in finance, namely the capital asset pricing model (CAPM), together with Lintner and Mossin [3-5]. The CAPM model describes the linear relationship between expected returns, and the systematic risk of market shares. Lintner [4] also developed various significant equilibrium properties within the risk asset portfolio. The nature of capital market equilibrium was explored by Black in deriving the capital asset pricing model and comparing two differing assumptions related to riskless asset. He found that in all cases, the expected return of the risky asset is a linear function of its risk [6]. Consequently, the Intertemporal Capital Asset Pricing Model (ICAPM) was established by Merton based on the CAPM equilibrium model. His study made a few assumptions to improve on the previous model [7].

The CAPM has continued to be the model utilized by many academic scholars and practitioners. Fama & French in their studies has identified a further two factors which influenced the market stock returns, namely the size of a firm, and a firm’s book-to-market value or equity [8]. Since then, many researchers have used both CAPM and Fama-French models as comparisons in their research. Bakshi et al. introduced the SVSI-J model (Stochastic-Volatility Stochastic-Interest Rate – Random Jumps), a model used to enhance the performance in pricing for short-term choice and upgrade the appropriate long-term portfolio selections [9].

Established by Sharpe-Lintner-Mossin and underpinned by the Fama-French model, the evolution of the CAPM has been extensive. Consumption-based models of stock returns have been proposed to capture key features of risk premiums in stock markets. However, many models failed to function. As a result, Nagel and Singleton constructed an optimal 'Generalized Method of Moments' (GMM) estimator, a generic method for estimating parameters in statistical models [10]. Blanco used a selection of portfolios for the North American Market, working with monthly returns from July 1926 to January 2006. His study used both the CAPM and Fama-French models. Blanco's empirical results showed that the Fama-French model outperformed the CAPM [11]. Similarly, a study carried out by Rossi on the Italian Stock Exchange found a firm’s size has a substantial effect on the markets stock returns [12].

Further improvements on the pricing model were made by Rufino who developed a model called the 'Robust Capital Asset Pricing Model' (RCAPM). The model gauged the 'uncertainty-return trade-off' in relation to asset returns [13]. Colacito and Croce proposed a new predictability test which then developed a novel recursive general equilibrium model for future global finance scenarios. This risk-sharing mechanism and their recursive preference-driven theory was supported by US and UK data [14]. Kan et al. studied the 'Two-Pass Cross-Sectional Regression' (CSR) R2 to measure the asset pricing models’ performance. The study found that ICAPM was the best overall performer, closely followed by the Fama-French ‘Three-Factor Model’ [15].

In real life, there are a lot of phenomenon or scenarios that need to be predicted or modelled for a better analysis and better predictive. For many years, pricing models used in finance studies have shown a positive linear relationship. However, previous research undertaken by Hwang et al. has shown the existence of a non-linear risk-return relationship in a traditional CAPM [16]. S-curve shaped is one of the examples of non-linear curves that is hoped to offer a better prediction or estimate with respect to time realistically. Thus, this paper proposes a new technique called an S-curve model on CAPM which offers realistic forecasts on the expected returns from an investment fund portfolio in order to help investment decision making.

2. EXPERIMENTAL DETAILS

Statistically, the risk of an investment asset is commonly measured by its standard deviation of return. Given a fixed dividend for a given year, the capital gain variations are left to the fluctuation of share prices. The standard deviation of the share prices within the year is taken as the risk of investment return on the particular share.

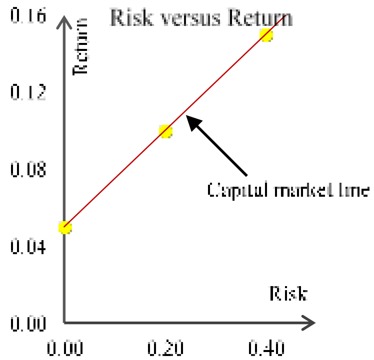

Fig (1) The market risk versus market return

A simple linear regression is the result of this equilibrium model portfolio, namely the Capital Asset Pricing Model (CAPM) as shown in Fig (1). Let (σ_i, r_i) be a point of risk and return on a portfolio *i*. A basic CAPM will be given as $r_i = \beta_0 + \beta_1\sigma_i + \varepsilon_i$ for 2 coefficient parameters β_0 and β_1. A rational mutual fund manager will form a portfolio *i* and have a point (σ_i, r_i) lies on the capital market line $r_i = r_f + \frac{r_M - r_f}{\sigma_M}\sigma_i$ where $\beta_0 = r_f$ and $\beta_1 = \frac{r_M - r_f}{\sigma_M}$. The expected return of an efficient portfolio $E\{r_i\} = R_i$ based on the risk, in terms of its standard deviation σ_i should ideally be on the regression line. The slope of the regression line represents an increase in the change of expected return r_i per unit increase in risk σ_i being consumed.

For a given portfolio *i*, $\sigma_i^{\,2}$ gives the variance associated to its price fluctuation or variation from its expected rate of return, independent from the general market portfolio *M*. If a portfolio *i* is uncorrelated to the market portfolio *M*, then the covariance $\sigma_{M,i}$ should be zero. A portfolio *i* that is highly related to the general market, such as blue-chip stocks, index related counters and active counters, is expected to have high variation $\sigma_i^{\,2}$. In an efficient stock market, a portfolio *i* is not rewarded by higher expected rates of return r_i for taking an extra risk that can be diversified away.

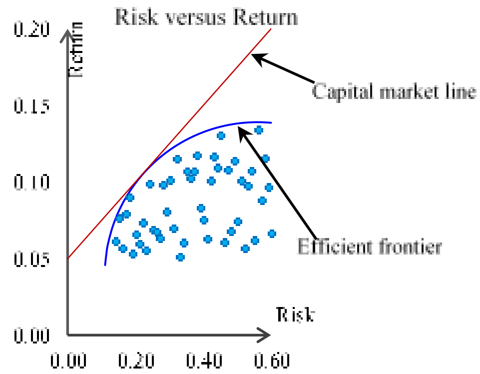

Fig (2) An efficient frontier of a risky asset

To build the CAPM, there is a need to investigate how various correlations among asset returns affect the trade-off between risk and return [17]. While risk is measured by the standard deviation which does not sum up linearly as the expected return of investment, the portfolio risk lowers without sacrificing the expected return. Using optimisation techniques, an efficient frontier can be computed at each level of expected return, on a combination of asset portfolios, to obtain the lowest risk as shown in Fig (2).

3. RESULTS AND DISCUSSION

In this ideal CAPM, there will be the same rate of increase in return *r* for a unit increase in the risk σ along the line which is represented by the first derivative:

$$r' = \frac{dr}{d\sigma} \qquad (1)$$

The composition of a mutual fund portfolio depends on the target investor's risk tolerance. In particular, there are more investors want to invest below the market risk σ_M which will in effect give higher rate of increase in the first derivation. This effect will eventually give a positive second derivative $r'' = \frac{d^2r}{d\sigma^2} > 0$ below market risk when $0 < \sigma_i < \sigma_M$, due to the risk averse nature of large corporations. It is prudent for large corporations to invest in a portfolio at a lower than market risk. At the same time, there are a smaller number of corporations who are willing to invest in a portfolio above market risk σ_M , which will in effect give a negative second derivative $r'' = \frac{d^2r}{d\sigma^2} < 0$ above the market risk when $\sigma_i > \sigma_M$, due to lower competition in terms of supply and demand.

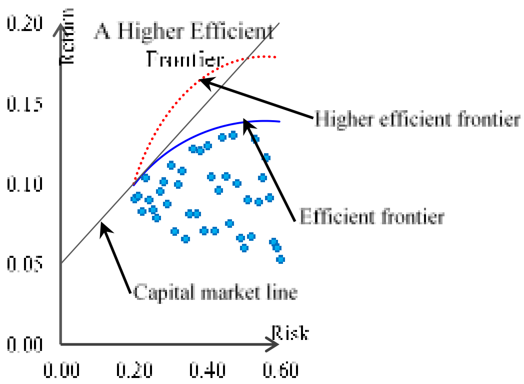

Fig (3) An efficient frontier will be naturally higher than the market line

It should be observed, that the typical efficient frontier of a risky asset has a negative second derivative on the right of market risk domain, as shown in Fig (3). In developing the S-efficient frontier curve, the following four assumptions will be considered.

1. Investors are risk averse and evaluate their investment portfolio solely in terms of its expected return and standard deviation over a short-term period.
2. Capital markets are relatively perfect and every investment asset is infinitely divisible. There is minimum cost of transaction, information gathering and taxes.
3. Every mutual fund manager has access to the same market investment opportunities and new investment opportunities arising from technological advancement.
4. Every mutual fund manager makes an investment analysis on the same market information explicitly on the expected return and risk involved. However, every fund manager chooses their investment portfolio based on his or her own risk tolerance.

These assumptions represent an ideal efficient market condition. A mutual fund manager will make an investment choice at a calculated risk above market risk for the highest instantaneous derivative r' as shown in Fig (3). Fund managers will push for the highest efficient frontier to go above the traditional linear CAPM line. At the same time, his or her risk tolerance has a limit depending on the investment portfolio he or she is managing. It is claimed that there will be a smaller number of investment portfolios as the risk gets higher beyond market risk σ_M . An increase in the rate of return will have a lower demand which imply the second derivative $r''(\sigma) < 0$ in concaving downward motion.

For the market to be in equilibrium, the efficient frontier curve will be pushed for higher returns on lower risk consumption as shown in Fig (3). Having the current market data on the risk and return model, any new investment portfolio must perform better than current high risk market performance in order to gain financial support from the investor venturing into it [18].

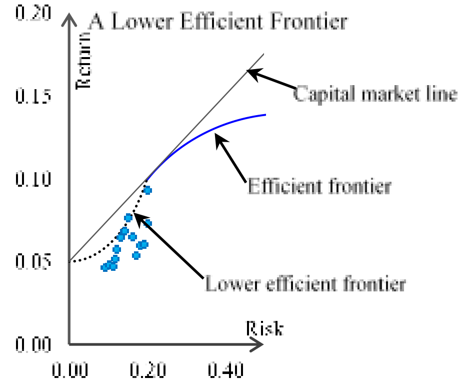

Fig (4) An efficient frontier being pushed below the linear CAPM

There are many low-risk assets which are perceived as risk free, one example is the Fixed Deposit (FD). Most investors can be considered depositors who do not pay attention to the return on investment, meaning, there is no pressure on the bank to increase the FD rates.

The lower efficient frontier is being pushed down along the market risk σ_M and moving up to the highest instantaneous derivative r' at the market risk σ_M itself, as shown in Fig (4). The lower efficient frontier will go below the traditional linear CAPM line. It is claimed that there will be a higher number of investment portfolios as the risk becomes higher, nearing market risk σ_M .

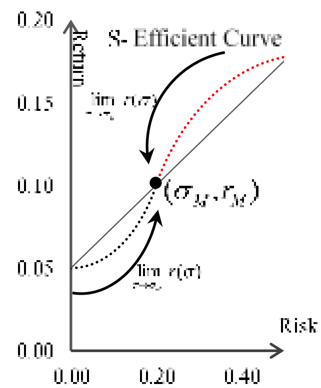

Fig (5) A continuously differentiable S-curve at the market risk and return point

In this S-curve model, while an increase in the investment risk will expect a higher rate of return $r'(\sigma) > 0$, an increase in the rate of return should also be positive in a concaving upward motion, that is $r''(\sigma) > 0$.

Normally, the return of investment at near market risk from the left should move dynamically toward the market rate of return r_M being slightly higher than the market risk from the right, as shown in Fig (5).

$$\lim_{\sigma \to \sigma_M^-} r(\sigma) = \lim_{\sigma \to \sigma_M^+} r(\sigma) = r_M \qquad (2)$$

Ideally, the rate of increase in the return of investment at near market risk approaching from the left, will also be equal to the rate of increase in the market return approaching market risk from the right.

$$\lim_{\sigma \to \sigma_M^-} r'(\sigma) = \lim_{\sigma \to \sigma_M^+} r'(\sigma) = r'_M \quad (3)$$

At the market risk and return point (σ_M, r_M), the first derivative $r'(\sigma_M)$ should be highest.

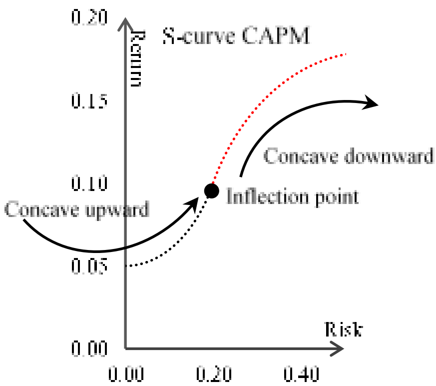

Fig (6) A newly proposed S-curve CAPM

Consequently, even though the Sharpe Ratio of any investment does not get higher than the Sharpe Ratio of the market portfolio, a mutual fund is expected to gain higher than market return r_M . Practically, the alternative derivative r' will move higher than the traditional Sharpe Ratio at slightly higher than market risk σ_M .

A new mutual fund manager, will increase his or her risk appetite for investment ventures based on new technological advancements. In the proposed S-curve model shown in Fig (6), higher risk σ_i^2 is expected to give a higher rate of return, simply for taking an extra risk near the market portfolio. Consequently, the highest rate of increase in the rate of return given by the derivative $r_i' = \frac{dr_i}{d\sigma_i}$ should be allocated near the market portfolio *M*.

There is no point buying a high-risk portfolio class, such as Top 20 active stocks, if they are not poised to outperform the underlying low risk portfolio class of non-active stocks. The rate of increase is the rate of return r_i' which will slow down as portfolio r_i moves away from the centre point (σ_M, r_M) . At a lower than market risk $\sigma_{M-\delta}$ on the left, the rate of return will concave upward, while at higher than market risk $\sigma_{M+\delta}$ on the right, the rate of return will concave downward. Thus, the highest changes in the portfolio performance can be observed within the neighbourhood of market portfolio. Ideally, the second derivative $r_i'' = \frac{d^2 r_i}{d\sigma_i^2}$ is zero at the centre point (σ_M, r_M), where an inflection point occurs during the changes of concavity. However, the inflection point may occur slightly to the right of market risk $\sigma_{M+\delta}$ for several small values of δ . For a small positive value δ, the stock market can be said to be bullish in condition, while a negative value of δ would indicate a bearish situation. The stock market pays to take extra risks above market risk during a bullish period, as indicated by a positive δ .

The proposed S-curve has several important implications.

1. Presuming the market is in equilibrium, traditional investment assets must be on the linear CAPM line. However, a new venture will improve upon an investment portfolio, by taking additional risk from average market risk. Improvements are made by injecting an extra niche and therefore, pushing for a higher efficient frontier curve.
2. Under the linear CAPM estimates on the future cash flow of an investment asset, or large corporation are simplified. The S-curve CAPM expects improvement and revision on the cash flow of an investment asset.
3. The S-curve CAPM embarks on non-linear asset pricing and an investor's risk appetite. The S-curve attempts to predict the behavior of future investors and broadens our understanding of the capital market phenomena.
4. Direct application of the S-curve CAPM measures the performance of mutual fund managers. A competitive fund is actively managed to outperform the market.

Suppose funds *A* and *B* take risks at 10% and 30% respectively, while obtaining returns at 7.5% and 12.5% respectively, as shown in Fig (7) and Table 1.

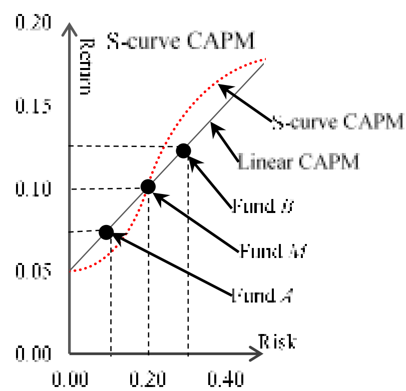

Fig (7) Risk versus return on linear CAPM and S-curve CAPM

Table 1 Example of riskless investment and its expected return on linear CAPM as in Fig (7)

Fund *i*	Risk σ_i	Return r_i
A	$\sigma_A = 10\%$	$r_A = 7.5\%$
M	$\sigma_M = 20\%$	$r_M = 10\%$
B	$\sigma_B = 30\%$	$r_B = 12.5\%$

Under the linear CAPM, funds *A* and *B* are considered performing well as expected. However, under the proposed S-curve CAPM, fund *A* is performing better than fund *B*. This new non-linear S-curve model demands a much higher return from fund B, while giving a much slacker to fund *A*.

On the lower risk below market risk σ_M , risk-free assets such as Treasury Notes, government bonds, and government papers, known as 'Sukuk' in Malaysia [19] are being looked at, as well as fixed deposits. In Malaysia, even though the fixed deposit is covered under the Malaysia Insurance Deposit Corporation, there is still some risk attached to bank closure [20-21]. The rate of return should be higher than the risk-free rate. However, the first derivative will be pushed down lower than Sharpe Ratio under linear CAPM. For instance, in Malaysia the risk free rate should typically be referred to as the base lending rate (BLR), which is controlled by the local central bank, namely the (BNM) Bank Negara Malaysia [22]. At the same time, the next riskless rate should be the fixed deposit rate at the local anchor banks.

4. CONCLUSION

The S-shaped curve pattern gives significant meaning on what is happening in the real situation. The S-curve model has the potential to be a model in the world of prediction or forecasting the real scenario on CAPM. The intention of this proposal is to provide an alternative dynamic view on the market return investment for a systematic market risk of a risky asset. Thus, the S-curve model can be used to give a more realistic forecast value on the return investment of asset portfolios and equity funds.

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