COMPUTATIONAL STUDY FOR MEGNETOHYDRODYNAMICS BUOYANCY FLOW VERTICAL PERMEABLE PLATE WITH CHEMICAL DIFFUSION AND THERMAL RADIATION

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ABSTRACT: *A computational effort for understanding of electrically conducting viscous fluid buoyancy flow along a porous vertical plate has been made in the presence of space concentration, applied magnetic field, external heat and radiative heat sources. Appropriate similarity transforms are employed to get transform the governing partial differential equations into ordinary differential form which are then coded for numerical treatment in Mathematica. The physical aspects of the study are revealed through several computations for various parameters of influence. The results for non-dimensional velocity, concentration and temperature functions are presented and discussed.*

Key words: Buoyancy flow, Similarity transforms, Porous medium, Magnetohydrodynamic flow, Thermal radiation

1. INTRODUCTION

The study of fluid flow and heat transfer across a porous medium is a subject of interest for many researchers working in the area of two dimensional flows. The reasons lies in the facts that such kinds of investigations have application in the manufacturing industry. The flow in the porous medium is use to study in the migration of underground water. The initial work was Madhusudhana and Viswanatha [1] thermal radiation and heat transfer effect on the steady MHD fluid flow past a vertical porous plate with injection.

The flow and heat transfer over an accelerated sheet is considered an important research area because of its applications in hot rolling, wire drawing, glass fiber production, manufacturing plastic films and extrusion of a polymer in a melt spinning process. Sakiadis [1, 2] was the first to propose and analyze the moving surface problem based on the boundary layer approximation. Crane [3] investigated the flow caused by the stretching of a sheet. Many researchers such as Dutta et al. [4], Sajjad et al. [5], Chen and Char [6] extended the work of Crane [3] by including the effect of heat and mass transfer analysis under different physical situations. Sajjad and Farooq [7] considered unsteady MHD flow and heat transfer for Newtonian fluids over an exponentially stretching sheet. Cortell [8-9] studied the flow and heat transfer of a fluid through a porous medium over a stretching surface. Hussain et al. [10] examined MHD stagnation point flow of micropolar fluids towards a stretching sheet. Farooq and Sajjad [11] considered MHD flow and heat transfer through a porous medium over a stretching/shrinking surface with suction. Xu and Liao [12], Hayat et al. [13] presented various aspects of such problem. Ahmad et al. [14] obtained closed form solution for a viscous, incompressible, MHD flow over a porous stretching sheet.

The study of magneto-hydrodynamic flow is of considerable interest in modern metal-working processes, fusing of metals, cooling of the first wall inside a nuclear reactor etc. Pavlov [15] studied the boundary layer flow of an electrically conducting fluid due to stretching of a plane elastic surface in the presence of a uniform transverse magnetic field. El-Hakiem et al. [16] analyzed the effect of viscous and Joule heating on the flow of an electrically conducting and micro polar fluid past a plate whose temperature varies linearly with the distance from the leading edge in the presence of a uniform transverse magnetic field. Further, Fang and his co-authors [17–21] discussed some other important aspects of shrinking flow. Arthur and Seini [22] investigated the hydromagnetic stagnation point flow of an incompressible viscous electrically conducting fluid towards a stretching sheet in the presence of radiation and viscous dissipation. Hussain et al. [23] obtained numerical solution for a similar flow between two disks in the presence of a magnetic field. Ahmad et al. [24] also obtained numerical solution for hydromagnatic fluid flow between two horizontal plates, both the plates being stretching sheets. Here, MHD buoyancy flow due to a vertical permeable sheet is considered to include thermal radiation and chemical diffusion. This work has a motivation to present a complete aspects of the study already made by Rao et al.[25]

2. Mathematical Analysis

Consider the mixed flow convection in a porous medium saturated with a Newtonian fluid on a vertical flat plate. The temperature of the ambient medium is T_∞ and the wall temperature is T_w. The flow along the vertical flat plate contains a species soluble in the fluid with concentration function C, The concentration at the plate surface is C_w and the solubility of species away from the plate is $C\infty$. An external heat source of strength Q_0 and radiative heat source of strength q_r are present.

The fluid and the porous medium are in local thermodynamic equilibrium, The flow is laminar, steady-state and two-dimensional. The velocity components along x-axis and y-axis are respectively denoted by u and v. A magnetic field of strength B is applied normally to the sheet. The governing equations describing the conservation of mass, momentum, energy and concentration can be written as follows:

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0 \qquad (1)$$

$$u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=\nu\frac{\partial^2 u}{\partial y^2}-(\frac{\sigma B^2}{\rho}+\frac{\nu}{K'})u \quad (2)$$

$$u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}=\frac{k1}{\rho C_p}\frac{\partial^2 T}{\partial y^2}+\frac{Q_0}{\rho C_p}(T-T_\infty)-\frac{1}{\rho C_p}\frac{\partial q_r}{\partial y} \quad (3)$$

$$u\frac{\partial C}{\partial x}+v\frac{\partial C}{\partial y}=D\frac{\partial^2 C}{\partial y^2} \quad (4)$$

where k_1 is thermal conductivity, D is mass diffusivity, C_p is specific heat at constant pressure, K′ is porosity of medium, Q_0 is heat source strength.

The boundary conditions of the problem are

$u=bx,\quad v=v_1,\quad T=T_w,\ C=C_w,$ at y=0

$u\to 0,\quad T\to T\infty,\quad C\to C_\infty$ as $y\to\infty$, (5)

where T_W, C_W, T_∞ and C_∞ are constant, v_1 is suction velocity.

Now using non dimensional functions:

$\psi=\sqrt{b\gamma x}f(\eta),\quad \eta=\sqrt{\frac{b}{\gamma x}}y,\quad \theta(\eta)=\frac{T-T\infty}{Tw-T\infty},\quad u=\frac{\partial\psi}{\partial y}=bf'(\eta)$, $v=-\frac{\partial\psi}{\partial x}=-[\frac{1}{2}\sqrt{\frac{b\nu}{x}}f(\eta)-\frac{1}{2}\frac{by}{x}f'(\eta)]$,

$\phi(\eta)=\frac{C-C_\infty}{C-C_\infty}$, and $q_r=-(\frac{4\sigma}{3\delta})\frac{\partial T^4}{\partial y}$

Where σ is the Stefan Boltzmann constant and δ is the Rosseland mean absorption coefficient. Temperature differences in the flow are assumed to be sufficiently small such that T^4 may be expressed as a linear function of temperature. Expanding T^4 about T_∞ in Taylor's series and neglecting higher order yield

$$T^4\cong 4T_\infty^3T-3T_\infty^3$$

The stream function ψ satisfies continuity equation (1) and the Eq (2) to Eq (4) and the governing equations becomes:

$$2f'''+ff''-(M+K)f'+Gr\,\theta=0 \quad (7)$$

$$(2+R_n)\theta''+Prf\theta'+Q\,\theta=0 \quad (8)$$

$$2\emptyset''+Scf\emptyset'=0 \quad (9)$$

The boundary conditions becomes

$f'=1, f=f_a, \theta=1, \phi=1$, at $\eta=0$

$f'\to 0,\quad \theta\to 0, \phi=0$ as $\eta\to\infty$ (10)

Where $S_c=\frac{\nu}{D}$ is Schmidt number, $M=\frac{2x\sigma B^2}{b\rho}$ is magnetic parameter, $K=\frac{2\gamma x}{K'b^2}$ is porosity parameter $p_r=\frac{\nu}{k1}$ Prandtl number

$Q=\frac{\nu x Q_0}{kb(T_w-T_\infty)}$ is heat source parameter, Rn is radiation parameter and $f_a=-2v_1\sqrt{\frac{x}{b\gamma}}$ is suction parameter.

3. RESULTS AND DISCUSSIONS

The influence of emerging parameters namely magnetic parameter m, porosity parameter K, Grasnof number, radiation parameter R_n , Parandtl number P_r, heat source parameter Q, schmit number S_c, wall suction parameter f_a, is observed on physical quantities namely. Velocity, heat and specie concentration functions. The graphs of these quantities are presented and discussed.

The set of coupled ordinary differential equations namely Eq (5) to Eq (8) is sought to be solved numerically as analytical solution seems vary difficultion the presence of non linear terms. Hence, these equations have been solved by the command ND solve of Mathematica

The increase in the parameter associated with magnetic field strength and porosity of medium namely M and K respectively demonstrated decreasing effects on $f'(\eta)$ as presented respectively in Fig.1 and Fig.2. Fig.3 shows that suction at wall represented by f_a also causes decrease in $f'(\eta)$. Fig.4 is mapped to show the increasing effect of Garshof number Gr on $f'(\eta)$. Fig.5 and Fig.6, shows that specie concentration $\phi(\eta)$ reduces with increase in S_c and f_a, respectively.

The temperature function $\theta(\eta)$ is decreasing with increasing strength of parameters M and P_r as shown respectively in Fig.7 and Fig.8. But Fig.9 and Fig.10 depict that $\theta(\eta)$ is increasing when R_n and Q are increased respectively.

The increment in f_a demonstrates decrease in $\theta(\eta)$ as presented in Fig.11. The increases in the values of Garshof number Gr maps the decreasing effect on $\theta(\eta)$ as depicted in Fig.12

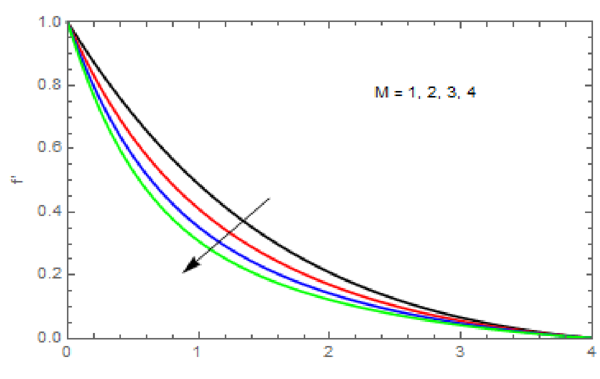

Fig 1: Graph of $f'(\eta)$ under the effect of M

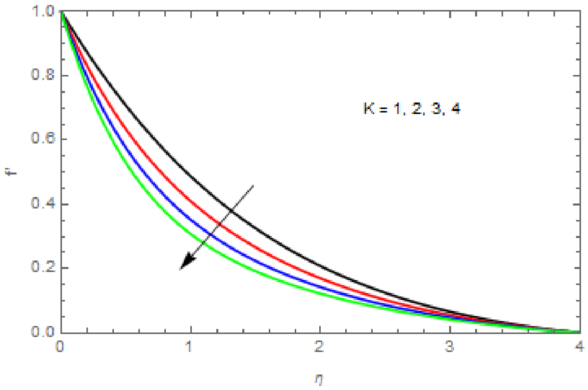

Fig 2: Graph of $f'(\eta)$ under the effect of K

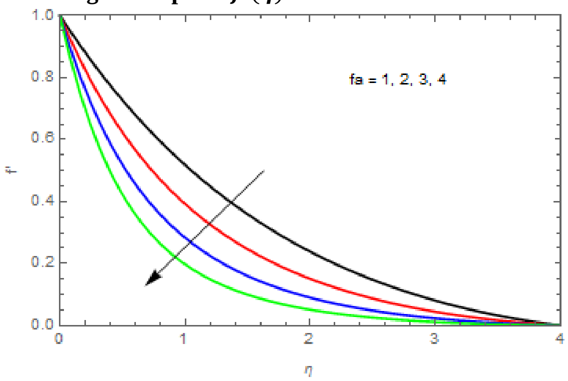

Fig 3: Graph of $f'(\eta)$ under the effect of fa

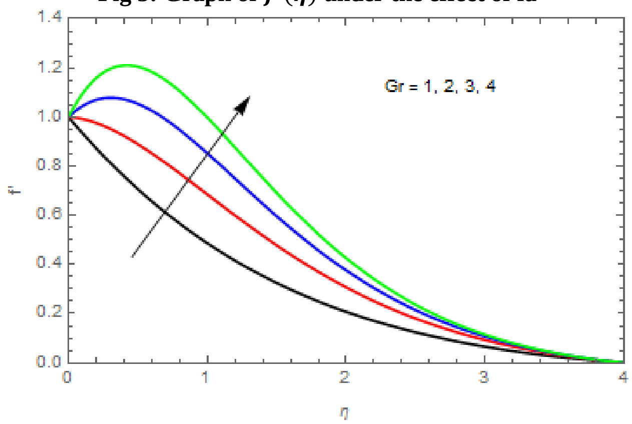

Fig 4: Graph of $f'(\eta)$ under the effect of Gr

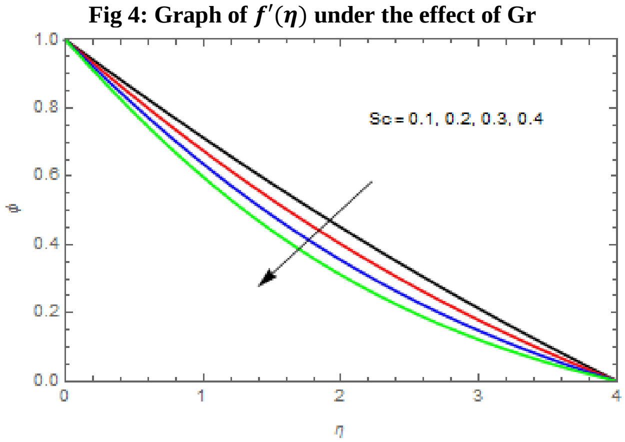

Fig 5: Graph of $\phi(\eta)$ under the effect of Sc

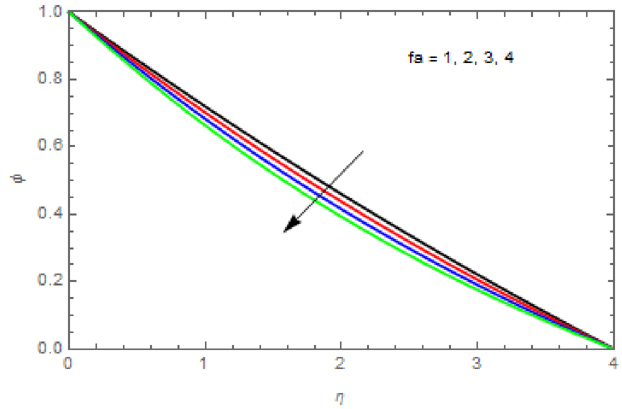

Fig 6: Graph of $\phi(\eta)$ under the effect of fa

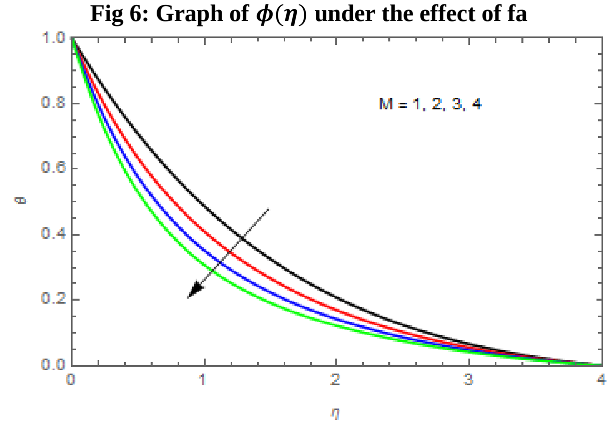

Fig 7: Graph of $\theta(\eta)$ under the effect of M

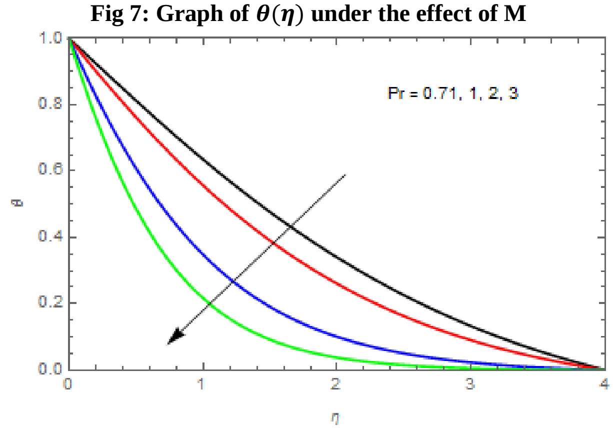

Fig 8: Graph of $\theta(\eta)$ under the effect of Pr

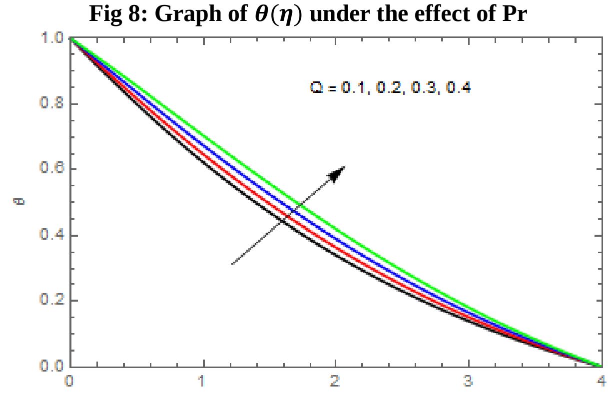

Fig 9: Graph of $\theta(\eta)$ under the effect of Q

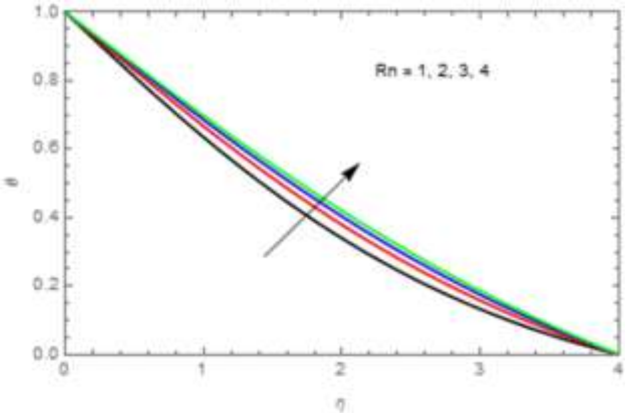

Fig 10: Graph of $\theta(\eta)$ under the effect of Rn

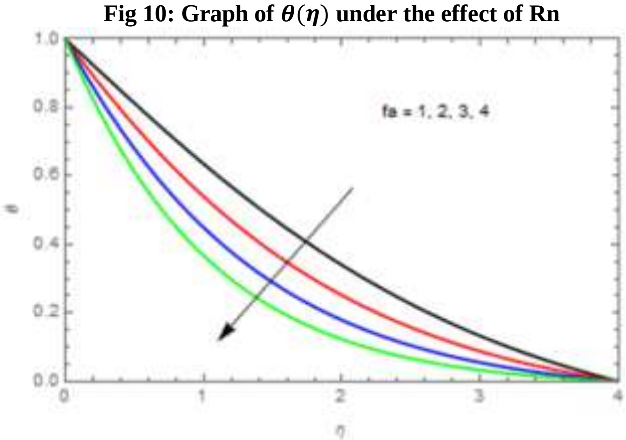

Fig 11: Graph of $\theta(\eta)$ under the effect of fa

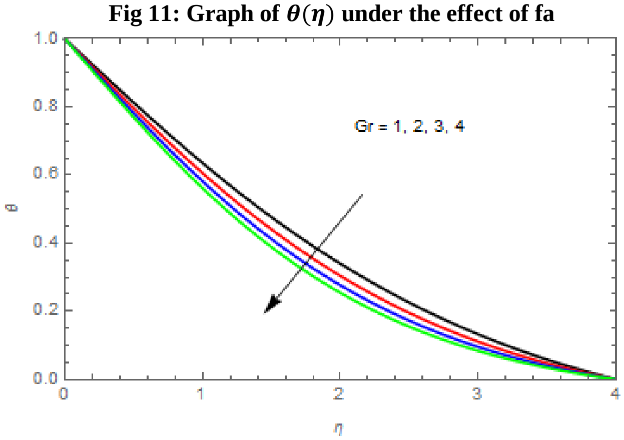

Fig 12 Graph of $\theta(\eta)$ under the effect of Gr

REFERENCES

[1] B.C. Sakiadis, Boundary layer behavior on continuous solid surface: I. Boundary layer equations fortwo dimensional and axisymmetric flow, AIChE Journal, vol. 7, no. 1, pp. 26–28, 1961.

[2] B.C. Sakiadis, Boundary layer behavior on continuous solid surface: II. Boundary layer equations for two dimensional and axisymmetric flow, AIChE Journal, vol. 7, no. 1, pp. 221–225, 1961.

[3] L. J. Crane,Flow past a stretching plate, Zeitschrift fur Ange- ¨ wandte Mathematik und Physik, vol. 21, no. 4, pp. 645–647, 1970.

[4] B. K. Dutta, P. Roy, and A. S. Gupta, Temperature field in flow over a stretching sheet with uniform heat flux, International Communications in Heat and Mass Transfer, vol. 12, no. 1, pp. 89–94, 1985.

[5] Sajjad Hussain, M. A. El-Hakeem, Farooq Ahmad,Numerical Solution of Micropolar Fluid Flow over a Stretching Sheet in aPorous Medium, J. Appl. Environ. Biol. Sci., 5(8S)43-51, 2015

[6] C. K. Chen and M. I. Char, Heat transfer of a continuous, stretching surface with suction or blowing, Journal of Mathematical Analysis and Applications, vol. 135, no. 2, pp. 568–580, 1988.

[7] Sajjad Hussain, Farooq Ahmad, Unsteady MHD Flow And Heat Transfer For Newtonian Fluids Over An Exponentially Stretching Sheet, Sci.Int.(Lahore), 27(2), 853-857, 2015.

[8] R. Cortell, Flow and heat transfer of a fluid through a porous medium over a stretching surface with internal heat generation/absorption and suction/blowing, Fluid Dynamics Research, vol. 37, no. 4, pp. 231–245, 2005.

[9] R. Cortell, Effects of viscous dissipation and work done by deformation on the MHD flow and heat transfer of a viscoelastic fluid over a stretching sheet, Physics Letters A, vol. 357, no. 4-5, pp. 298–305, 2006.

[10] S. Hussain, M. A. Kamal,, F. Ahmad, and M. Shafique, MHD stagnation point flow of micropolar fluids towards a stretching sheet, Sci.Int.(Lahore), 26(5), 1921-1929, 2014.

[13]. F. Ahmad, S. Hussain, M. Ali, An Analytical Solution of MHD Flow over Porous Stretching Sheet, J. Basic. Appl. Sci. Res., 4(3) (2014) 160-167.

[12] H. Xu and S. J. Liao, Series solutions of unsteady magnetohydrodynamic flows of non-Newtonian fluids caused by an impulsively stretching plate, Journal of Non-Newtonian Fluid Mechanics, vol. 129, no. 1, pp. 46–55, 2005.

[13] T. Hayat, Z. Abbas, and M. Sajid, "Series solution for the upper-convected Maxwell fluid over a porous stretching plate," Physics Letters A, vol. 358, no. 5-6, pp. 396–403, 2006.

[14] Farooq Ahmad, Sajjad Hussain, A. M. Alanbari and R. S. Alharbi (2015), MHD flow and heattransfer through a porous medium over a stretching/shrinking surface with suction, Sci.Int.(Lahore), 27(2), 931-935.

[15]. Pavlov, K. B., 1974. Magnetohydrodynamic flow of an incompressible viscous fluid caused by deformation of a surface, Magnitnaya Gidrodinamika, vol. 4, pp. 146-147.

[16]. El-Hakiem M.A., Mohammadein A. A., El-KabeirS.M.M.andGorlaR.S.R., 1999. Joule heating effects on magnetohydrodynamic free convection flow of a micropolar fluid, Int. Commun. Heat Mass Transfer, Vol. 26, pp. 219--227.

[17] Fang T, Yao S, Zhang J, Aziz A. Viscous flow over a shrinking sheet with a second order slip flow model. Commun Nonlinear Sci Numer Simul 2010;15:1831–42.

[18] Fang T, Liang W, Lee CF. A new solution branch for the Blasius equation – a shrinking sheet problem. Comput Math Appl 2008;56:3088–95.

[19] Fang T, Zhang J, Yao S. Slip magnetohydrodynamic viscous flow over a permeable shrinking sheet. Chin Phys Lett 2010;27:124702.
[20] Fang T, Zhang J. Thermal boundary layers over a shrinking sheet: an analytical solution. Acta Mech 2010;209:325–43.
[21] Yao S, Fang T, Zhong Y. Heat transfer of a generalized stretching/shrinking wall problem with convective boundary conditions. Commun Nonlinear Sci Numer Simul 2011;16:752–60.
[22]. Arthur, E. M., I. Y. Seini. Hydromagnetic stagnation point flow over a porous stretching surface in the presence of radiation and viscous dissipation. *Applied and Computational Mathematics* 3(5): 191-196(2014).
[23]. S. Hussain, M. A. Kamal, F. Ahmad, M. Ali, M. Shafique and S. Hussain, Numerical Solution for a Similar Flow between Two Disks in the Presence of a Magnetic Field, Appl. Math. 4 (8), 2013, 1163-1167.
[24]. Farooq Ahmad, Sajjad Hussain, Numerical Solution for Hydromagnatic Fluid Flow between Two Horizontal Plates, Both the Plates Being Stretching Sheets, J. Basic Appl. Sci. Res. 2014 4(5):181-185.
[25]B. Madhusudhana Rao, G. Viswanatha Reddy, M.C.Raju, Thermal Radiation and Heat Transfer Effects on the Steady MHD Fluid Flow past a Vertical Porous Plate with Injection. American-Eurasian Journal of Scientific Research 12 (4): 180-185, ISSN 1818-6785, 2017