INTERVAL TYPE 2 FUZZY CONTROL OF INVERTED PENDULUM

Tahseen Naeem[1] ,Huma Hanan[2] ,Bilal Ahmad[3] ,Afzal Ahmad[4] ,Dr. Sun Jun[5]
[1]Department of Computer Science, The Institute of Southern Punjab, Multan Pakistan, tahseenn15@gmail.com
[2]Department of Computer Science & Information Technology, Iqra Post Graduate College, RYK, Pakistan, humaafzal2017@gmail.com
[3]Department of Control Science & Engineering, Internet of Things ,Jiangnan University, China, bilalrouf@yahoo.com
[4] Ecological Dyeing and Finishing Laboratory , Internet of Things, Jiangnan University, China, afzalucte@vip.jiangnan.edu.cn
[5] Department of Computer Science, Internet of Things, Jiangnan University, China, Sunjun_wx@hotmail.com

ABSTRACT*:In this paper a interval type 2 fuzzy proportional derivative (IT2 FPD) controller is proposed for inverted pendulum (IP) system. Proposed controller uses the interval type 2 fuzzy sets with triangular member ship functions. Simulation results shows that the IT2 FPD controller has an ability to reduce the external disturbances applied to the system under observation and uncertainties associated with inputs and outputs of any fuzzy logic control system. The check the robustness of proposed controller IP system is observed under three different cases and there errors, root mean square error (RMSE), integral of square of errors (ISE) and mean average error (MAE), are calculated to estimate the effectiveness of proposed controller quantitatively. Proposed controller performs better where other controllers intend to be unstable.*

INTRODUCTION

The conventional linear proportional derivative (PD) & Proportional integral (PI) controllers are used to improve the transient response and steady state response of the system respectively. The advantage of PID controller is that it is very simple to design but it can only provide good performance to linear systems only. For highly uncertain and nonlinear systems PID controller is not a good choice [1,2]. Fuzzy logic controller (FLC) is a better approach for highly uncertain and nonlinear systems. There are two main advantages of FLC [5].

1: It can be used for those highly non-linear systems where it is difficult to model the system mathematically.

2: It can be used to apply heuristic rules that reflect the experience of human expert.

$$FOU(\widetilde{A}) = \bigcup_{\forall x \in X} J_x = \{(x, v) : v \in J_x \subseteq [0,1]\}$$

In fuzzy control design, choice of input is very important. Input to fuzzy control is either an errors or change of error. Depending upon the output of fuzzy control rules, fuzzy controller is PI or PD type controller. If the output of fuzzy control is change of control signal, it is called PI type fuzzy controller. If the output of fuzzy control is control signal, it is said to be PD fuzzy controller [6].

Type 2 fuzzy logic sets were first introduced by Zadeh in 1975 [10]. These sets can handle in much better way the above mentioned nonlinearities and uncertainties in the systems because of their membership functions [10]. In type 2 fuzzy sets, membership weight for each member of set is fuzzy set in the range of [0,1]. So, type 2 fuzzy sets provides extra degree of freedom which makes it possible to handle the nonlinearities and uncertainties in the system [10].

The inverted pendulum is a classic control problem in the field of control science research. It has lot of nonlinearities and high

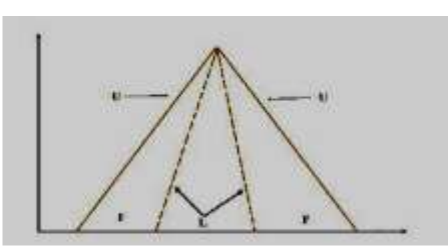

Figure 1. Interval type 2 Fuzzy Set

degree of uncertainty [28]. It provides a good opportunity to apply various control techniques and theories to test them. It is also important as it provides analogy for various natural systems like walking robot and missile stability etc. The uncertainty in the IP system can be categorize into two forms. First, nonlinear & un modelled dynamic uncertainty and second one is parameter uncertainty [30].

Parameter uncertainties involved with those parameter which are difficult to measure or tend to vary with time and temperature. As for as dynamic uncertainties are concerned, IP has un-modeled dynamics for example track inclination and bearings. Both, classical methods and modern techniques are

used to control the IP. Classical methods includes PID controllers and modern methods include predictive control and artificial neural networks [33-37].

In [38], Type 1 PD controller is used to control the IP system. This solution to control the IP system is very simple but it cannot handle the nonlinearities and uncertainties s and one output. Error signal and derivative of error signadirectly. So type 2 fuzzy logic sets are used in this paper to handle the nonlinearities. The proposed control consists of two inputl are inputs and control signal is the output. Proposed controller provides the better results in handling the nonlinearities. Results are compared with other controllers to check the robustness of proposed controller. Further paper is organized as follows. In section 2, proposed controller is introduced. In section 3, mathematical modeling of IP is presented. In section 4, simulation results are presented followed by conclusion and references.

Inverted Type-2 Fuzzy PD Controller

Interval type 2 Fuzzy PD controller (IT2-FPD). It has two input variables, e(k) and Δe(k), and one output variable, u(k). Two scaling factors $G_{\Delta e}$ and Ge are employed to scale input variables Δe(k) and e(k) , respectively.

$$\begin{aligned} E(k) &= G_e e(k) = G_e(y_r(k) - y(k)) \\ \Delta E(k) &= G_{\Delta e}\Delta e(k) = G_{\Delta e}(e(k) - e(k-1)) \end{aligned} \quad (1)$$

Where

K= Sampling instance

y(k)= system output, $y_r(k)$= Reference system output

The scaling factor G_u of output variable u(k) are given by the equation (2) where u(K) is the output of the IT2-FPD controller.

$$\boldsymbol{u(K) = Gu\ U(K)} \quad (2)$$

2.1 Interval type-2 fuzzy set

$$\check{A} = \int_{x\mathcal{E}X} \int_{v\mathcal{E}Jx\subseteq[0,1]} \frac{1}{(x,v)} = \int_{x\mathcal{E}X} \left[\int_{v\mathcal{E}Jx\subseteq[0,1]} \frac{1}{v}\right] /x \quad (3)$$

Where

$$X\ \mathcal{E}\ X \text{ and } v\ \mathcal{E}\ V \quad (4)$$

x and v are primary and secondary variables respectively. It has domain Jx. Jx is called primary membership of x and is given by equation no. 7. ∬ indicates union of x and v. When universe of docourse is discrete is be replaced with ∑. Foot print of uncertainty (FOU) of Ă is the union of all members of fuzzy set of Ă as shown in figure 1.

The lower membership function and upper membership function of Ă are two
type 1 membership functions that are bounded by FOU. The upper membership function is associated with upper bound and lower membership function is associated with lower bound respectively and is given by equation 5 and 6.

$$\bar{\mu}_{\tilde{A}}(x) = \overline{FOU(\tilde{A})} \quad \forall x \in X \quad (5)$$

$$\underline{\mu}_{\tilde{A}}(x) = \underline{FOU(\tilde{A})} \quad \forall x \in X \quad (6)$$

Note that Jx is an interval set represented as

$$J_x = \left\{(x,v) : v \in [\underline{\mu}_{\tilde{A}}(x), \bar{\mu}_{\tilde{A}}(x)]\right\} \quad (7)$$

The structure of mdamani type-2 fuzzy (T2-F) rule is given by

$$E(k) = \check{A}\ AND\ \Delta E(k) = \check{B}\ then\ U(k) = \check{C} \quad (8)$$

2.2 Fuzzification and inference

When $E(k) = (x\acute{}1)$ and $E(k) = (x\acute{}2)$ the vertical line at x1 and x2 incersetcs FOU(Ă) everywhere in the intervals of $[\underline{\cup}_{\check{A}}(x\acute{}1), \overline{\cup}_{\check{A}}(x\acute{}1)]$ and $[\underline{\cup}_{\check{A}}(x\acute{}2), \overline{\cup}_{\check{A}}(x\acute{}2)]$ respectively. Two firing angles, upper and lower firing angle, are then computed and given by

$$[\bar{f}\ (\acute{x}) = min\ \underline{\cup}_{\check{A}}(x\acute{}1), \underline{\cup}_B(x\acute{}2)]$$

and

$$[\underline{f}\ (\acute{x}) = min\ \underline{\cup}_{\check{A}}(x\acute{}1), \underline{\cup}_B(x\acute{}2)] \quad [9]$$

2.3 Output processing

In this research paper we used Wu-Mendel uncertainty bound for calculating the output of IT2- FPD controller [7]. Following steps are involved.

1. Calculation of centroids of M consequent IT2 FSs:

U_l and U_u are two end point of M consequent of IT2 fuzzy sets. These re generated by using Mendel and Krnik algorithms.

2. Calculation of boundary type-1 FLS centroids

$$U_l^{(0)}(x) = \sum_{i=1}^{M} \underline{f}^i U_l^i \Big/ \sum_{i=1}^{M} \underline{f}^i \qquad U_l^{(M)}(x) = \sum_{i=1}^{M} \bar{f}^i U_l^i \Big/ \sum_{i=1}^{M} \bar{f}^i$$

$$U_r^{(0)}(x) = \sum_{i=1}^{M} \bar{f}^i U_r^i \Big/ \sum_{i=1}^{M} \bar{f}^i \ , \quad U_r^{(M)}(x) = \sum_{i=1}^{M} \underline{f}^i U_r^i \Big/ \sum_{i=1}^{M} \underline{f}^i$$

3. Computation of uncertainty bounds:

$$\underline{U}_l(x) \leqslant U_l(x) \leqslant \overline{U}_l(x) \ , \quad \underline{U}_r(x) \leqslant U_r(x) \leqslant \overline{U}_r(x) \quad (10)$$

$$\overline{U}_l(x) = \min\{U_l^{(0)}(x), U_l^M(x)\} \ , \quad \underline{U}_r(x) = \max\{U_r^{(0)}(x), U_r^M(x)\} \quad (11)$$

$$\underline{U}_l(x) = \overline{U}_l(x) + \left[\frac{\sum_{i=1}^M(\bar{f}^i - \underline{f}^i)}{\sum_{i=1}^M \bar{f}^i \sum_{i=1}^M \underline{f}^i} \frac{\sum_{i=1}^M \underline{f}^i(U_l^i - U_l^1)\sum_{i=1}^M \bar{f}^i(U_l^M - U_l^i)}{\sum_{i=1}^M \underline{f}^i(U_l^i - U_l^1) + \sum_{i=1}^M \bar{f}^i(U_l^M - U_l^i)}\right] \quad (12)$$

$$\overline{U}_r(x) = \underline{U}_r(x) + \left[\frac{\sum_{i=1}^M(\bar{f}^i - \underline{f}^i)}{\sum_{i=1}^M \bar{f}^i \sum_{i=1}^M \underline{f}^i} \frac{\sum_{i=1}^M \bar{f}^i(U_r^i - U_r^1)\sum_{i=1}^M \underline{f}^i(U_r^M - U_r^i)}{\sum_{i=1}^M \bar{f}^i(U_r^i - U_r^1) + \sum_{i=1}^M \underline{f}^i(U_r^M - U_r^i)}\right] \quad (13)$$

4. Calculation of the approximate type reduction sets

$$[U_l(x), U_r(x)] \approx [\hat{U}_l(x), \hat{U}_r(x)] = [(\underline{U}_l(x) + \overline{U}_l(x))/2, (\underline{U}_r(x) + \overline{U}_r(x))/2] \quad (14)$$

5. Calculation of approximation of defuzzified output

$$U(x) \approx \hat{U}(x) = \frac{1}{2}[\hat{U}_l(x) + \hat{U}_r(x)] \quad (15)$$

Nonlinear inverted pendulum (IP) system

3.1 Mathematical model

Figure 2 shows the Inverted pendulum (IP) system, which consist of pendulum, cart and rail. The pendulum can rotate about its pivot point. It is supposed that no friction exit between the cart and pendulum or between the card and rail.

They dynamic equations of nonlinear inverted pendulum are given by [41,42]

$$\begin{bmatrix}\overline{x1}\\ \overline{x2}\end{bmatrix} = \begin{bmatrix}x1\\ f(x1,x2)\end{bmatrix} + \Delta A\begin{bmatrix}x1\\ x2\end{bmatrix} + \begin{bmatrix}0\\ g1(x1)\end{bmatrix}u \quad (16)$$

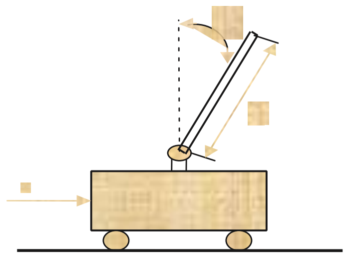

Figure 2 Inverted Pendulum on a cart

$$f(x1,x2) = \left[gSin(x1) - \frac{Xp*lX2x^2Sin(x1)*Cos(x1)}{mp+mc}\right] \Big/ b \quad (17)$$

$$g1(x1) = \frac{Cos(x1)}{ab}$$

$$a = mc + mp$$

$$b = \frac{4l}{3} - \frac{mplCosx1^2}{a} \quad (18)$$

Where

x_1 = Angle of IP system

$x2 = dx'$ is angular velocity

and u is input force applied to IP system.

l = half length of IP. It is measured in meters

m_p = mass of pendulum in Kg

m_c = mass of cart in Kg

g = 9.8 m/s^2 acceleration due to gravity

DA= structural nonlinearity of IP system which indicates the nonlinearities or uncertainties in the system which are caused by neglected parapets.

3.2. The proposed controller for IP system

The proposed controller IT2-FPD for IP is shown in figure 3. y_r indicates the desired position of IP. Objective is to balance the IP in vertical direction to upright position. At that point y_r will be equal to 0. The d and u are, respectively, the control signals of disturbance and force. The suggested controller for IP uses the triangular membership functions.

Figures 4-6 shows membership function of derivative of error signal (ΔE(k)) and simple error signal (E(k)) and the control signal. The Universe of discourse is divided into following five overlapping named

NS = Negative Small

Z=Zero

NL=Negative Large

PS=Positive Small

PL=Positive Large.

Base rule is mentioned in Table 1 and scaling factors G_e, G_{De}, and G_u are equal 1, 10, and 20. The parameters of IP system are give nin Table 2.

Table 1 The rule base.

Derivativeoferrorsignal	Errorsi gnal				
	NL	NS	Z	PS	PL
NL	PL	PL	PL	PS	Z
NS	PL	PL	PS	Z	NS
Z	PL	PS	Z	NS	NL
PS	PS	Z	NS	NL	NL
PL	Z	NS	NL	NL	NL

Simulation

In this section three different cases are tested on the IP system in order to check the robustness of the proposed controller.

4.1 Task 1: Normal case

Figure 7 shows the response of IP system with proposed controller with following initial conditions.

x_1 = 0.4 rad, and x_2 = 0.

Figure 7b and Figure 7c shows the response of the angular velocity and the force applied to the cart.

4.2 Task 2: Structure uncertainty

To observe the behavior of system in the presence of non linarites, after adding the value of ΔA to the system states as shown.

$$\Delta A = \begin{bmatrix} 0.076 & 0.076 \\ 0.076 & 0.076 \end{bmatrix}$$

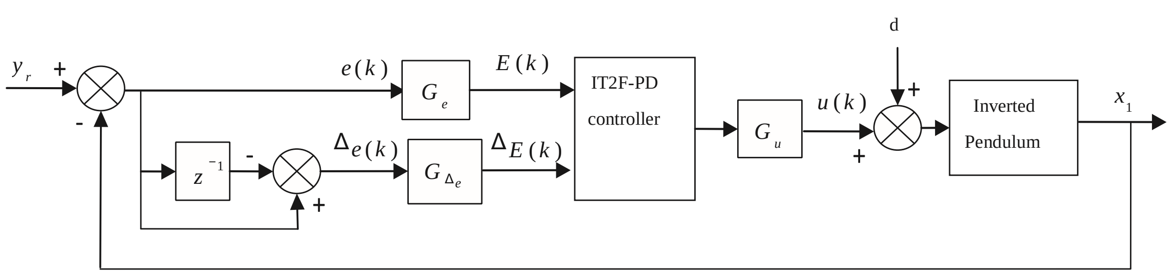

Figure 3 The interval type-2 fuzzy PD controller for IP system

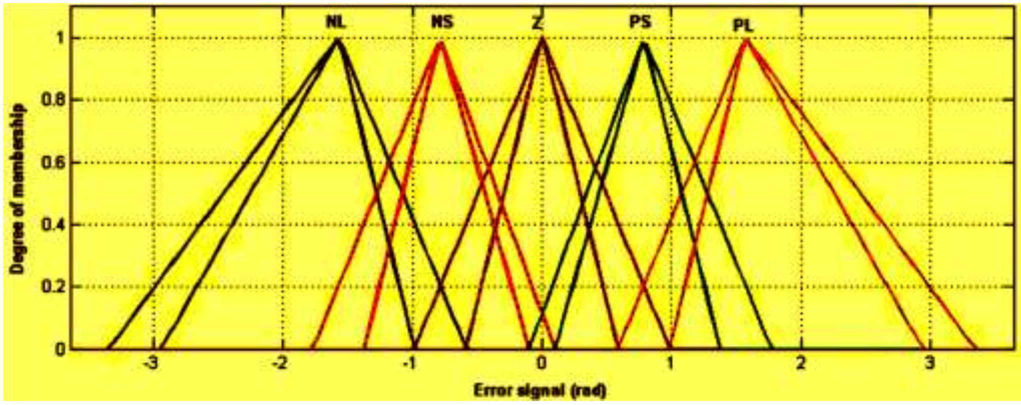

Figure 4 Membership function for error signals.

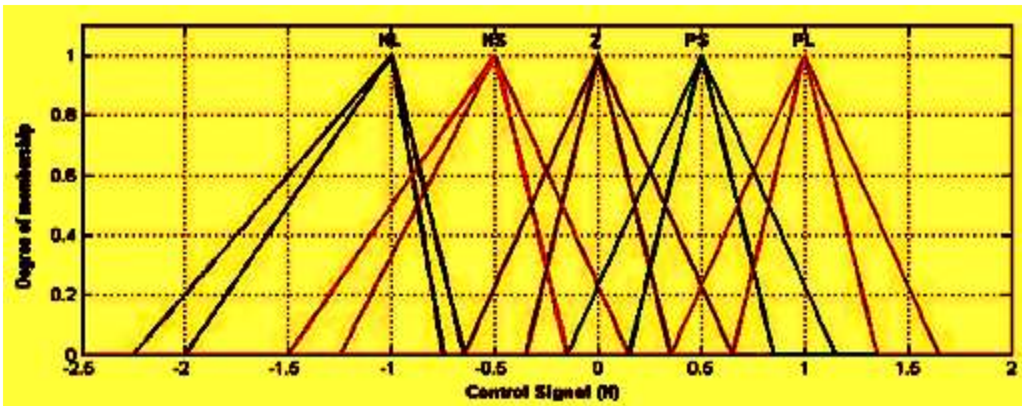

Figure 5 Membership function for derivative of error signal

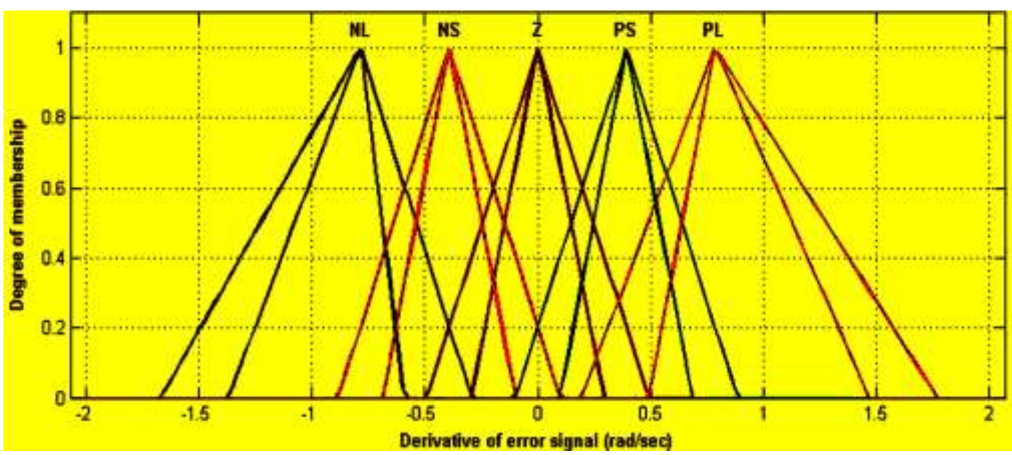

Figure 6 Membership functions of the control signal.

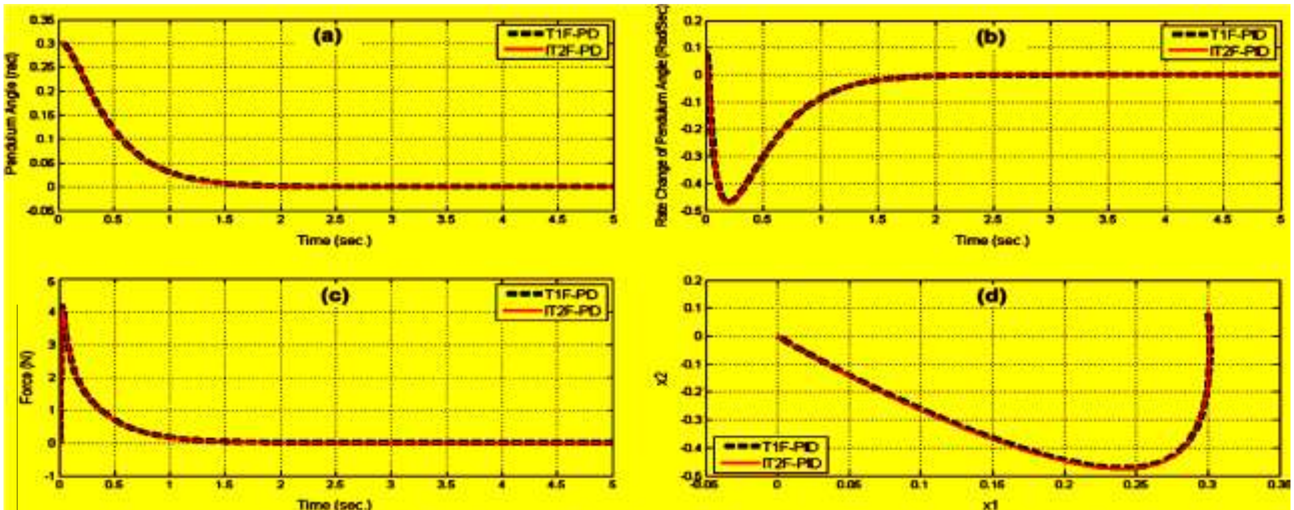

Figure 7 Response of the inverted pendulum system for normal case. (a) Pendulum angle. (b) Angular velocity. (c) Control signal. (d)Phase plane trajectory.

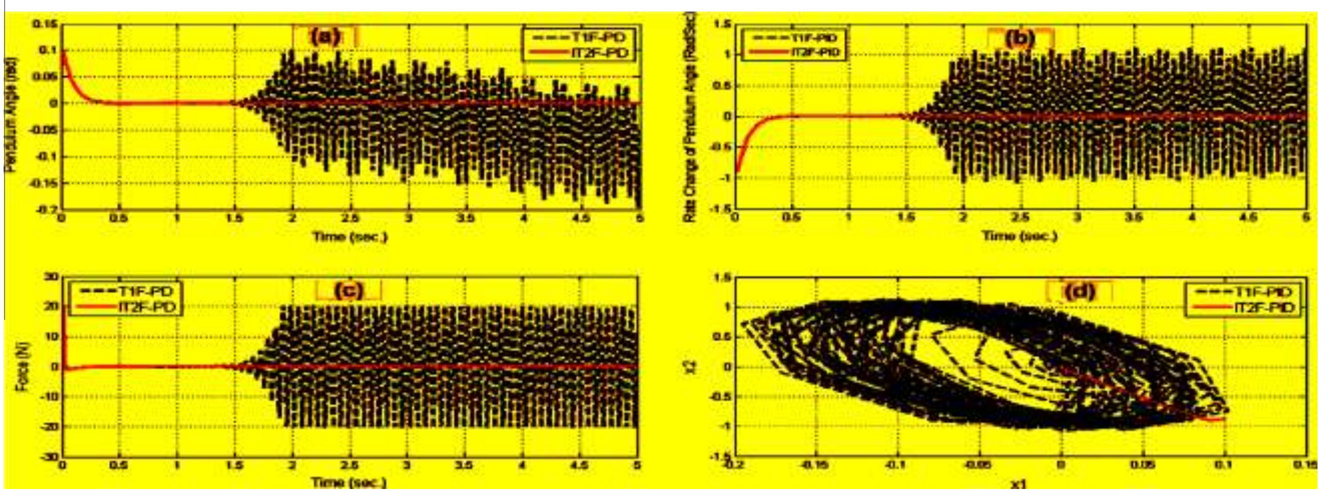

Figure 8 Response of the inverted pendulum system for structure uncertainty. (a) Pendulum angle. (b) Angular velocity. (c) Control signal. (d) Phase plane trajectory.

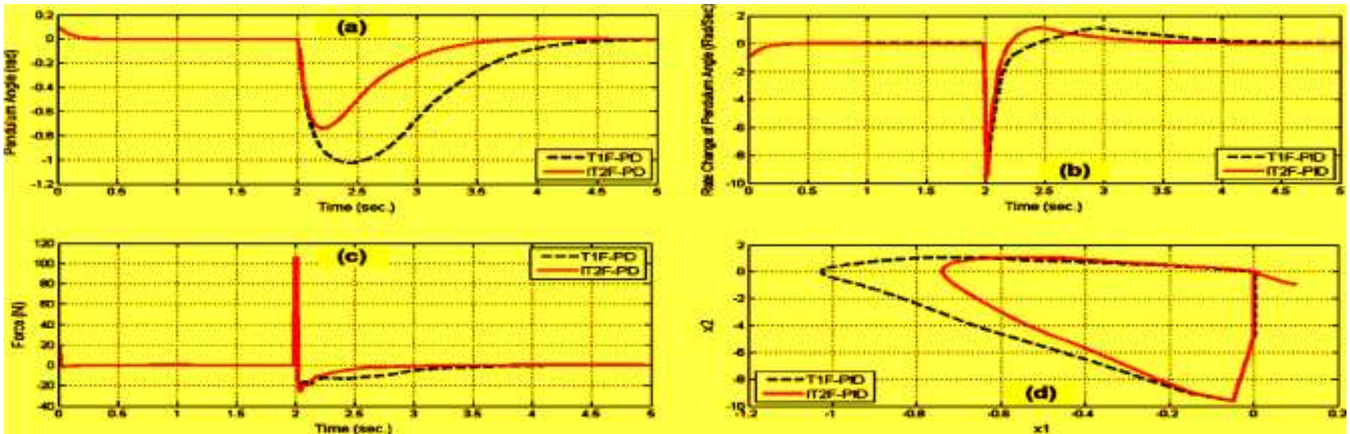

Figure 9 Response of the inverted pendulum system for angle disturbance. (a) Pendulum angle. (b) Angular velocity. (c) Control signal (d) Phase plane trajectory.

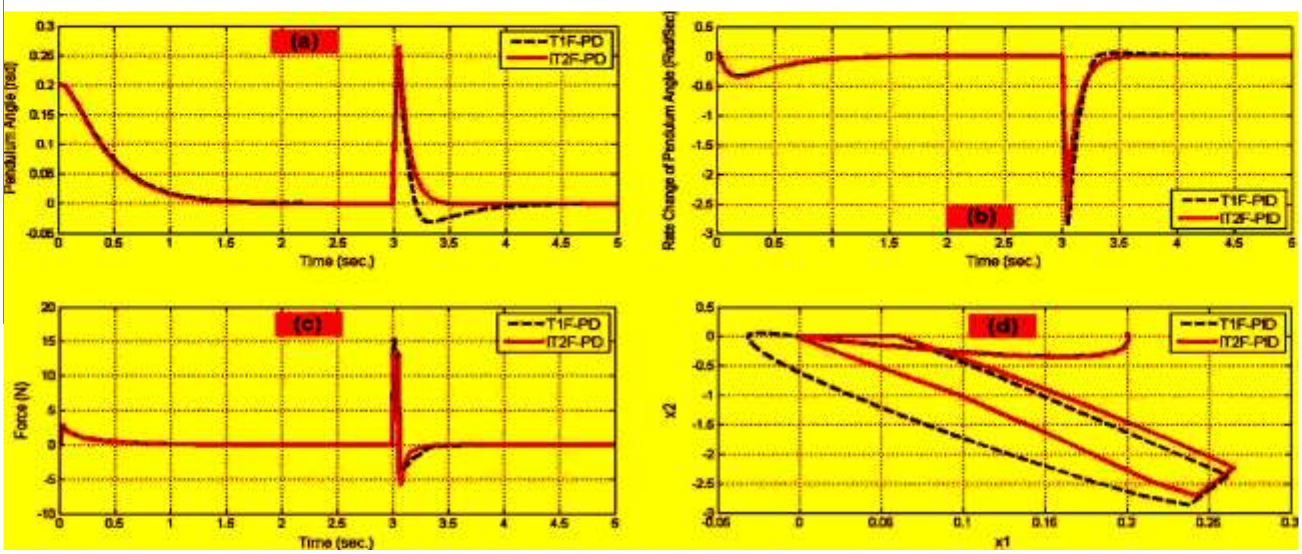

Figure 10 Response of the inverted pendulum system when the value of disturbance d=100 N. (a) Pendulum angle. (b) Angular velocity. (c) Control signal. (d) Phase plane trajectory.

Figure 8 shows the response of IP system by using the proposed controller and Type 1 FPD controller under following conditions.

$x_1 = 0.1$ rad, and $x_2 = 0$.

Figure 8 d shows the phase plane trajectory which gives a push to an oscillating system for Type 1 FPD controller but system moves to the origin for proposed controller indicating that system is stable.

Table 2 Parameters of IP system.

Symbol	Parameter	Values
m_c	Mass of cart	0.6kg
m_p	Mass of IP	0.3kg
g	Acceleration due to gravity	9.8m/s^2
L	Half length of IP	0.6m

4.3 Task 3: external disturbances

Figure 9 shoes the response of type 1 FPD and IT2-FPD responses when disturbance of .06 radian is added to the IP system. Proposed controller response was significantly faster than the Type 1 FPD controller. Phase plane trajectory of inverted pendulum indicates system is stable for both Type 1 FPD and IT2-FPD controllers. Responses of type 1 FPD and IT2-FPD controllers are shown in figure 10 when external disturbances were added 100 N . Here d indicates the disturbance.

The error performance was measured by using criterion of root mean square (RMS), mean average error (MAE) and integral of square of errors (ISE). The RMS, MAE and ISE are defined in following equations

$$RMSE = \sqrt{1/N \sum e(t)^2}$$

$$MAE = \int_0^\infty |e(t)| dt$$

$$ISE = \int_0^\infty e(t)^2 dt$$

Table 3-5 list the MAE, ISE and RMSE respectively for proposed controller, type 1 FPD controller and fuzzy sliding mode control. Fuzzy sliding mode control was proposed in

[36,37]. Error performance tables shows that proposed controller IT2-PFD performs better than others.

Table 3 ISE Values

	Structure uncertainty	Angle disturbance	Force disturbance (d=100N)	Force disturbance (d=110N)	(d
T1F-PD[38]	2.5548	1.6291	89.557	96,000	
FSMC[36,37]	0.1685	2.1485	29.126	33.077	
IT2F-PD	0.0628	1.6125	24.767	31.1328	

Table 4 MAE values.

	Structure uncertainty	Angle disturbance	Force disturbance (d =100N)	Force disturbance (d =110N)
T1F-PD[38]	24.883	13.605	119.355	4200
FSMC[36,37]	4.756	21.167	69.264	74.269
IT2F-PD	1.102	12.558	50.836	58.159

Table 5: RMSE values.

	Structure uncertainty	Angle disturbance	Force disturbance (d =100N)	Force disturbance (d =110N)
T1F-PD[38]	0.0715	0.0571	0.4232	13.856
FSMC[36,37]	0.0184	0.0656	0.2414	0.2672
IT2F-PD	0.0112	0.0571	0.2226	0.2495

CONCLUSION

In this research paper, IT2-FPD controller is proposed for IP system. The proposed controller is tested for three cases.

1:Normal case.

2:Structure uncertainties.

3:External disturbances.

In normal case system was stable and response was good for both controllers type 1 FPD and IT2-FPD. In structure uncertainty case, Proposed controller was stable where as Type 1 FPD became oscillatory. In case of external disturbances, proposed controller has an ability to reduce the effect of external disturbance as compared to Type 1 FPD. More over, error values for proposed controller are also low which indicates the best performance of proposed controller towards external disturbances.

REFERENCES

[1] J.H. Kim, S.J. Oh, A fuzzy PID controller for nonlinear and uncertainsystems, SoftComput.4(2000)123–129.

[2] B.M. Mohan, A. Sinha, Mathematical models of the simplest fuzzy PI/PD controllers with skewed input and output fuzzy sets,ISATrans.47(2008)300–310.

[3] J. Jantzen, Foundations of Fuzzy Control, John Wiley & Sons Ltd., WestSussex(England),2007.

[4] R.R. Yager, D.P. Filev, Essential of Fuzzy Modeling and Control, JohnWiley &SonsInc.,1994.

[5] C. Chao, C. Teng, A PD-like self-tuning fuzzy controller withoutsteady-stateerror,FuzzySetsSyst.87(1997)141–154.

[6] H. Boubertakh, M. Tadjine, P. Glorennec, S. Labiod, Tunning fuzzy PD and PI controllers using reinforcement learning, ISA Trans.49(2010)543–551.

[7] J.M.Mendel,Type-2fuzzysetsandsystems:anoverview,IEEE Comput.Intell.Mag.29(2007).

[8] J.M. Mendel, R.I. John, Type-2 fuzzy sets made simple, IEEE Trans.FuzzySyst.10(2002)117–127.

[9] H. Hagras,Type- 2FL Cs:a new generation of fuzzy controllers, IEEE Comput.Intell.Mag. (2007)30–43.

[10] O.Castillo,P.Melin,Type-2 fuzzy logic: theory and application, Studfuzz 223(2008).

[11] J.M. Mendel, R.I. John, L. Feilong, Interval type-2 fuzzy logic systems made simple, IEEE Trans. Fuzzy Syst. 14 (2006) 808– 821.

[12] Q. Liang, J.M. Mendel, Interval type-2 fuzzy logic systems: theory and design, IEEE Trans.FuzzySyst. 8(2000)535–550.

[13] E.A.Jammeh,M.Fleury,C.Wagner,H.Hagras,M.Ghanbari, Interval type-2 fuzzy logic congestion control for video streaming across IP networks, IEEE Trans. Fuzzy Syst. 17 (2009)1123–1142.

[14] R. Sepulveda, O. Montiel, O. Castillo, P. Melin, Embedding a high speed interval type-2 fuzzy controller for a real plant into an FPGA,Appl.SoftComput.12(2012)988–998.

[15] T.Kumbasar,I.Eksin,M.Guzelkaya,E.Yesil,Intervaltype-2 fuzzyinversecontrollerdesigninnonlinearIMCstructure,Eng. Applic.Artif. Intell.24(2011)996–1005.

[16] M. El-Bardini, A.M. El-Nagar, Direct adaptive interval type-2 fuzzy logic controller for the multivariable anaesthesia system, Ain ShamsEng.J. 2(2011)149–160.

[17] M.H. Fazel, A.D. Torshizi, I.B. Turksen, B. Rezaee, A new indirect approach to the type-2 fuzzy systems modeling and design,Inform.Sci. 232(2013)346–365.

[18] S.O. Olatunji, A. Selamat, A.A. Abdul, Raheem, Improved sensitivity based linear learning method for permeability prediction of carbonate reservoir using interval type-2 fuzzy logic system, Appl. Soft Comput.14(2014)144–155.

[19] Y. Maldonado, O. Castillo, P. Melin, Particle swarm optimization of interval type-2 fuzzy systems for FPGA applications,Appl.SoftComput.13(2013) 496–508.

[20] P. Melin, L. Astudillo, O. Castillo, F. Valdez, M. Garcia, Optimal design of type-2 and type-1 fuzzy tracking controllers for autonomous mobile robots underperturbed tor quesusing a new chemical optimization paradigm, Expert Syst. Applic. 40 (2013)3185–3195.

[21] O. Castillo, P. Melin, A review on the design and optimization of interval type-2 fuzzy controllers, Appl. Soft Comput. 12 (2012)1267–1278.

[22] O. Castillo, P. Melin, Optimization of type-2 fuzzy systems based on bio-inspired methods: a concise review, Inform. Sci.205(2012)1–19.
[23] O. Castillo, P. Melin, A.A. Garza, O. Montiel, R. Sepu´lveda, Optimization of interval type-2 fuzzy logic controllers using evolutionary algorithms, SoftComput. 15(2011)1145–1160.
[24] O. Castillo, P. Melin, W. Pedrycz, Design of interval type-2 fuzzymodelsthroughoptimalgranularityallocation,Appl. Soft Comput.11(2011)5590–5601.
[25] S. Wang, G.H. Huang, A two-stage mixed-integer fuzzy programming with interval-valued membership functions approach for flood-diversion planning, J. Environ. Manage. 117(2013)208–218.
[26] S. Wang, G.H. Huang, An interval-parameter two-stage stochastic fuzzy program with type-2 membership functions: anapplicationtowaterresourcesmanagement,Stoch.Environ. Res. RiskAssess.27(2013)1493–1506.
[27] S. Wang, G.H. Huang, H.W. Lu, Y.P. Li, An interval-valued fuzzy linear programming with infinite a -cuts method for environmental management under uncertainty ,Stoch. Environ. Res. RiskAssess. 25(2011)211–222.
[28] N. Patel, M.J. Nigam, Design of fuzzy PD controller for invertedpenduluminrealtime,in:ProceedingsofICAdC,AISC 174 ,pp. 955–962.
[29] M. Akole, B. Tyagi, Design of fuzzy logic controller for nonlinearmodel of invertedpendulum-carsystem,in: National SystemsConference,NSC2008.
[30] H. Niemann, J. K. Poulsen, Analysis and design of controllers foradoubleinvertedpendulum,in:ProceedingsoftheAmerican ControlConferenceDenver,Colorado,2003,2803–2808.
[31] J.Wang,SimulationstudiesofinvertedpendulumbasedonPID controllers, Simul. Model.Pract. Theory19 (2011)440–449.
[32] V. Sukontanakarn, M. Parnichkun, Real-time optimal control forrotaryinvertedpendulum,Am.J.Appl.Sci.6(2009)1106 – 1115.
[33] S. Kizir, Z. Bingul, C. Oysu, Fuzzy control of a real time inverted pendulum system, in: 12th International Conference, KES2008,pp.674–681.
[34] H. Lu, M. Chang, C. Tsai, Adaptive self-constructing fuzzy neural network controller for hardware implementation of an inverted pendulum system, Appl.SoftComput.11(2011)3962– 3975.
[35] M. Akole, B. Tyagi, Design of fuzzy logic controller for nonlinear model of inverted pendulum-cart system, in: NationalSystemsConference,NSC 2008 ,pp. 17–19.
[36] M.R.Dastranj,M.Moghaddas,Y.Ghezi,M.Rouhani,Robust control of inverted pendulum using fuzzy sliding mode control and genetic algorithm, Int. J. Inform. Electron. Eng. 2 (2012) 773–776.
[37] W. Chang, J.B. Park, Y.H. Joo, G. Chen, Design of robust fuzzy-model-based controller with sliding mode control for SISO nonlinear systems,FuzzySetsSyst. 125(2002)1–22.
[38] N. Patel, M.J. Nigam, Design of fuzzy PD controller for inverted pendulum minrea ltime,in:Proceedings of International Conferenceon Advancesin, Computing,2012,pp.955–962.
[39] N.N. Karnik, J.M. Mendel, Centroid of a type-2 fuzzy set, Inform.Sci.132(2001)195–220.