PID CONTROLLER DESIGN AND HARDWARE IMPLEMENTATION OF IP

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ABSTRACT: *inverted pendulum (IP) is a classic control design problem. A number of controllers have been designed for the stability and robustness of this control problem. It is the area of interest for the researcher because modern world has many practical problems which are directly related to IP. In this paper, we develop a mathematical model of IP. A PID controller with the help of root locus is designed. Stability and robustness were confirmed with the help of simulation as well as hardware setup.* For this experiment, we used kit of GOOGOLTECH company linear inverted pendulum series GLIP2001 (V1.00)

INTRODUCTION

A.   Brief introduction of PID controller

PID controller is the abbreviation of the integral controller, differential controller and proportional controller.it is the old and basic control technique in the development history of the automatic control system.[1] The controller technique in which we use both PI and PD techniques to get the parameters of our own choice is called PID controller technique. Our main concern is with the improvement of steady state error and the transient parameter improvement. We can improve steady state error by adding pole like we add in PI technique. Pole effects the response of the closed loop transfer function and improves the steady state error. [2] While when we have to improve the transient response parameters of the closed loop transfer function like percentage overshoot, settling time or peak time we add zero in the transfer function as in PD to get our desired parameters. When we have to improve both the response we have to add both zero and pole to the closed loop transfer function. This technique is called PID technique.it is widely used control technique because it has an advantage, that occupies 85% to 90% of the industrial control algorithm. [2] .PID controller has the history of above than 70 years and it is still one of the main technique because of its good reliability, good structure, easy to regulate and stability in industrial control.[5,6]

A relative controller (Kp) will have the impact of dropping the rise time and will diminish, yet not ever expel, the steady state error. An integral control (Ki) will have the consequence of expelling the steady-state error, yet it might make the transient response poorer. A derivative control (Kd) will have the impact of developing the dependability of the system, dropping the overshoot, and refining the transient response. Impacts of each of controllers Kp, Kd, and Ki on a closed loop system are summarized in table 1. [2,3]

These connections may not be accurately precise, because Kp, Ki, and Kd are conditional on each other. Altering one of these variables can alter the effect of the other two. For this reason, the table should only be used as a reference when you are determining the values for Ki, Kp and Kd. [3].

Table 1.	characteristic of kp,ki,kd			
	RISE TIME	**% OS**	**SETTLING TIME**	**S-S ERROR**
Kp	Decrease	Increase	Small Change	Decrease
Ki	Decrease	Increase	Increase	Eliminate
Kd	Small Change	Decrease	Decrease	Small Change

These connections may not be precise, because Kp, Ki, and Kd are conditional on each other. Altering one of these variables can alter the effect of the other two. For this reason, the table should only be used as a reference while determining the values for Ki, Kp and Kd. [3]

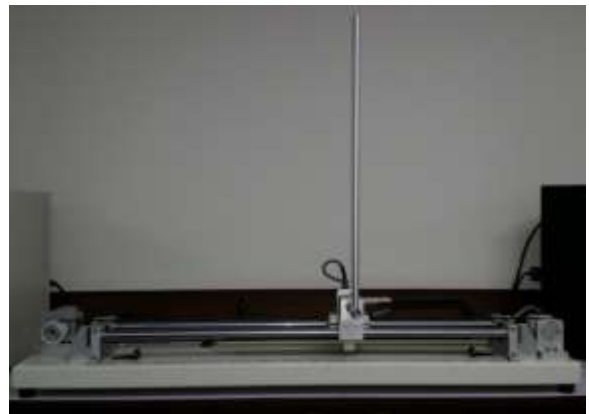### B.   Brief introduction of the inverted pendulum

Figure 1 Inverted pendulum setup

An inverted pendulum is a pendulum that has its focal point of mass over its pivot point. It is regularly connected with the pivot point appended on a truck that can move level and might be known as a truck and post Most applications bound the pendulum to 1 level of freedom by settling the shaft to a hub of turn. While a typical pendulum is steady when hanging downwards, a rearranged pendulum is actually

unsteady, and must be powerfully adjusted keeping in mind the end goal to stay vertical; this should be possible either by applying torque at the pivot point or by moving the hub point level as a feature of an input system. A straightforward demo of moving the pivot point in an input system is accomplished by adjusting an altered shaft on the end of one's finger. The modified pendulum is a common issue in elements and control hypothesis and is utilized as a standard for testing control procedures. [5]. An inverted pendulum is shown is Figure 1.

C.   Applications

Basically, in the case of inverted pendulum it is not naturally stable we have to make it stable by monitoring the position and angle of the pendulum pole with the help of feedback control and in return, we balance the IP in the vertical direction with the help of PID controller. A moving cart helps us to achieve this balance. The cart inverted pendulum is widely used in educational and research institute whose mechanical model gives resemblance to the missile and rocket launchers guidance control.[1] In these systems, in spite of the disturbance, the rocket booster have to keep the rocket stable [8] and keep it in some specific position and angle to get the desired result. The controlling of the inverted pendulum can be viewed as like the controlling of the under-actuated automated arm, which has less number of the actuator in contrast with the level of flexibility.

Design and application of moveable wheeled inverted pendulum systems have captured a lot of attention recently [4] and at least one commercial artefact, the Segway, see Fig 2, is offered. Such vehicles are of interest because they have a small trail.

In this paper, we will achieve the stability of inverted

Figure 3 HRP-4C Humanoid: stable walking under linear inverted pendulum model [4]

pendulum using PID controller design technique. In section 2 and 3 mathematical modelling and controller design of IP is done respectively. Then results are counter confirmed in hardware setup.

Mathematical Modeling of IP

The description of the model of the inverted pendulum was acquired by utilizing Lagrange Equation, which is one of the techniques that can be utilized to get a numerical displaying for an extremely complex and non-linear mechanical system like an inverted pendulum, [3]. The free body outline of the system is as shown in Fig. 4.

The model of an inverted pendulum on a cart was acquired utilizing Lagrange Equation which depended on the distinction between Kinetic (K_E) and Potential energy (P_E) of the system. The numerical model is fundamentally required for recreation in

Figure 2   The Segway: A mobile wheeled inverted pendulum[4]

Another example of an inverted pendulum is the humanoid robots that can balance on their foot and walk and run like a human being.as shown in Figure 3.
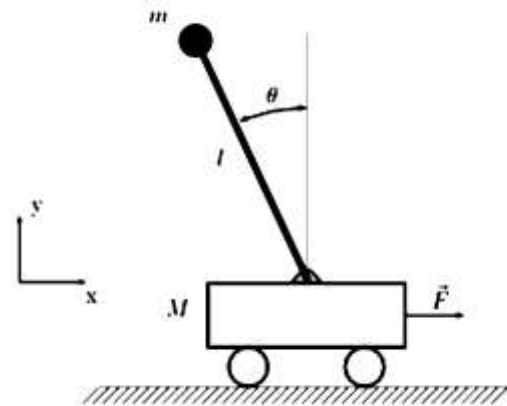

Figure 4 Inverted pendulum on Cart.

MATLAB/Simulink environment furthermore for the controller improvement for the system. The numerical conditions of both the position of the pendulum and angle of the cart are shown in differential conditions as:

Langragian (L) natural form is given by[7]

$$L=K_E-P_E \qquad\qquad (1)$$

$$L = \frac{1}{2}(M+m)\dot{x}^2 - ml\dot{x}\dot{\theta}cos\theta + \frac{1}{2}ml^2\dot{\theta}^2$$
$$- mglcos\theta \qquad (2)$$

x denotes carts position so by using equations (1) and (2)

$$F = \frac{d}{dt}\frac{\partial L}{\partial \dot{x}} - \frac{\partial L}{\partial x} \qquad (3)$$

$$0 = \frac{d}{dt}\frac{\partial L}{\partial \dot{\theta}} - \frac{\partial L}{\partial \theta} \qquad (4)$$

So,

$$\ddot{\theta} = \frac{Fcos\theta - (M+m)gsin\theta + ml(sin\theta cos\theta)\theta^2}{mlcos^2\theta - (M+m)l} \qquad (5)$$

$$\ddot{x} = \frac{u + ml(sin\theta)\theta^2 - mgcos\theta sin\theta}{M + m - mcos^2\theta} \qquad (6)$$

By linearizing about equilibrium point's equation (5) and (6) gives

$$(\theta = 0, sin\theta \rightarrow \theta, cos \rightarrow 1 \ and \ \dot{\theta}^2 \rightarrow 0)$$

ε

εThe equation will become

$$\ddot{\theta} = \frac{F - (M+m)g\theta}{Ml} \qquad (7)$$

ε

$$\ddot{x} = \frac{F - mg\theta}{M} \qquad (8)$$

In the form of state space

$$\dot{x}(t) = Ax(t) + Bu(t) \qquad (9)$$
$$y(t) = Cx(t) + Du(t) \qquad (10)$$

$$\frac{d}{dx}\begin{bmatrix}\theta \\ \dot{\theta} \\ x \\ \dot{x}\end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ \frac{(M+m)g}{Ml} & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ \frac{-mg}{M} & 0 & 0 & 0 \end{bmatrix}\begin{bmatrix}\theta \\ \dot{\theta} \\ x \\ \dot{x}\end{bmatrix}$$

$$+ \begin{bmatrix} 0 \\ \dfrac{-1}{Ml} \\ 0 \\ \dfrac{1}{M} \end{bmatrix}F \qquad (11)$$

$$y = \begin{bmatrix}\theta \\ x\end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}\begin{bmatrix}\theta \\ \dot{\theta} \\ x \\ \dot{x}\end{bmatrix} \qquad (12)$$

Table 2 SYSTEM PARAMETERS	
Parameters	Values
Length of the pendulum(L)	0.25m
Mass of cart (M)	1.096kg
Mass of pendulum rod(m)	0.109kg
Acceleration due to gravity(g)	9.8ms⁻²

After putting values from the table into a system of equations, we get.

$$A = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 32.667 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ -1.633 & 0 & 0 & 0 \end{bmatrix}$$

$$B = \begin{bmatrix} 0 \\ -2.381 \\ 0 \\ 0.833 \end{bmatrix}$$

$$C = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}$$

$$D = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

CONTROLLER DESIGN & SIMULATION RESULTS
As we have derived the transfer function of an inverted pendulum. The response. $G(s) = \frac{3}{s^2 - 29.4}$, there are thus two poles located at +5.42 and -5.42 ,as shown in figure 6. This system is unstable as one pole lies on the right half plane. Now our centre of interest is to compensate and stabilize this system by adding poles or zeros to the system.
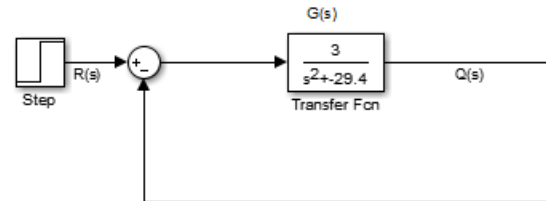

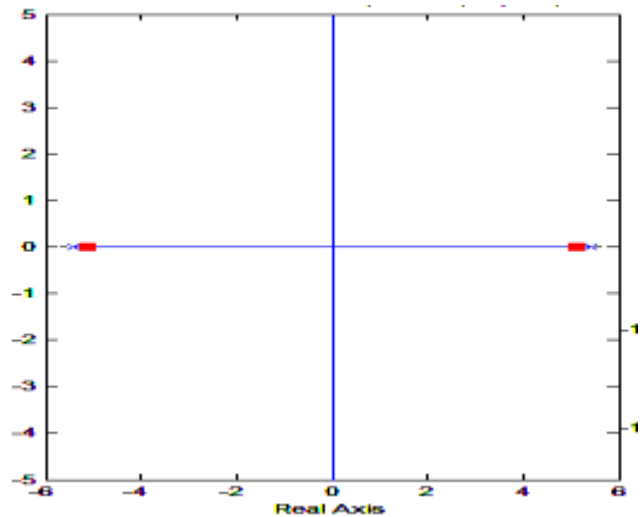**Figure 5 Block diagram of the Transfer function**

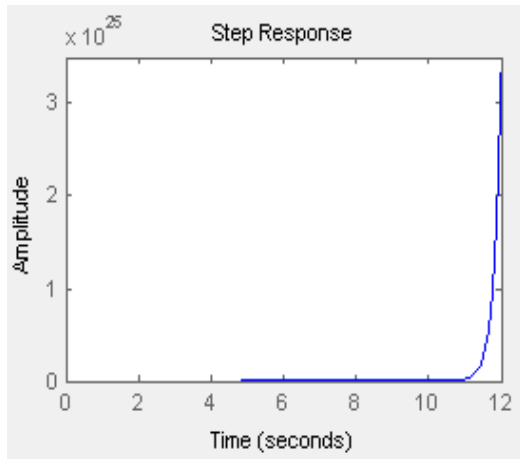**Figure 6 unstable system without controller design.**

Figure 7 Step response of the uncompensated system

Table 3 Table for uncompensated system	
Uncompensated system	$\dfrac{3}{s^2 - 29.4}$
T_s	Undefined
T_p	Undefined
$E(\infty)$	Undefined
Location of poles	-5.42, +5.42

These system parameters are undefined because we have an unstable system.

1.1   Steps Involved in PID Design

Step 1.   Requirements

T_S	0.5sec
%O.S	11.5%
$e(\infty)$	0
Z	0.55

Step 2.   Add a pure integrator in the system.
Step 3.   Stimulate the system and verify that it satisfies the requirements or not. In the given system after adding the pure integrator at the forward path we are achieving the steady-state error of our classic.
Step 4.   Add the zeros in the forward path to achieve the transient parameters. Now we have to achieve our requirement by adding the dominant zeros. For this, our gain should be equal to 30. For the sack of finding the location of dominant zeros, we have to calculate its angle contribution to make sure that it must lie in the root locus. By calculation, we find the location of dominant zeros at   -10±3i.
Step 5.   Again Stimulate the system by adding zeros of the given location and verify that our system requirement satisfies or not. Further explanation of the effect of zeros and poles were discussed in the next section.

1.2   Effect of poles on system type

By increasing system type we can achieve our goal of minimizing the steady state error. As this system focuses on a unity negative feedback system. The values of the static error constants depend upon the form of G(s), substantially the number of pure integration in the forward path. Since steady state errors are dependent upon the number of integrations in the forward path, we increase the type of our system [1]. After increasing the system type, new pole locations are -5.42, 0, and 5.42. By inspecting the system we came to know that our system is still unstable as shown in figure8. By increasing the system type we can achieve the less or zero static error but systems parameters are disturbed i.e. %overshoot, settling time, rise time and peak time. This parameter can be easily attained by adding zeroes to the system. The given graph figures show the
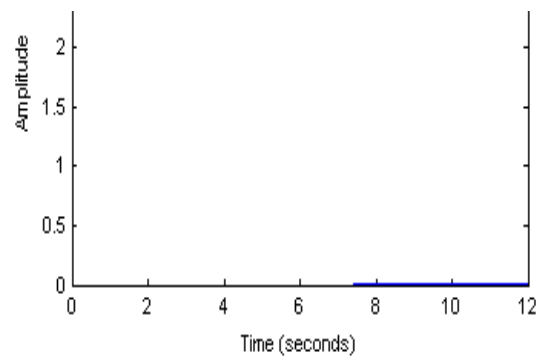

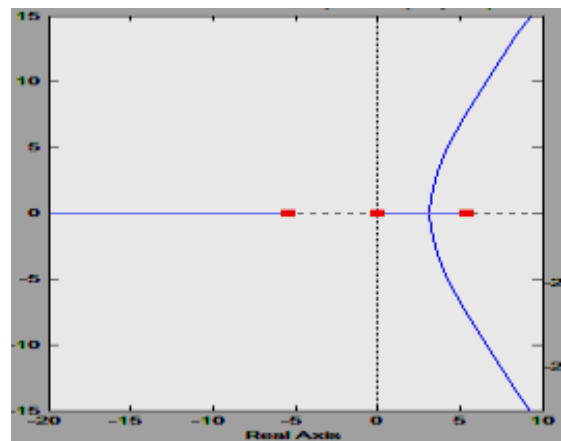**Figure 8 (a) Effect of adding a pole**

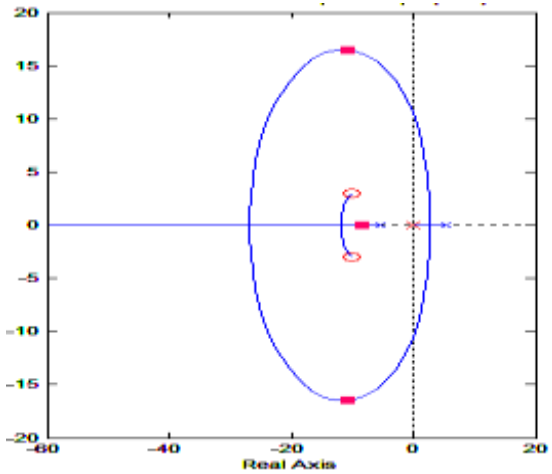**Figure 8 (b) Effect of adding a pole at the origin**

Figure 9   Effect of adding zeros and poles

1.3   Effect of Zeros on the System.

The target is to design a response that has sensible percent overshoot and a shorter settling time than the uncompensated system. This can be attained by adding zeros to the system [2]. In this section, we add dominant zeros to the system located at   -10$\pm$3i . After adding poles and zeros to the system, if the point is on the root locus, then a simple gain tuning is all that is required in order to meet the transient response specification. The effect of adding zeros and pole to the system , in figure 9, 10. In the figure, it is clear that the point is on the root locus and by tuning the gain we have improved the transient parameters.

1.4   Parameters compensated system:

In the previous section, we have seen the combined effect of poles and zeros to the system and after compensation we get compensated transfer function $G_c(s)$

$$G_{c(s)} = \frac{K(s-10+3i)(s-10-3i)}{s} \qquad (13)$$

The block diagram of the compensated system is shown in figure 11.
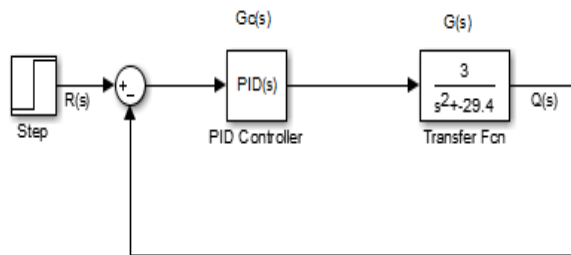

Figure 11   Block diagram of the compensated system

In this section, we are interested in the PID controller's gain values i.e. K_p , K_d and K_i . Assuming K=30 we have $G_c(s)$

$$G(s) = \frac{10(s^2-200s+1090)}{s} \qquad (14)$$

As general transfer function of PID controller is

$$G(s) = \frac{K1S+K2+K3S^2}{s} \qquad (15)[9]$$

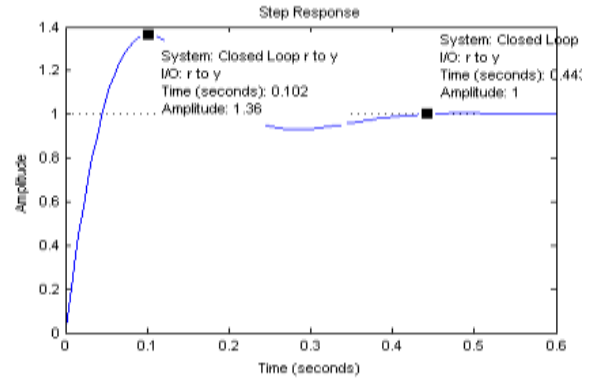Where $K_1=K_p$, $K_2=K_i$ and $K_3=K_d$ Comparing equation 14 and 15 we get

Figure 10   Step response of the theoretically compensated system

Kp =200, Kd = 10 and Ki=1090
Controller gain can be tuned to obtain the desired response. Keeping in mind the following Table I.

Hardware Representation

For this experiment, we are using kit of GOOGOLTECH company Linear inverted pendulum series GLIP2001 (V1.00)
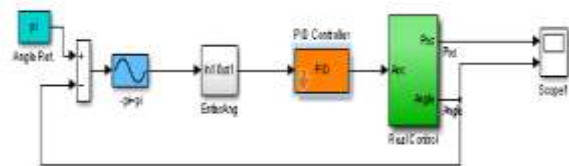

Figure 12 Kit of GOOGOLTECH

Figure 13 PID Simulink block diagram for the system

Following parameters were set in Simulink interface of hardware setup.
Kp=200, Ki=1090, Kd=10
Response of the angle of the system on giving the values of kp,kd and ki is shown in figure 14. Initially IP started swinging and after getting enough push to go vertical, it balanced itself on given parameters of Kp, Ki and Kd. Once system get balanced, it is capable to reject the external disturbance.

CONCLUSION

Control of inverted pendulum (IP) is one of the classis problems of control systems. Number of techniques and algorithms are developed to control the inverted pendulum. Because of its practical applications in many fields it is still considered to be the basic tool to test the algorithm or design technique. In this research, simulation of IP control is done in Simulink Matlab. Then simulation parameters are used in hardware setup to test the stability and robustness of system practically. Successfully obtained stability and disturbance rejection of hardware IP confirmed that simulation design is stable and robust
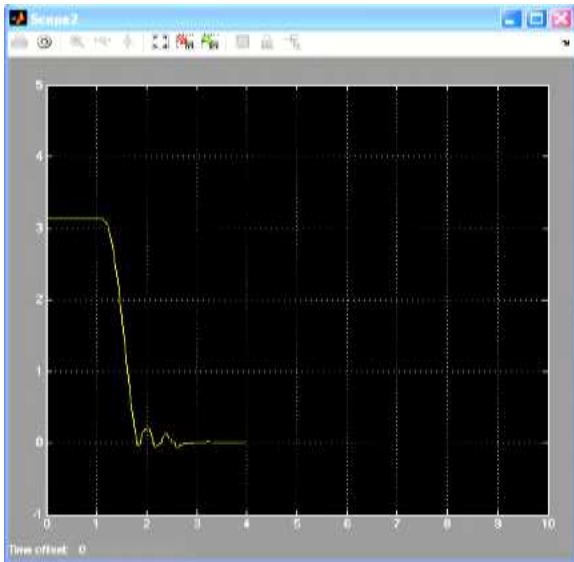

Figure 14   Angle response of the PID controlled system

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