COMPUTATIONAL INVESTIGATION FOR STAGNATION POINT FLOW TOWARDS A STRETCHING VERTICAL SHEET WITH SLIP EFFECTS

[1]Asmat Ullah Yahya, [2]Sajjad Hussain, [3]Danial Habib, [4]Farooq Ahmad

[1]National College of Business Administration & Economics Lahore (Sub Campus Multan),Pakistan.
[2]Department of Mathematics, Govt Postgraduate College, Layyah, Pakistan.
[3]Department of Mathematics Virtual University Lahore, Pakistan,
[4]Department of Mathematics, Govt Islamia Civil Lines College, Lahore, Pakistan.
Asmatyahya786@gmail.com, sajjadgut@gmail.com, danisheikh4006@gmail.com, farooqgujar@gmail.com,

ABSTRACT: This article presents computational study of mixed convection stagnation point flow of viscous fluids. The flow is considered owing to a vertical shrinking/stretching sheet with slip boundary. Mathematical formulation involves similarity variables in order to transform the model problem consisting of partial differential form equations into ordinary differential type. The resulting highly non-liner differential equations are solved by using computational software Mathematica. Several computations are made for enough ranges of the pertinent parameters slip parameter δ, stretching/shrinking velocity parameter ε, mixed convection parameter λ and Prandtl number P_r to reveal the physical nature of the problem of velocity and temperature function. The obtained results are presented in graphical patterns of velocity and temperature functions with detailed discussion. The comparison of present results with the previous available results is excellent.

1. INTRODUCTION

In industrial and manufacturing processes the study of stretching sheet is very important. For example, the study of the biological fluids in the presence of magnetic fields flow towards a stretching sheet is important in bio engineering and medical use Sharada and Shankar, [2] dealt mixed convection in MHD stagnation point flow due to stretching surface. This research effort considers the flow and heat transfer of viscous fluids with various conditions such as magnatohydrodynamic, radiation, suction at wall, shrinking of the surface and slip boundary. The study of slip condition on convective boundary layer flow beside a vertical plate embedded in a porous medium has accepted considerable practical and theoretical awareness. Niranjan and Sivasankaran, [3] analyzed the numerical study on magneto convection stagnation-point flow in a porous medium with chemical reaction, radiation, and slip effects.

In theoretical and applications point of view the stagnation point flows in fluid dynamics over a stretching surface are important. Makinde, [4] studied the effects of internal heat generation and radiation on MHD mixed convection stagnation-point flow in a perpendicular plate embedded in a porous medium. . Ibrahim and Makinde, [5] studied numerically the stagnation point flow in the presence of magnetic field with heat transfer of an incompressible electrically conducting fluid near a stagnation point. Sajjad et al [6] investigated MHD stagnation point flow of micropolar fluids towards a stretching sheet. Sajjad et al. [6] investigated hydromagnetic micropolar fluid flow between two parallel plates, the lower plate is stretching. Farooq et. al. [7] obtained numerical solution for magnatohydrodynamic stagnation point flow towards a stretching sheet. Oahimirea and Olajuwonb, [8] disused the hydro-magnetic flow of a viscous fluid near a stagnation point on a linearly stretching sheet with variable thermal conductivity and heat source. Seini and Makinde, [10] investigated the effects of radiation and first order homogeneous chemical reaction on hydromagnetic boundary layer flow of a viscous, steady, and incompressible fluid over an exponential stretching sheet. Singh and Kumar, [12] considered the result of fluctuating heat and mass transfer on unsteady MHD free convection flow of radiating and reacting fluid past a vertical porous plate in slip flow regime.

Zaimi and Ishak [1] investigated heat transfer for stagnation point flow with partial slip effect. These authors considered the flow due to a stretching vertical sheet. In this artical, we reconsidered the above mentioned problem to solve it by using coding of Mathematica and present a details of the work. .

2. MATHEMATICAL ANALYSIS

The mixed convection flow problem has been formulated with the assumptions that the fluid is viscous and incompressible, stagnation point flow is steady and two dimensional. The flow is due to stretching vertical sheet that is placed in plan y = 0, when slip boundary conditions of momentum are considered valid. Cartesian coordinates are being used, the flow is confined to y > 0. The fluid velocity is **V** = V (u, v),the fluid temperature is T, The straining velocity of the stagnation point is U(x).The surface temperature $T_w(x) = T_\infty + bx$, where T_∞ is free stream temperature and b is positive constant.

The equation of fluid motion that govern the flow as given by Zaimi and Ishak [1] are as follows:

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0 \quad (1)$$

$$u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=U\frac{dU}{dx}+\upsilon\frac{\partial^2 u}{\partial y^2}+g\beta(T-T_\infty) \quad (2)$$

$$u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}=\alpha\frac{\partial^2 T}{\partial y^2} \quad (3)$$

The boundary conditions are

$$u = cx + L\left(\frac{\partial u}{\partial y}\right), \quad \boldsymbol{v=0}, \quad T = T_w \quad at\ y = 0 \quad (4)$$

$$u \to U(x) = ax, T \to \boldsymbol{T}_\infty, \quad as \quad y \to \infty$$

where g is the gravity acceleration, υ is the kinematic viscosity, c is straining rate parameter, β is the thermal

expansion coefficient, α is the thermal diffusivity, a is a constant, and L denotes the slip length/coefficient.
The velocity components u and v that are expressed in terms of the stream function $\psi(x,y)$ as given below:

$$u=\frac{\partial\psi}{\partial y},\quad v=-\frac{\partial\psi}{\partial x}$$

Using similarity transformations:

$$\psi(x,y)=\sqrt{\upsilon x U}\,f(\eta),\ \eta=y\sqrt{\frac{U}{\upsilon x}}$$

$$u=axf',\ v=-\sqrt{a\upsilon}\,f$$

The equation of continuity (1) is satisfied identically.

The equations (2) and (3) respectively become as:

$$f'''+ff''+1-f'^2+\lambda\theta=0 \qquad (5)$$

$$\theta''+\Pr(\theta' f-\theta f')=0 \qquad (6)$$

The boundary conditions (4) become:

$$f(0)=0,\quad f'(0)=\varepsilon+\delta f''(0),\quad \theta(0)=1$$
$$f'(\eta)\to 1,\theta(\eta)\to 0\quad as\quad \eta\to 0 \qquad (7)$$

where $\lambda=\frac{g\beta b}{a^2}$ is the buoyancy parameter, $\varepsilon=\frac{c}{a}$ is the stretching parameter, $\Pr=\frac{\nu}{\propto}$ is the Prandtl number and $\delta=L\left(\frac{a}{\nu}\right)^{\frac{1}{2}}$ is the velocity slip parameter.

3. RESULT AND DISCUSSION

The coupled equations (5) and (6) together boundary conditions (7) are solved numerically by NDsolve command of Mathematica to addressed the inherent difficulty of non linear terms involved in the mathematical model of the problem. Results have been obtained for several values of parameters, namely λ, ***Pr***, δ and ε**.** The impacts of these parameters have been observed on temperature and velocity distributions. The results of the parametric study presents excellent resemblance with physical conditions. This shows that our computational technique works very well. We reduced the higher order of derivatives in the above mentioned equations as follows:

$$f=f_1,\ f'=f_2,\ f''=f_2'=f_3,$$
$$\theta=f_4,\ \theta'=f_5,\ \theta''=f_5' \qquad (8)$$

Then the equations (5) and (6) respectively become as:

$$f_3'=f_2^2-f_1 f_2-1-\lambda f_4 \qquad (9)$$

$$f_5'=\Pr(f_2 f_4-f_1 f_5) \qquad (10)$$

The boundary conditions of (7) can be written as:

$$\eta=0,\ f_2=1,\ f_1=0,\ f_4=1$$
$$\eta\to\infty,\ f_2=\varepsilon+\delta f_3,\ f_4=0 \qquad (11)$$

The set of eight equations namely given by Eq.(8) to (10) together five boundary conditions as in relations (11) have been solved by using ND solve command and discussed as below:
Table 1 and 2 is presented to provide comparison for skin friction coefficient and heat transfer coefficient at the surface $f''(0)$ and $-\theta'(0)$ respectively. When the sheet is stretching and the flow is buoyancy aided ($\lambda=1$). While the table 3 and 4 present the comparison of the above mentioned quantities namely $f''(0)$ and $-\theta'(0)$ respectively, when the sheet is stretching ($\varepsilon>0, \varepsilon=1$) and the flow is bouncy opposing ($\lambda=-1$). It is seen from these results that the comparison is excellent and hence it ensures the validity of the computational techniques used in our work.
The graphical pattern of the results have been presented. The effect of slip parameter δ on horizontal velocity f' is demonstrated in Fig. (1) The flow is accelerated with rising values of δ. Similarly the magnitude of $f'(\eta)$ is augmented with increase in values of ε (ε > 0 or ε< 0) as shown in Fig. (2). Fig. (3) maps buoyancy opposing effect on $f'(\eta)$. It is seen that velocity $f'(\eta)$ decreases. The buoyancy parameter λ (λ < 0) decreases the curve of temperature function $\theta(\eta)$ as illustrated in Fig. (4). Here in increase in the Prandtl number causes reduction in the temperature distribution. The curves of $\theta(\eta)$ comes down significantly increase in Pr as depicted in Fig. (5).

Table 1:- Comparison of the values of skin friction coefficient $f''(0)$ when $\varepsilon=1,\ \lambda=1$ and various values of *Pr*

Pr	$f''(0)$		
	Results	Results	Present Results
0.72	0.3645	0.36449	0.36446
6.8	0.1804	0.18041	0.18022
10		0.15563	0.155511
20	0.1175	0.11750	0.117114
30		0.09889	0.0987143
40	0.0873	0.08724	0.0871086
50		0.07903	0.079035
60	0.0729	0.07284	0.072846
70		0.06794	0.0676336
80	0.0640	0.06394	0.0636706
90		0.06059	0.0603487
100	0.0578	0.05772	0.0575103

Table 2:- Comparison of the value of heat transfer coefficient $-\theta'(0)$ when $\epsilon = 1, \lambda = 1$ and various values of Pr

Pr	$-\theta'(0)$		
	Results	Results	Present Results
0.72	1.031	1.09310	1.09315
6.8	3.2902	3.28957	3.28553
10		3.98240	3.98189
20	5.6230	5.62013	5.61984
30		6.87771	6.87754
40	7.9463	7.93830	7.93818
50		8.87292	8.87282
60	9.7327	9.71801	9.71793
70		10.49524	10.4952
80	11.2413	11.21874	11.2187
90		11.89831	11.8983
100	12.5726	12.54109	12.5410

Table 3:- Comparison of the values of skin friction coefficient $f''(0)$ when $\varepsilon = 1, \lambda = -1$ and various values of Pr

Pr	$f''(0)$		
	Results Ishak et al [40]	Results Zaimi and Ishak [1]	Present Results
0.72	-0.3852	-0.38518	-0.38513
6.8	-0.1832	-0.18323	-0.183232
10		-0.15747	-0.155019
20	-0.1183	-0.11831	-0.117158
30		-0.09938	-0.0986298
40	-0.0876	-0.08758	-0.0870219
50		-0.07929	-0.0788456
60	-0.0731	-0.07304	-0.0726723
70		-0.06810	-0.0677913
80	-0.0642	-0.064047	-0.0638026
90		-0.06070	-0.0604615
100	-0.0579	-0.05782	-0.0576082

Table 4:- Comparison of the value of heat transfer coefficient $-\theta'(0)$ when $\epsilon = 1, \lambda = -1$ and various values of Pr

Pr	$-\theta'(0)$		
	Results	Results	Present Results
0.72	1.0293	1.02925	1.02934
6.8	3.2466	3.24608	3.24609
10		3.94370	3.94434
20	5.5923	5.58959	5.58988
30		6.85149	6.85168
40	7.9227	7.91489	7.91503
50		8.85153	8.85164
60	9.7126	9.69818	9.69827
70		10.47665	10.4767
80	11.2235	11.20117	11.2012
90		11.88161	11.8817
100	12.5564	12.52515	12.5252

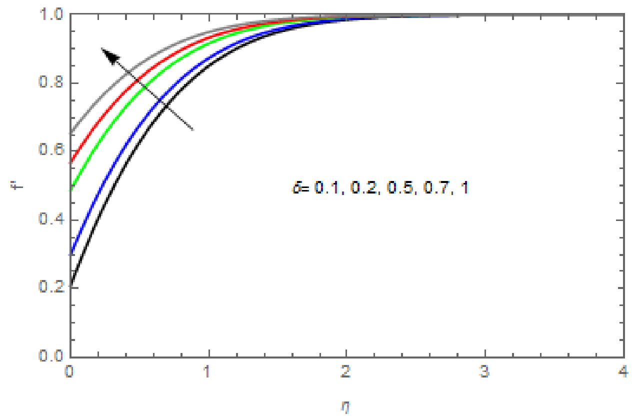

Fig. 1: Graph of f' under the effect of δ

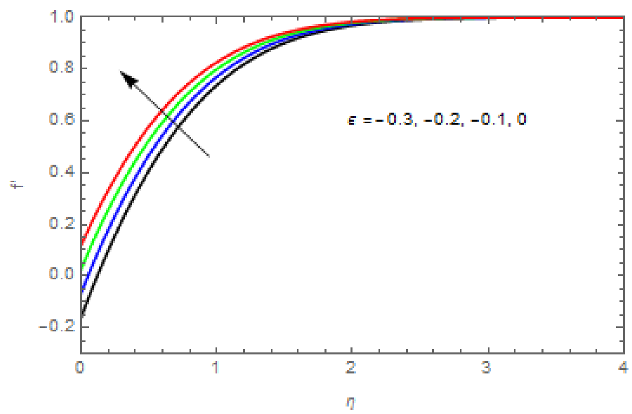

Fig. 2: Graph of f' under the effect of ϵ

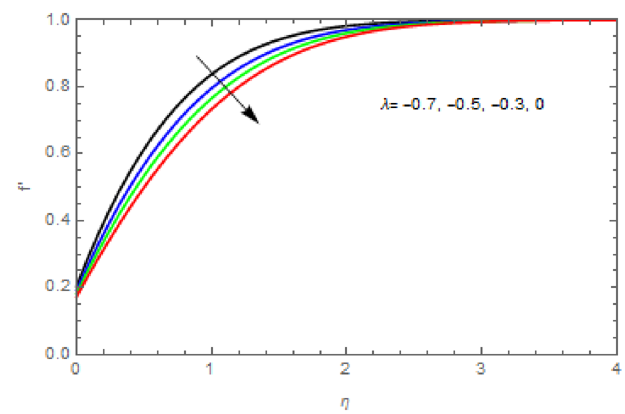

Fig. 3 Graph of f' under the effect of λ

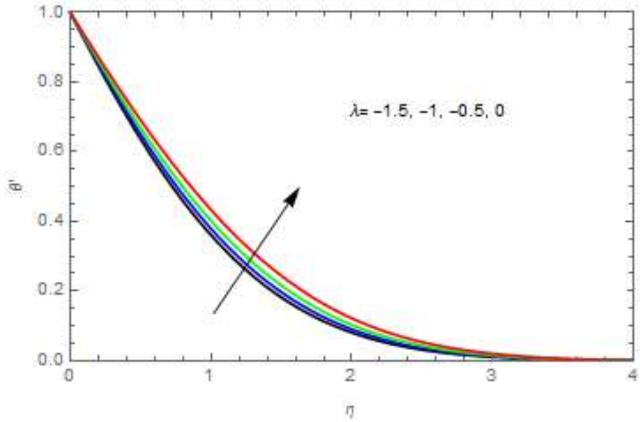

Fig. 4: Graph of $\theta(\eta)$ under the effect of λ

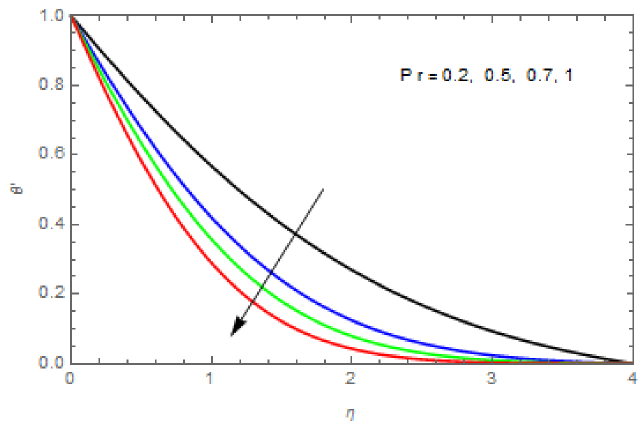

Fig. 5: Graph of $\theta(\eta)$ under the effect of Pr

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