EXPLICIT P-STABLE HYBRID TECHNIQUES FOR SIMULATION OF PERIODIC EQUATIONS

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ABSTRACT:-*A differential equation of second order with initial states which does not involve any term of first order is solved by many authors. It can be resolved by transforming it into a class of equations having order 1 and then exercising it or by directly exercising it using altered techniques. In the following paper we recommend technique that can be use for simulation directly on equations having less function evaluations as compare to techniques from literature. This approach noticeably minimizes the time and expense of the system.*

Key words: Hybrid, Second Order Differential Equation, Explicit

1. INTRODUCTION

We need to obtain the result by simulating the differential equation which are periodic having order two

$$\hat{\boldsymbol{z}}''(\hat{t}) = \boldsymbol{f}(\hat{t}, \hat{\boldsymbol{z}}(t)), \quad \hat{\boldsymbol{z}}(\widehat{t_0}) = \hat{\boldsymbol{z}}_{\boldsymbol{0}} \ , \quad \hat{\boldsymbol{z}}'(\widehat{t_0}) = \hat{\boldsymbol{z}}'_{\boldsymbol{0}} \ , \hat{\boldsymbol{z}}(\hat{t}), \boldsymbol{f}(\hat{t}, \hat{\boldsymbol{z}}) \in R^n \ , \qquad (1)$$

and also above equation does not have 1st order term. $\boldsymbol{f}(\hat{t}, \hat{\boldsymbol{z}}): [\widehat{t_0}, T] \times R^m \rightarrow R^m$ is differentiable appropriately. To simulate (1), many mathematical techniques have been explored extensively recently. Such a obstacles are result of solicitation of atomic fluctuations, astrodynamics, the branches of science concerned with earthquakes and related phenomena, these are believed as tough integration difficulties usually.Much of the time, when the issue has a vast measurement, or the assessment of right hand side capacity is extremely extravagant, or reaction time is amazingly critical, for instance in recreation process, we need to acquire a precise arrangement in a sensible timeline. Therefore, there is an extraordinary interest of effective systems for (1).When all is said in done, the numerical strategies for (1) can be sorted into two fundamental sorts. One of them comprises of such routines in which coefficients rely on upon issue under thought, that is, the recurrence of the issue is known as an earlier.While the second one embodies the systems whose coefficients are consistent and are free to the issue concerned [2].Lambert and Watson [10] presented the ideas of P-stability and interim of periodicity for second order intermittent introductory worth issues for numerical systems. Costabile [6] introduced a structure for an awesome assortment of the routines through levelheaded estimate to the cosine for (1) to examine the interims of periodicity and requests of stage slack (scattering).

2. PRELIMINARIES

Assumeinitial problem

$$\hat{\boldsymbol{z}}''(\hat{t}) = \boldsymbol{f}(\hat{t}, \hat{\boldsymbol{z}}(t)), \hat{t} \in [a, b] \ , \hat{\boldsymbol{z}}(a) = \boldsymbol{\gamma} \ , \hat{\boldsymbol{z}}'(a) = \hat{\boldsymbol{\gamma}}, \qquad (2)$$

having solution $\hat{\boldsymbol{z}}(\hat{t})$. At that point general type of Linear Multistep technique is

$$\sum_{j=0}^{k} \eta_j \hat{\boldsymbol{z}}_{n+j} = h^2 \sum_{j=0}^{k} \mu_j \boldsymbol{f}_{n+j} \qquad (3)$$

whereη_j and μ_j are constants subject to the conditions

$$\eta_k = 1, |\eta_0| + |\mu_0| \neq 0 \qquad (4)$$

with first and second characteristic polynomials $\boldsymbol{\phi}$ and $\boldsymbol{\psi}$ respectively, where

$$\boldsymbol{\phi}(r) = \sum_{j=0}^{k} \eta_j r^j \ , \boldsymbol{\psi}(r) = \sum_{j=0}^{k} \mu_j r^j \qquad (5)$$

DEFINITION 2.1

Linear multistep technique (3) is related withlinear difference operator $[\hat{\boldsymbol{z}}(\hat{t}) \ ; h]$, defined as

$$l \ [\hat{\boldsymbol{z}}(\hat{t}) \ ; h] = \sum_{j=0}^{k} [\eta_j \hat{\boldsymbol{z}}(\hat{t} + jh) - h^2 \mu_j \hat{\boldsymbol{z}}''(\hat{t} + jh)] \qquad (6)$$

where$\hat{\boldsymbol{z}}(\hat{t})$ is random and differentiable function continuously on $[\acute{a}, \acute{b}]$. By expansion Taylor seriesabout $\hat{t}$, equation (6) gives

$$l \ [\hat{\boldsymbol{z}}(\hat{t}) \ ; h] = \hat{C}_0 \hat{\boldsymbol{z}}(\hat{t}) + \hat{C}_1 h \widehat{\boldsymbol{z}'}(\hat{t}) + \cdots + \hat{C}_q h^q \hat{\boldsymbol{z}}^{(q)}(t) + \cdots \qquad (7)$$

where

$$\hat{C}_0 = \eta_0 + \eta_1 + \eta_2 + \cdots + \eta_k$$
$$\hat{C}_1 = \eta_1 + 2\eta_2 + \cdots + k\eta_k$$
$$\hat{C}_2 = \frac{1}{2!}(\eta_1 + 2\eta_2 + \cdots + k^2 \eta_k) - (\mu_0 + \mu_1 + \mu_2 + \cdots + \mu_k)$$
$$\hat{C}_q = \frac{1}{q!}(\eta_1 + 2^q \eta_2 + \cdots + k^q \eta_k) - \frac{1}{(q-1)!}(\mu_1 + 2^{q-2}\mu_2 + \cdots + k^{q-2}\mu_k), \ \ q = 3,4, \ldots$$

Therefore the technique (3) is then have the order p with error constant$\hat{C}_{p+2}$.

DEFINITION 2.2

Equation (6) is named as local truncation error for system (2) with order p, satisfying

$$\widehat{\boldsymbol{T}}_{n+k} = \hat{C}_{p+2} h^{p+2} \hat{\boldsymbol{z}}^{(p+2)}(\hat{t}_{\boldsymbol{n}}) + O(h^{p+3})$$

With at point $\hat{t}_n$,$\hat{C}_{p+2} h^{p+2} \hat{\boldsymbol{z}}^{(p+2)}(\hat{t}_{\boldsymbol{n}})$as principal localtruncation error.

THEOREM

A linear multistep technique is called convergent if it is both zero stable as well as consistent, Henrici [9].Now we discuss Techniques of Fourth Order for simulations of system (1).

3. FOURTH ORDER TECHNIQUES

The fundamental strategy has structure

$$\hat{\boldsymbol{z}}_{n+1} - 2\hat{\boldsymbol{z}}_n + \hat{\boldsymbol{z}}_{n-1} = h^2\{\hat{\beta}_0[\hat{\boldsymbol{z}}''_{n+1} + \hat{\boldsymbol{z}}''_{n-1}] + \hat{\gamma}\hat{\boldsymbol{z}}''_n + \hat{\beta}_1[\hat{\boldsymbol{z}}''_{n+\alpha_1} + \hat{\boldsymbol{z}}''_{n-\alpha_1}]\} \qquad (8)$$

Where off-step qualities are characterized as

$$\hat{\boldsymbol{z}}_{n\pm\alpha_1} = X_\pm\hat{\boldsymbol{z}}_{n+1} + Y_\pm\hat{\boldsymbol{z}}_n + Z_\pm\hat{\boldsymbol{z}}_{n-1} + h^2\{a_\pm\hat{\boldsymbol{z}}''_{n-1} + b_\pm\hat{\boldsymbol{z}}''_n + c_\pm\hat{\boldsymbol{z}}''_{n-1}\} \qquad (9)$$

(13)

And

$$\hat{\boldsymbol{z}}''_n = \boldsymbol{f}(\hat{t}_n, \hat{\boldsymbol{z}}_n), \hat{\boldsymbol{z}}''_{n\pm1} = \boldsymbol{f}(\hat{t}_n \pm h, \hat{\boldsymbol{z}}_{n\pm1}), \hat{\boldsymbol{z}}''_{n\pm\alpha_1} = \boldsymbol{f}(\hat{t}_n \pm \alpha_1 h, \hat{\boldsymbol{z}}_{n\pm\alpha_1}) \qquad (10)$$

CONDITIONS OF FORTH ORDER TECHNIQUES

For approaches (8), (9) & (10), local truncation error is represented as

$$L[\hat{\boldsymbol{z}}(\hat{t}_n; h)] = \hat{\boldsymbol{z}}_{n+1} - 2\hat{\boldsymbol{z}}_n + \hat{\boldsymbol{z}}_{n-1} - h^2\{\hat{\beta}_0[\hat{\boldsymbol{z}}''_{n+1} + \hat{\boldsymbol{z}}''_{n-1}] + \hat{\gamma}\hat{\boldsymbol{z}}''_n + \hat{\beta}_1[\hat{\boldsymbol{z}}''_{n+\alpha_1} + \hat{\boldsymbol{z}}''_{n-\alpha_1}]\} \qquad (11)$$

Expanding $\hat{\boldsymbol{z}}(\hat{t}_{n\pm1})$and $\hat{\boldsymbol{z}}''(\hat{t}_{n\pm1})$, $\hat{\boldsymbol{z}}''(\hat{t}_{n\pm\alpha_1})$using taylor'sexpansion. for simplicity we set

$$X_\pm + Y_\pm + Z_\pm = 1,$$

Eq(11)⟹

$$L[\hat{\boldsymbol{z}}(\hat{t}_n; h)] = h^2(1 - 2\hat{\beta}_0 - 2\hat{\beta}_1 - \hat{\gamma})\hat{\boldsymbol{z}}'' - h^3[\hat{\beta}_1(X_+ - Z_+ + X_- - Z_-)\frac{\partial f}{\partial \hat{z}}\hat{\boldsymbol{z}}' + h^4\{[\frac{1}{12} - \hat{\beta}_0 - \hat{\beta}_1\alpha_1^2]\frac{\partial^2 f}{\partial t^2} + [\frac{1}{6} - 2\hat{\beta}_0 - \hat{\beta}_1\alpha_1(X_+ - Z_+ + X_- - Z_-)]\frac{\partial^2 f}{\partial t \partial \hat{z}}\hat{\boldsymbol{z}}' + \cdots \qquad (12)$$

coefficients of h^0, h^1, h^2, h^3, h^4 and h^5 from(12) are zero to obtain accuracy and coefficients of h^6must not. Therefore for fourth order, fundamental and adequate conditions

Case 1

$$\hat{\beta}_1 = 0,\ \hat{\beta}_0 = \frac{1}{12},\ \hat{\gamma} = \frac{5}{6} \qquad (18)$$

Case 2

$$\hat{\gamma} = \frac{5}{6} + 2\hat{\beta}_1(\alpha_1^2 - 1), \hat{\beta}_0 = \frac{1}{12} - \hat{\beta}_1\alpha_1^2, Y_+ = 1 + \alpha_1 - 2X_+, Y_- = 1 - \alpha_1 - 2X_-,$$

$$Z_+ = X_+ - \alpha_1, Z_- = X_- + \alpha_1, a_+ + a_- = c_+ + c_-$$

And either

$$\alpha_1 = 0\ ,\ , b_+ + b_- = -(X_+ + X_-) - 2(c_+ + c_-)$$

or

$$\alpha_1 \neq 0,\ b_+ = \frac{1}{12}\alpha_1^2 + \frac{1}{12}\alpha_1 - X_+ - c_+ - a_+,\ b_- = \frac{1}{12}\alpha_1^2 - \frac{1}{12}\alpha_1 - X_- - c_+ - 2c_- + a_+ \qquad (13)$$

CONDITIONS FOR P-STABILITY

To get these conditions for approaches (8), (9), (10) & (13), employing it to scalar test equation, we have

$$\hat{\boldsymbol{z}}_{n+1} - 2\hat{\boldsymbol{z}}_n + \hat{\boldsymbol{z}}_{n-1} = -m^2h^2\left\{\left(\frac{1}{12} - \hat{\beta}_1{\alpha_1}^2\right)[\hat{\boldsymbol{z}}_{n+1} + \hat{\boldsymbol{z}}_{n-1}] + \left(\frac{5}{6} + 2\hat{\beta}_1({\alpha_1}^2 - 1)\right)\hat{\boldsymbol{z}}_n + \hat{\beta}_1[(X_+ + X_-)\hat{\boldsymbol{z}}_{n+1} + (Y_+ + Y_-)\hat{\boldsymbol{z}}_n + (Z_+ + Z_-)\hat{\boldsymbol{z}}_{n-1}]\right\} + \hat{\beta}_1 m^4h^4\{(a_+ + a_-)\hat{\boldsymbol{z}}_{n+1} + (b_+ + b_-)\hat{\boldsymbol{z}}_n + (c_+ + c_-)\hat{\boldsymbol{z}}_{n-1}\}$$

We state essential and adequate conditions for P-stability taking after Thomas [12]

$$\hat{\beta}_1(\alpha_1^2 - X_+ - X_-) \leq 0$$

And

$$\{\frac{2}{3} + 4\hat{\beta}_1(\alpha_1^2 - X_+ + \hat{A}_-)\}^2 < 16\hat{\beta}_1\{(\alpha_1^2 - X_+ + X_-) - 4(a_+ + a_-)\}$$

At every step three functions need to per solved for these P-stable techniques of fourth order.

4. COCLUSION

In this paper we have examined a group of explicit hybrid techniques which are P-stable fourth order strategies. We have determined conditions for fourth order approach and conditions for P-stability taking after Thomas [12,13] and Khiyal [10].Additionally it has been seen that utilization of these techniques non-linear differential frameworkascent to understand non-linear logarithmic framework which must be illuminated at every step. We need to evaluate three functions per step in these techniques and hence are three evaluation approaches.

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