

# THERMAL RADIATION WITH SORET AND DUFOUR EFFECT FOR MHD FLOW PASSING THROUGH POROUS MEDIUM

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**ABSTRACT:** Computational analysis of magneto hydrodynamic flow of viscous fluids is considered owing to moving boundary vertical and parametric. The problem examines the thermal radiations sort and Dufour effects along with other influential parameters due to magnetic field. Thermal diffusivity and porosity of surfaces and medium. The computational results for flow speed, temperature and specie concentrations are determines for various values of above mentioned parameters. The computational techniques ND solve instruction in Mathematica has been employed on the transformed set of ordinary differential equations. The results are presented graphical patterns and have been discussed.

**Key words:** Mixed convection, Porous medium, Magnetohydrodynamic, Soret and Dufour effects, Porous plate, Thermal radiation.

## 1. INTRODUCTION

Most of beneficial studies have been found in useful manner to analyze the influence of the combined heat and mass transfer process by natural convection in a thermal radiation with Soret and Dufour effect passing through porous medium. This is due to its various applications, such as development of advanced technologies as well as fluid flow in MHD with Soret and Dufour effects through porous medium and also use in nuclear waste management, etc. Here stratified porous medium means that the ambient concentration of dissolved constituent and the temperature is not uniform and varies as a linear function. It has been found that an energy flux can be generated not only by temperature gradients but by composition gradients and also radiation as well. The mass fluxes can be created by temperature gradient and this is the Soret or thermal-diffusion effect. On the other hand, The energy flux caused by a composition gradient is called the Dufour or diffusion-thermo effect. These effects are considered as second-order phenomena, on the basis that they are of smaller order of magnitude than the effects described by Fourier's and Fick's laws, but they may become significant in areas such geosciences or hydrology. Madhusudhana *et al* [1] studied magneto hydrodynamic transient free convection and chemically reactive flow past a porous vertical plate with radiation and temperature gradient dependent heat source with the boundary conditions in slip flow regime. [2] analyze the effects of Dufour and Soret and magneto hydrodynamic on Darcy-Forchheimer mixed convection flow with heat and mass transfer from a vertical flat plate embedded in a saturated porous medium taking into the influence thermophoresis, viscous dissipation and radiation. Other relevant work to this paper could be found in [3].

Magneto hydrodynamic flow and heat and mass transfer processes occur in many industrial applications such as the geothermal system, aerodynamic processes, chemical catalytic reactors and processes, electromagnetic pumps, and magneto hydrodynamic power generators. Many studies have been carried out to investigate the magneto hydrodynamic transient free convective flow. Gupta [4] first studied

transient free convection of an electrically conducting fluid from a vertical plate in the presence of magnetic field. Free convection effects on flow past an exponentially accelerated vertical plate was studied by Singh and Kumar [5]. Jha *et al.* [6] analyzed mass transfer effects on exponentially accelerated infinite vertical plate with constant heat flux and uniform mass diffusion. Muthucumaraswamy *et al.* [7] studied the flowpast an exponentially accelerated infinite vertical plate in the presence of variable surface temperature. Again Muthucumaraswamy *et al.* [8] analysed the combined effects of heat and mass transfer on exponentially accelerated vertical plate in presence of uniform magnetic field. Rajesh *et al.* Hassan *et al.* [9] studied the micropolar fluids near the stagnation point flow of electrically conducting due to a surface with the boundary in motion (stretching/shrinking). Ali *et al.*[10] obtained numerical solution of a viscous, incompressible, electrically conducting fluid flow and heat transfer over porous stretching sheet with injection. Sajjad *et al.* [11] studied MHD stagnation point flow of micropolar fluids towards a stretching sheet is studied in the presence of variable surface temperature. Reddy [12] analysed the effects of magnetohydrodynamic force and buoyancy on convective heat and mass transfer flow past a moving vertical porous plate in the presence of thermal radiation and chemical reaction. Hayat *et al* [13] analyzed the effects of thermal-diffusion and diffusion-thermo on MHD three-dimensional axisymmetric flow of a viscous fluid between radially stretching sheets in the presence of Hall and ion-slip currents, viscous dissipation, Joule heating and first order chemical reaction. Babu *et al.* [14] studied the effects of radiation and heat source/sink on the steady two dimensional magnetohydrodynamic (MHD) boundary layer flow of heat and mass transfer past a shrinking sheet with wall mass suction. Hassan *et al.* [15] worked on mathematical analysis and numerical solution for micropolar fluids flow due to a shrinking porous surface in the presence of magnetic field and thermal radiation.

This computational study examines thermal radiation with Soret and Dufour effect for MHD flow passing through porous medium to extend the problem already discussed by Okedoye and Akinrinmade.[16].

## 2. MATHEMATICAL ANALYSIS

Consider the mixed convection flow in a porous medium saturated with a Newtonian fluid on a vertical flat plate. The temperature of the ambient medium is  $T_\infty$  and the wall temperature is  $T_w$ . The flow along the vertical flat plate contains a species, slightly soluble in the fluid, The concentration at the plate surface is  $C_w$  and the solubility far away from the plate is  $C_\infty$ . Several assumptions are used in the present paper viz, the fluid and the porous medium are in local thermodynamic equilibrium, The flow is laminar, steady-state and two-dimensional, The porous medium is isotropic and homogeneous, The properties of the fluid and porous medium are constant, The Boussinesq approximation is valid and the boundary layer approximation is applicable.

In-line with these assumptions, the governing equations describing the conservation of mass, momentum, energy and concentration can be written as follows:

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \quad (1)$$

$$U \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} = \nu \frac{\partial^2 u}{\partial y^2} - \frac{\sigma \beta_0^2}{\rho} u - \frac{\nu}{K'} u + \frac{g\beta}{\rho} (T - T_\infty) + \frac{g\beta}{\rho} (C - C_\infty) \quad (2)$$

$$\rho C_p \left( u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} \right) = K_1 \frac{\partial^2 T}{\partial y^2} - \frac{\partial q_r}{\partial y} + \frac{D_m K_T}{C_S} \frac{\partial^2 C}{\partial y^2} \quad (3)$$

$$\rho \left( u \frac{\partial C}{\partial x} + v \frac{\partial C}{\partial y} \right) = D_m \frac{\partial^2 C}{\partial y^2} + \frac{D_m K_T}{D_T} \frac{\partial^2 C}{\partial y^2} + A(C - C_\infty) \quad (4)$$

Where all quantities are defined in the list of symbols.  $K_1$  is thermal diffusivity,  $\rho$  is density,  $T$  is temperature of fluid,  $u, v$  are velocity components,  $x, y$  are Cartesian coordinates,  $\Theta$  is dimensionless temperature,  $C$  is concentration function of specie,  $D_m$  is mass diffusivity. The boundary conditions of the problem are

$$\begin{aligned} y=0, \quad v &= -V_0, \quad T=T_w, \quad C=C_w \\ y \rightarrow \infty, \quad u &\rightarrow 0, \quad T \rightarrow T_\infty, \quad C \rightarrow C_\infty \end{aligned} \quad (5)$$

Where  $T_w, C_w, T_\infty$  and  $C_\infty$  have constant values,  $V_0$  is velocity due to suction at wall.

Equations (1) to (4), are now nondimensionalized using the following quantities,

$$\psi = \sqrt{U_0} x^{3/4} f(\eta), \quad \eta = \sqrt{\frac{U_0}{\nu}} x^{-1/4} y, \quad \theta(\eta) = \frac{T - T_\infty}{T_w - T_\infty}, \quad \phi$$

$$(\eta) = \frac{C - C_\infty}{C_w - C_\infty},$$

$$\alpha = \frac{K_1}{\rho C_p}, \quad q_r = -\left(\frac{4\sigma}{38}\right) \frac{\partial T^4}{\partial y}$$

Where  $\sigma$  is the Stefan Boltzmann constant and  $\delta$  is the Rosseland mean absorption coefficient. Temperature differences in the flow are assumed to be sufficiently small such that  $T^4$  may be expressed as a linear function of temperature. Expanding  $T^4$  about  $T_\infty$  in Taylor's series and neglecting higher order yield

$$T^4 \cong 4T_\infty^3 T - 3T_\infty^4$$

And also stream function  $\psi$  satisfied continuity equation (1)

The Eq. (2) to Eq.(4) become as below:

$$f''' = \frac{1}{2} f'^2 - \frac{3}{4} f f'' + (M + K) f' - Gr (\theta + N\phi) \quad (6)$$

$$\theta'' = -\frac{1}{(1+R_n)} \left[ \frac{3}{4} Pr f \theta' + Du \phi'' \right] \quad (7)$$

$$\phi'' = -\frac{3}{4} Sc f \phi' - Du \theta'' \quad (8)$$

The boundary conditions (5) become:

$$\begin{aligned} f(0) &= f_w, \quad f'(0) = 1, \quad \theta(0) = 1, \quad \phi(\eta) = 1, \\ f'(\eta) &= 0, \quad \theta(\eta) = 0 \rightarrow, \quad \phi(\eta) = 0 \text{ as } \eta \rightarrow \infty \end{aligned} \quad (9)$$

Where

$$M = \frac{\beta_0^2 \sigma}{U_0 \rho}, \quad Gr = \frac{g\beta(T_w - T_\infty)}{\rho U_0^2}, \quad N = \frac{\beta C(C_w - C_\infty)}{\beta(T_w - T_\infty)},$$

$$K = \frac{\nu}{U_0 K'}, \quad Pr = \frac{\nu}{\alpha}, \quad Sc = \frac{\mu}{D_m}$$

$$Du = \frac{D_m(C_w - C_\infty)}{C_S(T_w - T_\infty)}, \quad Sr = \frac{K_T(T_w - T_\infty)}{T_m(C_w - C_\infty)}$$

We notice that  $N$  is positive for thermally assisting flows, negative for thermally opposing flows and zero for thermal-driven flows.  $Gr$  is Grashof number,  $Sr$  is Soret number,  $Du$  is Duffer number,  $K$  is the porosity parameter,  $P$  is Prandtl number,  $M$  is magnetic parameter,  $R_n$  is radiation parameter,  $f_w$  is suction parameter and  $Sc$  is Schmidt number.

## 3. RESULTS AND DISCUSSION

In the above section of this study the finally governing equation of the problem namely Eq. (6) to Eq. (8) with associated condition of boundary Eqs.(9) required some viable solution these completed equation involve nonlinear terms and Larger order derivatives any close form solutions of these equations seems difficult to find. Therefore, a numerical treatment by namely NDSolve command of computational software Wolfram Mathematica (11.1.0) has been harnessed. A vigorous parametric analysis has been carried out for computational results of flow speed heat and mass distributions

The effect of mass buoyancy parameter  $N$  ( $N > 0$ ) and Grashof number  $Gr$  ( $Gr > 0$ ) are served on zero dimensional flow velocity  $f'(\eta)$  respectively in Fig. 1 and Fig. 2. Both these parameters show increasing influence on  $f'(\eta)$ . But the porosity parameter ' $K$ ' and magnetic parameter ' $M$ ' indicate decreasing effects on  $f'(\eta)$  as depicted respectively in Fig. 3 and fig. 4. Fig. 5 is shown that the increasing in Dufore number  $Du$  implies increasing in flow speed  $f'(\eta)$  but the flow speed  $f'(\eta)$  decreases with rise in the values of  $Pr$  as seen in Fig. 6. The increasing in radiation parameter  $R_n$  shows slight increment in  $Sc$  indicates slight reduction in  $f'(\eta)$  as parameter respectively in Fig.7 and Fig. 8. Fig. 9 demonstrates the decreasing impacts of suction variable  $F_w$  ( $F_w > 0$ ) on  $f'(\eta)$ . The specie concentration  $\phi(\eta)$  decreases with increment in the values of mass buoyancy parameter  $N$  and Grashof number  $Gr$  as demonstrated respectively in fig. 10 and Fig. 11. Fig. 12 and Fig. 13 respectively show that rise in curve of  $\phi(\eta)$  with increment in the value of parameter  $K$  and  $M$ . The dufore number  $Du$  implies slight increases in  $\phi(\eta)$  but Schmidt number  $Sc$  causes significant decreases in  $\phi(\eta)$  as shown respectively in Fig. 14 and Fig.15. The increment in suction at boundary ( $F_w > 0$ ) causes decreases in the values of  $\phi(\eta)$  as shown in Fig. 16. Both mass buoyancy parameters  $N$  and Grashof number  $Gr$  both show a decreasing effect on  $\theta(\eta)$  as demonstrated respectively in Fig. 17 and Fig. 18. The temperature  $\theta(\eta)$  increases with increment in Dufore number as shown in Fig. 19. Fig 20 indicates that the increases in Prandtl number causes reduction in  $\theta(\eta)$ . The temperature function  $\theta(\eta)$  reduces with increment in suction  $F_w$  as depicted in Fig.21

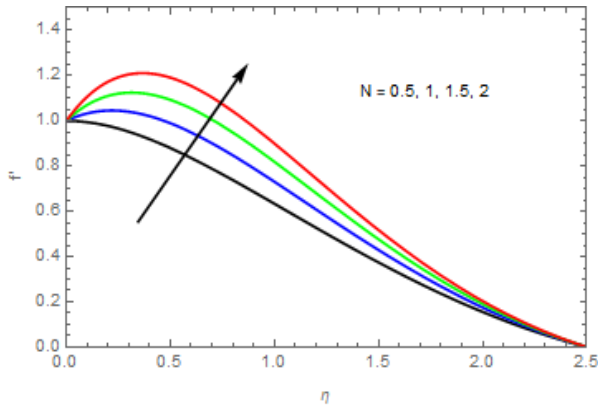


Fig 1: Graph of  $f'(\eta)$  under the effect of  $N$

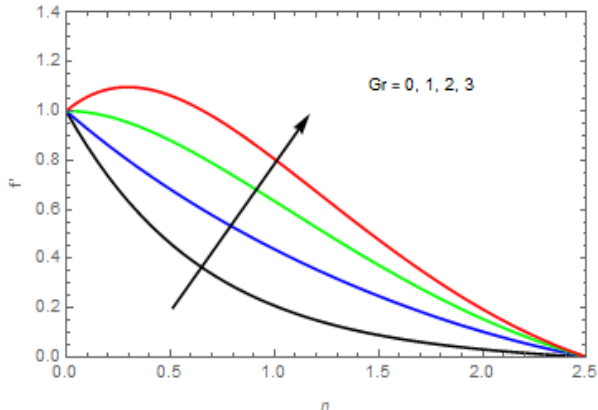


Fig 2: Graph of  $f'(\eta)$  under the effect of  $Gr$

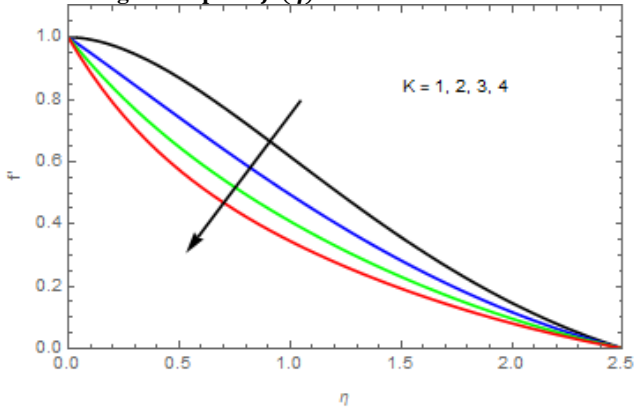


Fig 3: Graph of  $f'(\eta)$  under the effect of  $K$

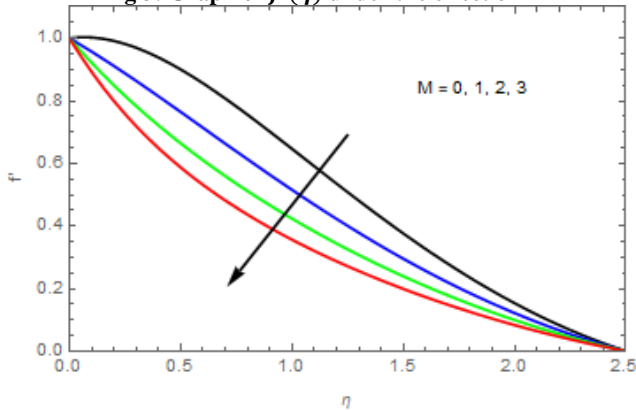


Fig 4: Graph of  $f'(\eta)$  under the effect of  $M$

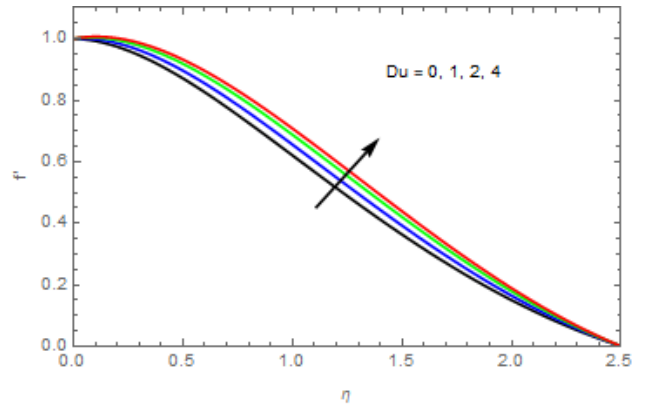


Fig 5: Graph of  $f'(\eta)$  under the effect of  $Du$

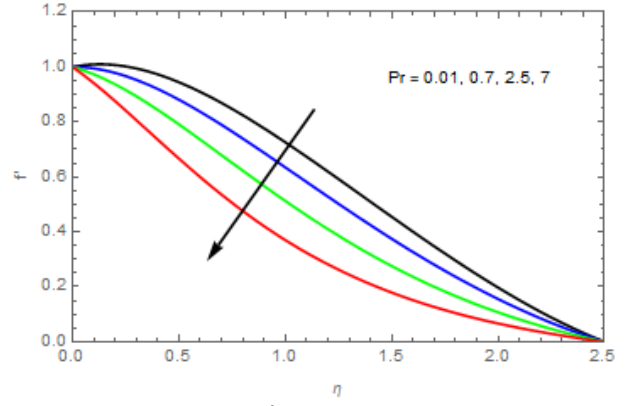


Fig 6: Graph of  $f'(\eta)$  under the effect of  $Pr$

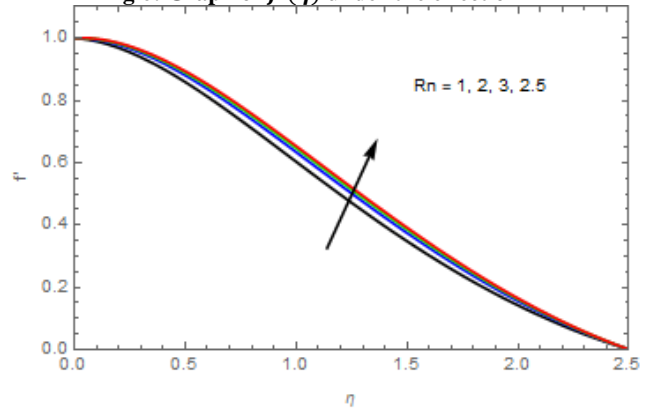


Fig 7: Graph of  $f'(\eta)$  under the effect of  $Rn$

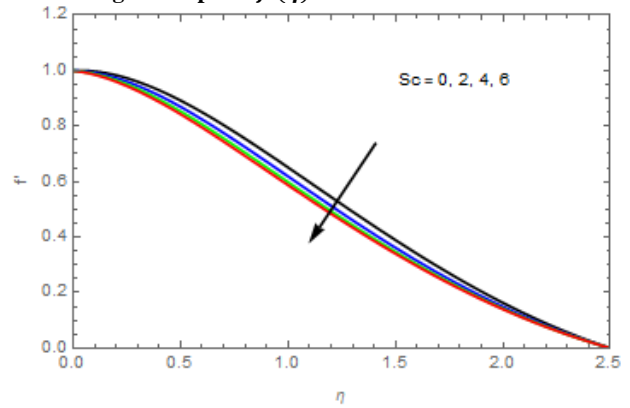


Fig 8: Graph of  $f'(\eta)$  under the effect of  $Sc$

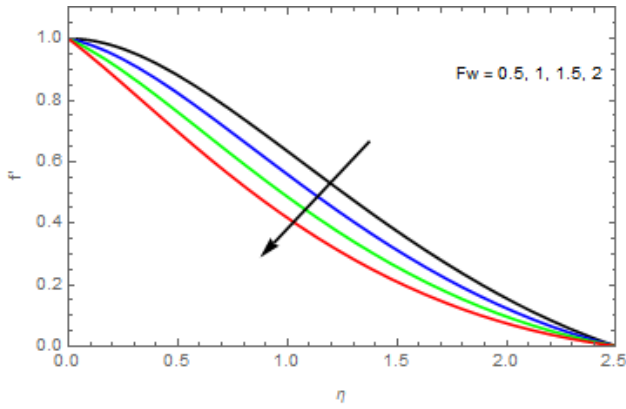


Fig 9: Graph of  $f'(\eta)$  under the effect of  $Fw$

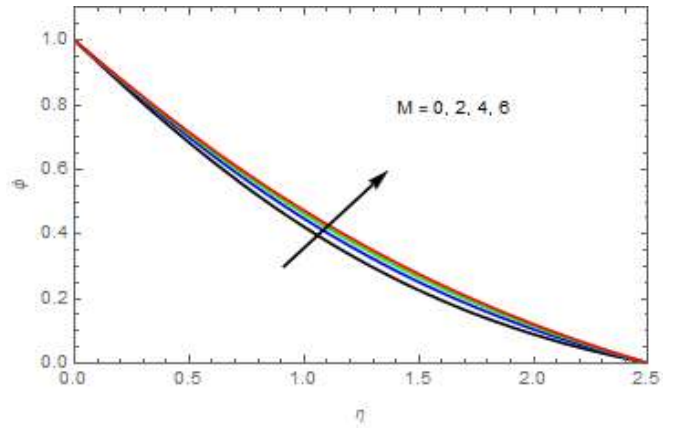


Fig 13: Graph of  $\phi(\eta)$  under the effect of  $M$

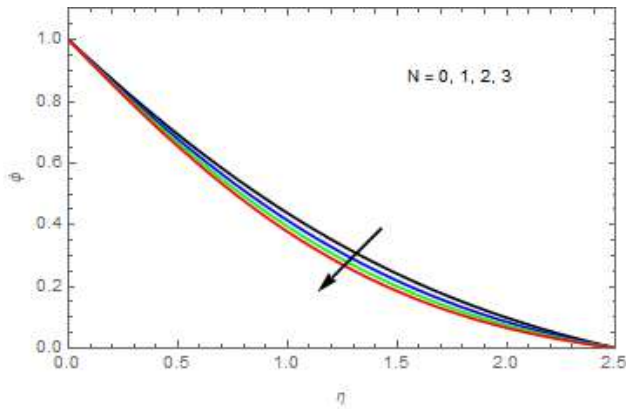


Fig 10: Graph of  $\phi(\eta)$  under the effect of  $N$

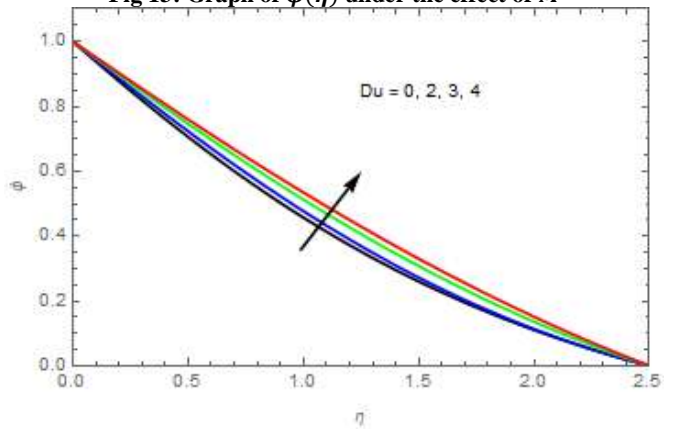


Fig 14: Graph of  $\phi(\eta)$  under the effect of  $Du$

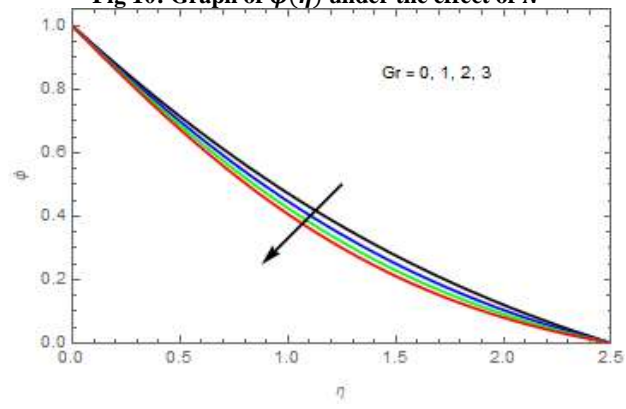


Fig 11: Graph of  $\phi(\eta)$  under the effect of  $Gr$

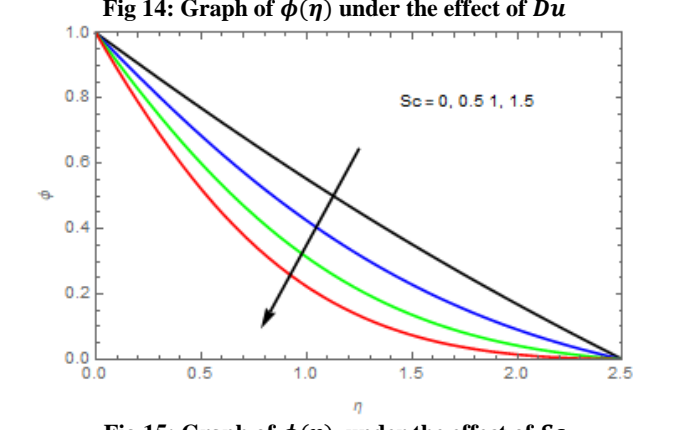


Fig 15: Graph of  $\phi(\eta)$  under the effect of  $Sc$

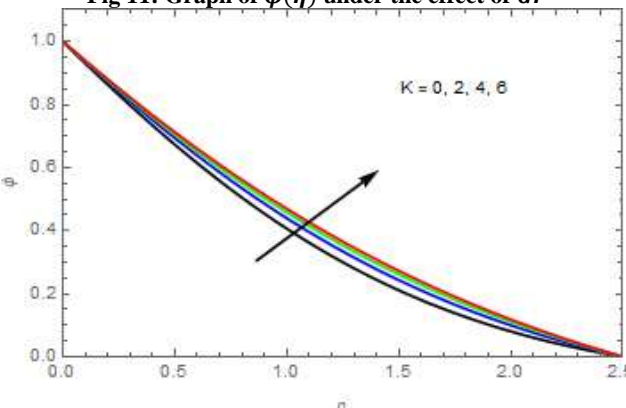


Fig 12: Graph of  $\phi(\eta)$  under the effect of  $K$

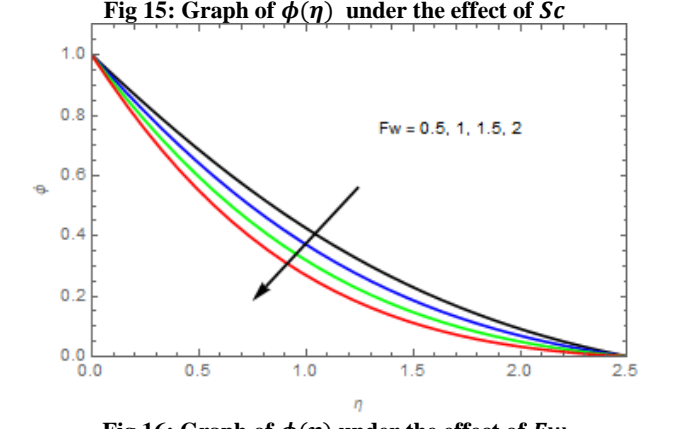


Fig 16: Graph of  $\phi(\eta)$  under the effect of  $Fw$

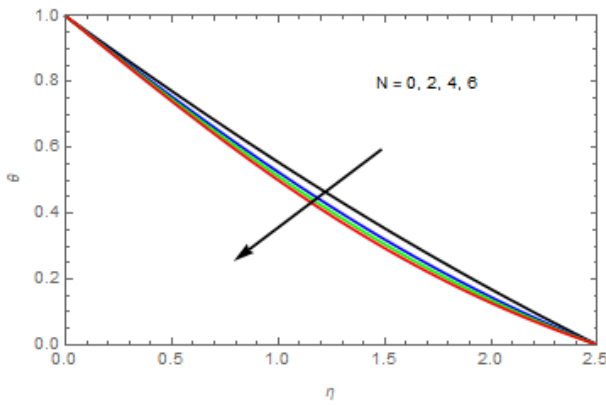


Fig 17: Graph of  $\theta(\eta)$  under the effect of  $N$

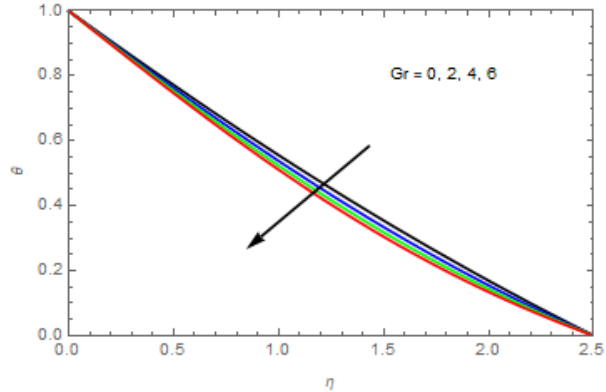


Fig 18: Graph of  $\theta(\eta)$  under the effect of  $Gr$

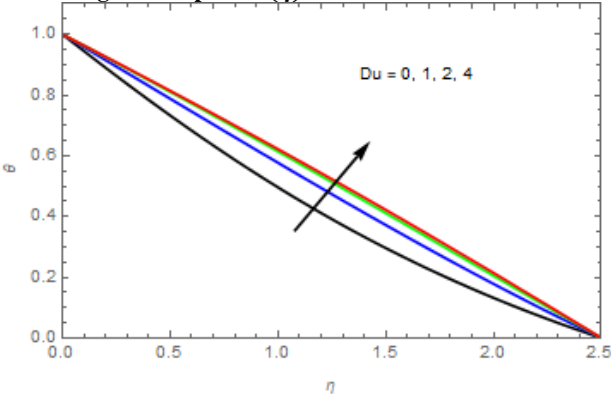


Fig 19: Graph of  $\theta(\eta)$  under the effect of  $Du$

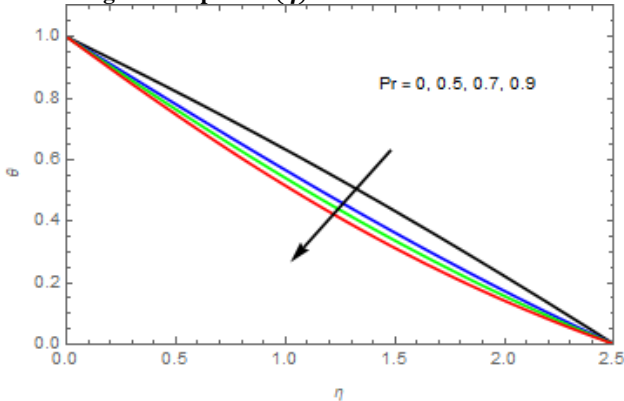


Fig 20: Graph of  $\theta(\eta)$  under the effect of  $Pr$

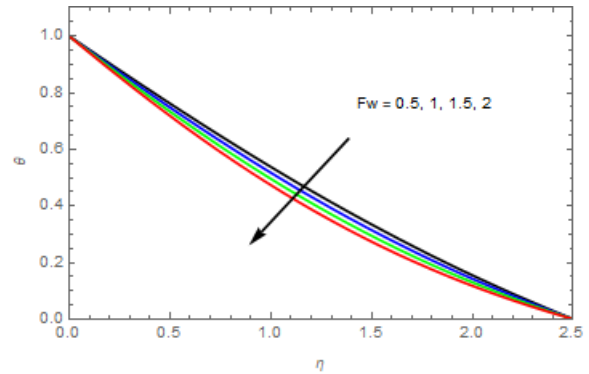


Fig 21: Graph of  $\theta(\eta)$  under the effect of  $Fw$

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