

OPTIMAL DG PLACEMENT AND SIZING IN DISTRIBUTION NETWORK WITH RECONFIGURATION

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ABSTRACT: Nowadays there is an increasing demand for using DG in low and medium voltage rates and the need for accurate coordination and planning grows every day. With increasing of consumer's distance in distribution networks, losses increases and voltage profile decreases. DG is a proper solution to solve and improve this issue in the network. The present article works on the issue of optimal DG placement and sizing in the distribution network with the possibility of reconfiguration with time-varying loads and DG output power. To optimize in a 33 bus IEEE network, genetic algorithm and harmony search algorithm have been used. The study of results shows that if DG is connected to the optimal sizing and in the optimal place in a network with proper structure, voltage profile is improved and also, at the same time, losses in the network decrease and the harmony search algorithm goes toward the answer with higher speed and accuracy, compared to the genetic algorithm.

Key Words: Optimal placement, distribution network, genetic algorithm, harmony search algorithm, losses, voltage profile, reconfiguration

1. INTRODUCTION

Using distributed generation (DG) in distribution network brings along advantages for the producing companies and the consumers, some of which are decrease in lines losses, voltage profile improvement, network security progress, power quality improvement, increasing of distribution and transmission systems capacity and delaying investment for network development [1].

The above mentioned advantages brings along an increasing use of DG in distribution networks, so problems complexity increases in designing and developing networks. Distribution networks are made as loop and radial and with open switch or close switch. Switches determine the path of energy in different hours toward the consumers at the end of the line by opening and closing. So, due to dispersed production sources and reconfiguration switches in the distribution network, the need for a reconsideration of classic methods of problem solving in the network arise [2].

Many studies have been done so far on DG placement and reconfiguration in distribution networks, and various methods and ideas to reach different goals have emerged. Decrease of losses, increase of capability of security and voltage profile, placement of FACTS and cost are among the goals set for networks with DG. Wang *et al* [3] have presented an analytic method for optimal placement for DG in radial networks to minimize the losses. Farashbashi *et al* [4] presented the function of proper aim to improve voltage profile and decrease of losses in lines and supporting the voltage in subtle points. Borges *et al* [5] have set security capability, voltage profile and losses as goals. Mori *et al* [6] have stated optimal placement of FACTS in radial distribution networks. Phonrattanasak *et al* [7] have stated the issue of locating DG units in feeders as multi objective optimization.

DG units placement and sizing problem in networks is a multi objective function. To solve the problem of optimization in distribution networks different algorithms

have been used, such as genetic algorithms [8], simulated annealing [9], tabu search [6], particle swarm optimization [10], differential [11], harmony search [12]. Many studies have been done by genetic algorithm to optimize DG placement, finding optimal DG sizing and finding the proper structure of distribution network. Haesen *et al* [8] have studied the effect of DG presence on voltage profile at different load levels using the genetic algorithm. Sing *et al* [13] have presented optimal DG units placement and sizing with the aim of decreasing losses in the network with changing loads using the genetic algorithm. Gareth *et al* [14] have examined scattered generation resources placement by optimal load distribution and genetic algorithm. Pisica *et al* [15] have presented optimal DG placement in distribution networks with the aim of decrease of losses using genetic algorithm.

In many studies harmony search algorithm has been used for optimization in power networks. Kazemi *et al* [12] have stated harmony search algorithm with the aim of improvement in power network security and have used active and reactive power controllers to improve voltage profile and have compared the results of optimizing using genetic algorithm and confirmed this algorithm's use. Coelho [16] stated harmony search algorithm improvement in optimal load distribution. Verma *et al* [17] have used the harmony search algorithm to schedule transportation network development and compared this algorithm with genetic and bacteria and confirmed this method's potentials.

Placing DG on the side of the load or along the feeder affects the power passing the lines and buses voltage. This effect can be along with, or against network condition improvement. So it is necessary to examine the effect of DG on the voltage profile before placing it. The study of the effect

of DG on the network and the voltage of each bus is possible through an index defined as voltage profile improvement (VQI). As the DG nominal power increases in a bus, voltage.

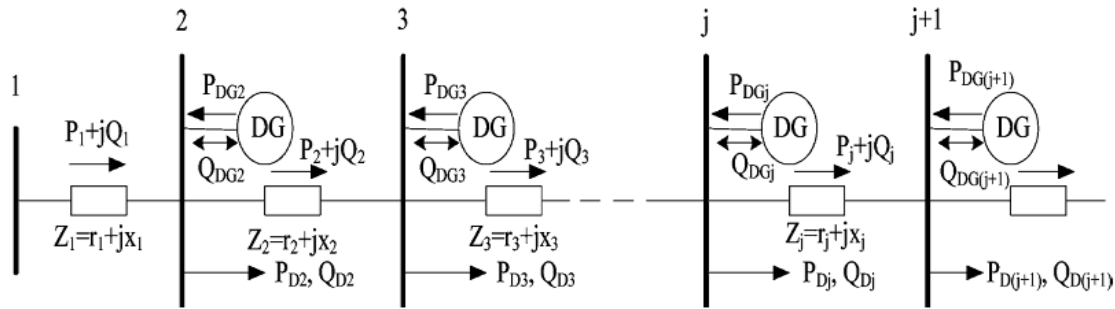


Figure 1. A simple network model

profile and losses change in the network and different powers become the maximum or minimum

This article focuses on a way to calculate the optimal DG placement and sizing in radial networks with switching ability. A network with switching ability consists of on and off switches in network's branches, in a way that the network radial structure is kept preserved. The method suggested, is used to improve voltage profile, network losses and finding the proper situation of the switches. The results of simulating in the test network confirm this method's usability.

To estimate the advantages of this method, the IEEE 33 bus distribution network is considered. Also, to calculate the accuracy of the method, DG optimal place and sizing have been calculated in six scenarios and the voltage profile state and network losses have been compared to the state of load distribution in absence of DG and reconfiguration. The results of the calculations are presented too.

The article structure is as follows: section II defines primary concepts, the objective function, the function's parts and the statement of restrictions. In section III optimization method is suggested. In section IV analysis of test radial network is stated to show load distribution optimization operation and calculation of losses and profile improvement index, and ultimately in section V the results are presented.

2. PROBLEM FORMULATION

The main goal of this article is finding the optimal DG place and size and proper structuring of distribution network to minimize the losses and network's voltage profile improvement.

2.1. Voltage Profile Improvement (VQI)

Voltage profile is one of the main indexes and factors to introduce scattered generations resources. One of the goals of using DG systems is voltage profile improvement and keeping the buses in the acceptable area. Since DG can support a part of active and reactive power to the load, it can also improve the voltage profile by dropping the current and consequently increasing the rate of voltage in the distributing system.

Voltage profile for node i is defined as [2]:

$$VO_i = \frac{(V_i - V_{min})(V_{max} - V_i)}{(V_{nom} - V_{min})(V_{max} - V_{nom})} \tag{1}$$

Voltage in bus_i is in pu and V_{nom} shows nominal voltage in each bus which is usually 1. V_{max} shows the maximum allowable voltage in each bus which is considered nearly 1.05pu. V_{min} is the minimum voltage in each bus which is 0.95 pu. Now if we multiply VQI of each bus by the same bus weight factor, it will equal the voltage profile index:

$$VQI = \sum_{i=1}^N VQ_i * K_i \tag{2}$$

In which K_i is the weight-giving factor for the bus_i .

$$\sum_{i=1}^N K_i = 1 \tag{3}$$

Now if all buses weight the same it can be said that:

$$K_1 = K_2 = \dots = K_N = \frac{1}{N} \tag{4}$$

So the relationship of voltage profile index can be rewritten as [2]:

$$VQI = \frac{1}{N} \sum_{i=1}^N VQ_i \tag{5}$$

2.2. Loss Reduction

In this article lines current values are calculated in each section using distribution load flow. The lost active power is defined as [3]:

$$P_{loss}^{i,j} = R_{i,j} * conj(I_{i,j}) \tag{7}$$

The total loss is defined as:

$$P_{loss} = \sum_{i=1}^N \sum_{j=i}^N P_{loss}^{i,j} \tag{8}$$

2.3. Reconfiguration in the Distribution Network

In reconfiguring the network, operational constraints such as network's radial structure, all loads being fed and voltage drop constraints should be observed, otherwise the developed structure will not be acceptable. In this article different acceptable structures defined in the system are studied and the optimal structure is distinguished. Each two complementing structures in a reconfiguration in the network are defined as a variable and the proper structure for DG place and size is calculated. Each structure consists of two switches; open-switch and close-switch. Optimal changes of this switches, improved the voltage profile of buses and reduce the total losses in the network.

2.4. Network Constraints

Network constraints are defined as below:

$$V_{\min} < V_i < V_{\max} \quad i \in N_N \quad 9$$

$$0 < I_j < I_{j\max} \quad j \in N_L \quad 10$$

$$S_{DG_i} < S_{DG_{\max}} \quad i \in N_N \quad 11$$

In which V_i is the voltage of bus_i , V_{\max} , V_{\min} the size of voltage area in the network, and N_N the number of the buses. I and I_{\max} are the current and the maximum current allowed to pass through the $branch_j$. N_i is the number of the branches. S_{DG_i} and $S_{DG_{\max}}$ are the output DG in each hour and the highest DG size capable of being placed at the level of network's voltage [2].

Also, three constraints must be applied in reconfiguration:

- The network topology must permanently be radial.
- The configuration process cannot work in systems with islanding.
- All loads must be fed.

3. SOLUTION METHOD

Figure 1 shows the proposed stochastic simulation process to solve the problem. Two algorithms are presented to solve the problem in this figure:

3.1. Genetic Algorithm

Genetic algorithm enjoyed Darwin's Theory of evolution in survival and natural evolution process of generation reform to find the proper solution to a problem. Genetic algorithm starts without knowing the right solution and completely relates to the answer through the environment and evolution process like reproduction, gene crossover and gene mutation, to get to the best solution [18]. These steps for optimization using genetic algorithm are shown below:

Step 1: Random generation of primary population strings.

Step 2: Calculation of fitness value for each string in the population and choosing the fittest members from among the chosen population.

Step 3: Reproduction using a probabilistic method

Step 4: Creating a pool after choosing

Step 5: Creating offspring by crossover and mutation operation.

Step 6: Assessment of the offspring and estimation of the fitness value for each of the solutions.

Step 7: If the search goal is achieved, or an allowable generation is attained, we can return the best chromosome as the solution; otherwise we should go back to step 4 [18].

Here, the probabilistic method "roulette wheel" is used due to its being simple and also enjoying crossover method with a probability of 0.8 out of 1. Probability of mutation is between 0.5 and 1 and the maximum of recurrence in the group is adjusted on 20, when the population size is 20.

3.2. Harmony Search Algorithm

Recently, Jay et al. [19] suggested the meta-heuristic new

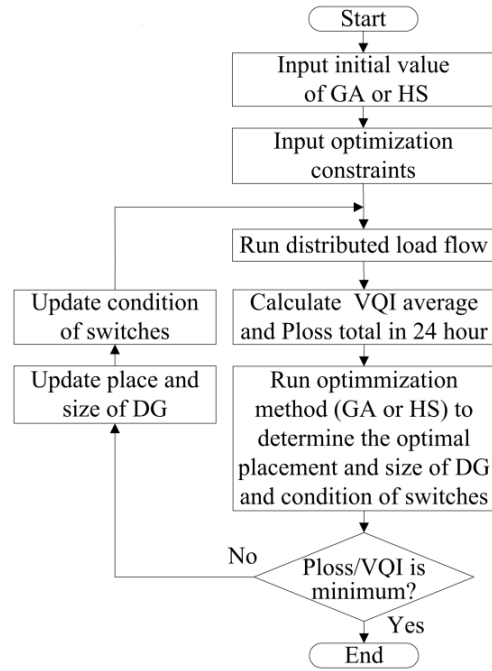


Figure 2. Simulation optimization process

HS algorithm which is designed using the search processes in music to get to the best harmony to solve optimization. In optimization methods harmony in music is comparable to the vector of optimization solution, and music improvement to global and local search program [20]. HS algorithm doesn't need initial values for the variables. Furthermore, instead of searching the gradient, HS algorithm operates using a random search based on the rate of attention to the harmony's memory and step adjustment.

In HS algorithm players of the music look for the best kind of harmony from the aesthetic viewpoint, and optimization algorithms also look for the best case by extracting the value of the objective function. This algorithm has successfully answered optimization problems in different applied areas of calculations and engineering [16].

Optimization using the HS algorithm is as follows:

Step 1: entering the initial value in the optimization problem and the algorithm parameters

Step 2: entering the initial value to the harmony memory (HM)

Step 3: Improvisation of the new harmony from HM

Step 4: updating HM

Step 5: repetition of steps 3 and 4 until the terminating scales are satisfied [16].

Algorithm parameters are as follows:

BW parameter is the band width of the variables and is defined as:

$$BW_{k+1} = BW_{\max} - rand_k * (BW_{\max} - BW_{\min}) \quad 12$$

If a new harmony doesn't have a better result, the value will be considered as:

$$BW_{k+1} = BW_k$$

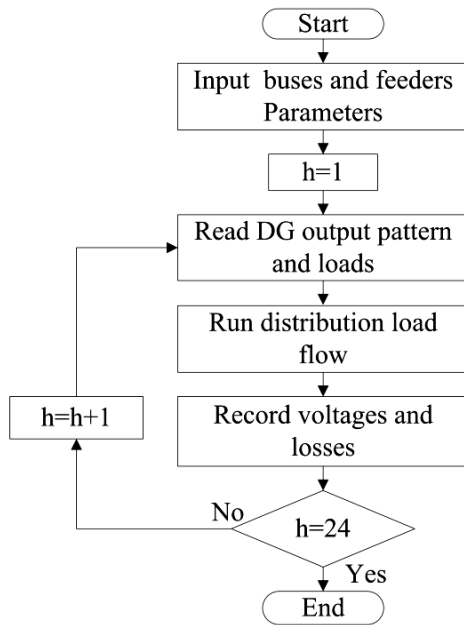


Figure 3. Load flow simulation process

$Rand_k$ is a number between 0-1 and BW_{max}, BW_{min} are fixed values.

Par_k is the adjustment fee for each step of X_{new} and is defined as:

$$PAR_k = \begin{cases} \frac{f_{max}^k - f_{min}^k}{f_{max}^{k-1} - f_{min}^{k-1}} & \text{if } 0 < \frac{f_{max}^k - f_{min}^k}{f_{max}^{k-1} - f_{min}^{k-1}} < 1 \\ 1 & \text{other} \end{cases} \quad 13$$

f_{min}^k is the minimum value of the objective function from among all vectors of the harmony answer in repetition of k , and f_{max}^k is the maximum value of the objective function from among all vectors of the harmony answer [21]. Here, we chose

$$PAR_{max} = 1, \quad PAR_{min} = 0.3, \\ BW_{max} = 0.8, \quad BW_{min} = e^{-20}, \quad HM = 7$$

In the process of optimization shown in figure 2, the initial values and the constraints of optimization are requested. In each step, the load flow has been run and the values of the voltage profile for each bus and the losses of each feeder has been determined. Then the VQI average and the total losses in the network are calculated and recorded. After that GA or HS optimization are run, then the location and the capacity of the DGs and the condition of each switch is calculated. The load flow with the new values of the parameters is run again and. After some iteration the best value of the objective function will be calculated. Figure 3 shows the distribution load flow procedure with time-varying loads and DG outputs. Table 1 presents the daily loads and the daily wind turbine power output patterns. The Load flow is run for each hour and the voltage and the losses in the network will be recorded. For each iteration, the load flow is run for 24 times.

TABLE 1. LOADS AND WIND TURBINE POWER OUTPUT PATTERNS

Time	Pat1 (pu)	Pat2 (pu)	Pat3 (pu)	Load (pu)
1	0.50	0.60	0.2	0.78
2	0.55	0.65	0.23	0.76
3	0.40	0.75	0.26	0.72
4	0.92	0.00	0.29	0.70
5	0.00	0.95	0.32	0.68
6	0.00	1.00	0.36	0.66
7	0.00	0.95	0.40	0.64
8	0.50	0.90	0.43	0.65
9	0.70	0.90	0.48	0.83
10	0.92	0.85	0.52	0.86
11	0.95	0.70	0.55	0.89
12	1.00	0.60	0.58	0.92
13	0.95	0.50	0.58	0.90
14	0.90	0.40	0.55	0.98
15	0.90	0.32	0.52	1.00
16	0.84	0.30	0.50	0.97
17	0.70	0.35	0.47	0.92
18	0.61	0.32	0.43	0.90
19	0.64	0.23	0.40	0.91
20	0.70	0.00	0.37	0.92
21	0.6	0.00	0.32	0.91
22	0.65	0.00	0.27	0.89
23	0.60	0.40	0.25	0.85
24	0.50	0.55	0.22	0.80

TABLE 2. SCENARIOS

Scenario number	1	2	3	4	5	6
Algorithms	-	GA	GA	GA	GA	GA
	-	HS	HS	HS	HS	HS
Number of DG	0	1	2	0	1	2
Reconfiguration	No	No	No	Yes	Yes	Yes

4. SIMULATIONS AND RESULTS

The method presented in the IEEE 33 bus distribution network in figure 1, is coded in MATLAB. Line voltage in the network is 23 kV and the data needed for the lines and loads in the distribution network to simulate is taken from [22]. Switches shown in figure 4, are capable of putting all the six feeders in on or off positions in a way that the network remains radial. In table 1 properties of load and the generation power of the wind power station and three patterns of generation power of the wind power according to environmental features as a percentage of DG nominal power in pu in different hours of the day are shown. ‘Load’ shows the rate of variations in load of each bus and ‘Pat’ shows the pattern of DG generation power in pu in a day.

The effect of DG nominal capacity on voltage profile improvement index and the losses in the distribution network in the three models of DG output power are calculated for each bus each hour. In figures 5 and 6 the results of this simulation are shown. It is seen in the first

model, being in proportion with the network consumed load in 24 hours, that if DG capacity is increased gradually, voltage profile improvement index first increases, but later decreases. In other models this changes differ. At the same time it is seen in the Figure 5, that as DG nominal capacity increases, the system losses decrease. Since minimum and maximum points of loss and VQI are not in the same place, therefore an algorithm is required which is capable of finding the optimal network structure in a way that it keeps losses at minimum and improves the voltage profile.

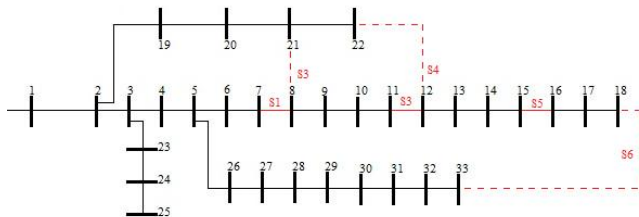


Figure 4. 33-bus IEEE distribution network [22]

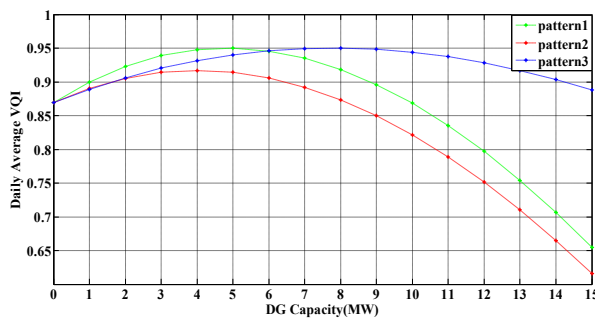


Figure 5. Daily VQI under three DG output patterns and. Capacities

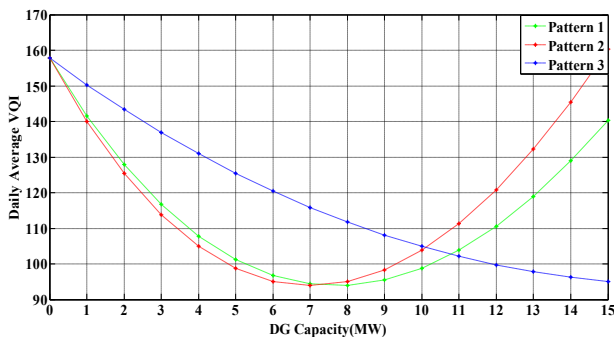


Figure 6. Daily average of losses under DG output pattern 1

In this article the optimal values are calculated using genetic and harmony search algorithms in a couple of different scenarios. In these scenarios the first pattern of wind turbine power is used. The scenarios are defined based on the number of DGs and the existence of reconfiguration in the network, and the presented algorithms have been used to optimize. These scenarios presented in table 2. The results of simulation are seen in tables 3, 4, 5.

5. CONCLUSIONS

This paper proposes a new DG interconnection planning method. This method uses time-varying loads and DG output

power and presents a process of optimization for DG placement and sizing in distribution network with

TABLE Scenarios one to three are defined without reconfiguration in network and a number of DG from 0 to 2. With a raise in the number of DG, decrease of losses and voltage profile improvement is witnessed. The presented algorithm will go toward the appropriate result after 20 iterations in GA. In scenarios four to six there is reconfiguration in the network and the number of DGs varies from 0 to 2. Despite reconfiguration in the network a considerable decrease of losses appears and voltage profile improves too.

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TABLE Table 5 shows calculated optimum capacity and placement of DG, loss power and VQ in each scenario. Figure 7 shows voltage profile improvement with genetic algorithm in scenario 1 to 3 in 33 buses, and figure 8 shows voltage profile improvement with hormone search algorithm in scenario 4 to 6 in 33 buses.

Table 6 and figure 9 show comparing the conditions without reconfiguration and with reconfiguration in network. In the second condition more losses decrease is calculated. In the two conditions of genetic and harmony search algorithms, the highest voltage profile improvement is seen in the third scenario and the least loss in the sixth scenarios. The least value of the objective function is calculated in the sixth and in the presence of 2 DGs and reconfiguration in the network. In table 6 and figure 9 the comparison of the calculated results of the two algorithms are shown. It is observed that harmony search algorithm converged faster to better solutions in comparison with results using GA.

TABLE 3. RESULT OF OPTIMIZATION WITH GA

Scenario	DG1		DG2		Closed switch	Ploss (kW)	VQI
	Bus	P	Bus	P (kW)			
1	-	-	-	-	-	157.7	0.870
2	29	2.88	-	-	-	77.86	0.919
3	32	1.45	14	3.90	-	71.82	0.932
4	-	-	-	-	6-4-2	121.4	0.892
5	31	2.41	-	-	6-4-1	65.5	0.913
6	3	2.09	24	2.35	6-4-1	61.5	0.923

TABLE 4. RESULT OF OPTIMIZATION WITH HS

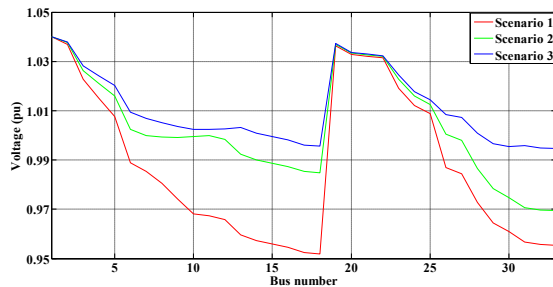
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reconfiguration ability in order to reduce the losses and increase the voltage profile improvement index. In this study, GA and HS algorithms have been used to optimize the objective function in 33-bus distribution network. Numerical results reveal that HS algorithm will converge to a better result in comparison with results using GA.



Scenario	DG1		DG2		Closed switch	Ploss (kW)	VQI
	Bus	P	Bus	P (kW)			
1	-	-	-	-	-	157.7	0.870
2	26	4.25	-	-	-	82.21	0.938
3	30	1.55	16	0.62	-	71.53	0.913
4	-	-	-	-	6-4-2	121.4	0.892
5	30	2.15	-	-	6-4-1	60.52	0.905
6	19	1.39	31	1.92	5-4-1	58.63	0.903

Figure 7. Voltage profile of the 33-bus system with Scenario 1 to 3

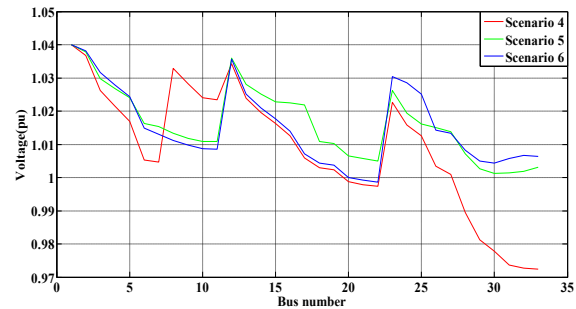


Figure 8. Voltage profile of the 33-bus system with Scenario 4 to 6

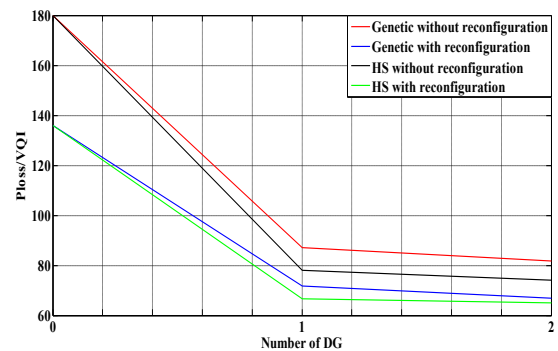


Figure 9. Comparison the results with the GA and HS algorithm

TABLE 6
RESULTS OF SIMULATION WITH THE GA AND HS ALGORITHM

Algorithm	1 DG						2 DG					
	Without interconnection			With interconnection			Without interconnection			With interconnection		
	VQI	Ploss (kW)	Ploss/VQI	VQI	Ploss (kW)	Ploss/VQI	VQI	Ploss (kW)	Ploss/VQI	VQI	Ploss (kW)	Ploss/VQI
Genetic	0.908	80.21	88.3	0.913	65.5	71.7	0.919	77.86	84.7	0.905	60.52	66.8
HS	0.913	71.53	78.3	0.923	61.5	66.6	0.932	71.82	77.0	0.903	58.63	64.9

REFERENCES

[1] El-Khattam, W. Salama, M. "Distributed generation technologies, definitions and benefits", *Electric Power Systems Research*, **71**(2): 119-128 (2004).
 [2] Su, S. Y. et al., "Distributed generation interconnection planning: A wind power case study", *IEEE Transactions on Smart Grid*, **2**(1): 181-189 (2011).
 [3] Wang, C. Nehrir, M. H. "Analytical approaches for optimal placement of distributed generation sources

in power systems", *IEEE Transactions on Power Systems*, **19**(4): 2068-2076 (2004).
 [4] Farashbashi-Astaneh, S. Dastfan, A. "Optimal placement and sizing of dg for loss reduction, voltage profile improvement and voltage Sag mitigation", in *International Conference on Renewable Energies and Power Quality (ICREPQ'10) Granada (Spain)*, (2010).
 [5] Borges, C. L. Falcao, D. M. "Optimal distributed generation allocation for reliability, losses, and

- voltage improvement", *International Journal of Electrical Power & Energy Systems*, **28**(6): 413-420 (2006).
- [6] Mori, H. Tani, H. "Two-staged tabu search for determining optimal allocation of D-FACTS in radial distribution systems with distributed generation", in *Transmission and Distribution Conference and Exhibition: Asia Pacific. IEEE/PES* (2002).
- [7] Phonrattanasak, P. "Optimal placement of DG using multiobjective particle swarm optimization", in *2nd International Conference on Mechanical and Electrical Technology (ICMET)* (2010).
- [8] Haesen, E. et al, "Optimal placement and sizing of distributed generator units using genetic optimization algorithms", *Electrical Power Quality and Utilisation Journal*, **11**(1): 97-104 (2005).
- [9] Jeon, Y. J. et al, "An efficient simulated annealing algorithm for network reconfiguration in large-scale distribution systems", *IEEE Transactions on Power Delivery*, **17**(4): 1070-1078 (2002).
- [10] Batrinu, F. Carpaneto, E. Chicco, G. "A novel particle swarm method for distribution system optimal reconfiguration", in *IEEE Power Tech, Russia* (2005).
- [11] Chiou, J. P. Chang, C. F. Su, C. T. "Variable scaling hybrid differential evolution for solving network reconfiguration of distribution systems", *IEEE Transactions on Power Systems*, **20**(2): 668-674 (2005).
- [12] Kazemi, A. Parizad, A. Baghaee, H. "On the use of harmony search algorithm in optimal placement of FACTS devices to improve power system security", in *IEEE EUROCON* (2009).
- [13] Singh, D. et al, "GA based optimal sizing & placement of distributed generation for loss minimization". *International Journal of Intelligent Technology*, **2**(4): 263-269 (2007).
- [14] Harrison, G. P. et al, "Hybrid GA and OPF evaluation of network capacity for distributed generation connections", *Electric Power Systems Research*, **83**(3): 392-398 (2008).
- [15] Pisica, I. Bulac, C. Eremia, M. "Optimal distributed generation location and sizing using genetic algorithms", *15th International Conference on Intelligent System Applications to Power Systems, ISAP'09* (2009).
- [16] Coelho, L. D. S. Mariani, V. C. "An improved harmony search algorithm for power economic load dispatch", *Energy Conversion and Management*, **50**(10): 2522-2526 (2009).
- [17] Verma, A. Panigrahi, B. Bijwe, P. "Harmony search algorithm for transmission network expansion planning", *IET Generation, Transmission & Distribution*, **4**(6): 663-673 (2010).
- [18] Abou El-Ela, A. Allam, S. Shatla, M. "Maximal optimal benefits of distributed generation using genetic algorithms", *Electric Power Systems Research*, **80**(7): 869-877 (2010).
- [19] Geem, Z. W. Kim, J. H. Loganathan, G. "A new heuristic optimization algorithm: harmony search", *Simulation*, **76**(2): 60-68 (2001).
- [20] Lee, K. S. Geem, Z. W. "A new meta-heuristic algorithm for continuous engineering optimization: harmony search theory and practice", *Computer methods in applied mechanics and engineering*, **194**(36): 3902-3933 (2005).
- [21] Xia, H. Chen, D. Gao, L. "Modified harmony search algorithm for power economic load dispatch", *Journal of Computational Information Systems*, **2**(5): 2103-2110 (2013).
- [22] Baghzouz, Y. Ertem, S. "Shunt capacitor sizing for radial distribution feeders with distorted substation voltages", *IEEE Transactions on Power Delivery*, **5**(2): 650-657 (1990).